

Preliminary and Incomplete

Banks Behaving Badly*

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Abstract

We model an economy in which banks can hold assets on their balance sheets, or move them off into the shadow banking sector. Banks have projects of varying productivity and face financing frictions. They choose which projects to invest in and how to finance the assets. We identify the nature of the assets a bank holds on and off the balance sheet. All banks hold worse assets off balance sheets, while weaker banks hold proportionally more of their assets off balance sheet. Changing the cost of financing changes the mix of on-and-off balance sheet assets. In particular, regulating hedge funds may increase the size of the shadow banking sector.

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1 Introduction

“In the United States, for example, shadow banking is now considerably larger than the traditional banking system; in Europe, it is roughly half the size; and in China, at 25-35 percent, it is the fifth largest shadow banking sector in the world.”

Christine Lagarde (Managing Director of the IMF) on October 2, 2014, in a speech at Georgetown University.¹

Shadow banking is a large and persistent part of the modern financial architecture. We illustrate this for the U.S. in Figure 1. For the purposes of our paper, we consider shadow banking to be any credit transformation that banks undertake for which the liabilities or funding are determined in the absence of any direct government intervention. Shadow banking activities are activities by regulated entities that do not appear on their balance sheets and are thus lightly regulated, if at all.

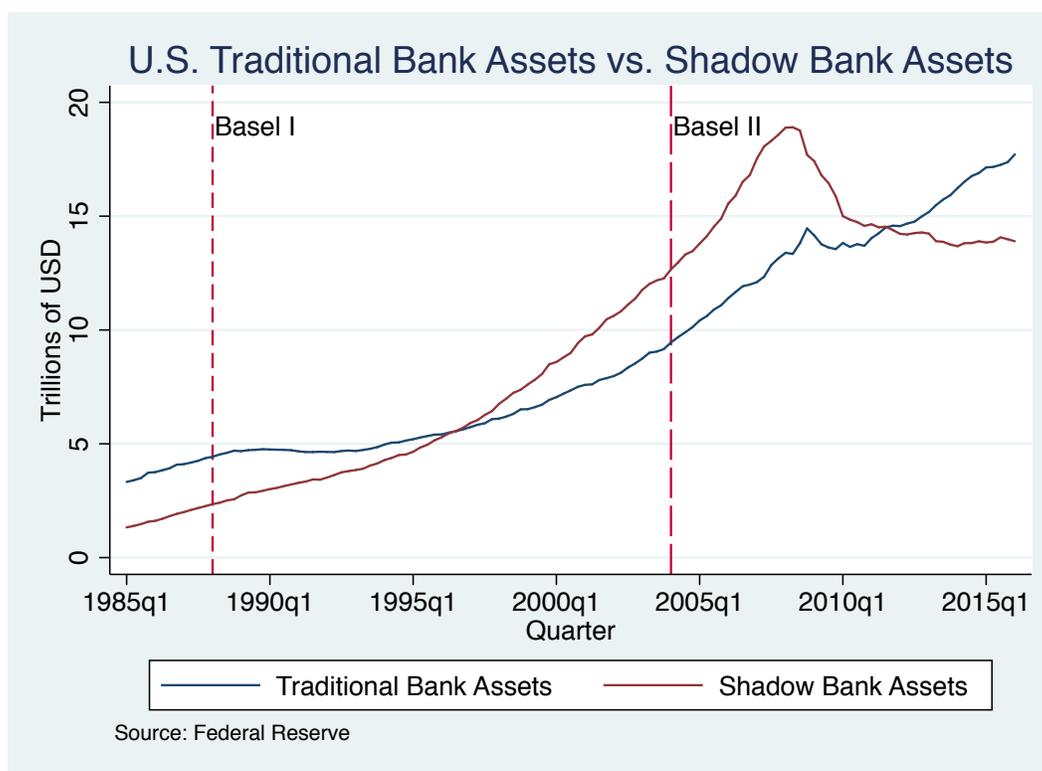


Figure 1: **Traditional bank assets vs. shadow bank assets in U.S.**

We are interested in a few basic questions about the shadow banking sector. First, what affects its size, and how is the size related to the regulatory stance and the private market costs of funds? Second, what is the risk of this sector and how does it compare to the risk of the on-balance-sheet

¹Transcript available at <https://www.imf.org/external/np/speeches/2014/100214.htm>

assets exhibited by the bank? Third, given the feasibility of shadow banking, what is the optimal regulatory policy toward on-balance-sheet activities? Finally, although the regulator can directly control measurable aspects of a banks' balance sheet, are there other aspects of market design (i.e., the source of financing for off balance sheet loans) that affect the relative size and risk of the shadow banking sector?

We present a parsimonious model in which banks, facing financing frictions decides which of their available projects they wish to fund and keep on balance sheet, and which assets they wish to move off balance sheet. Retaining assets on the balance sheet is costly because regulators require that for each dollar invested in risky projects, a percentage is placed in a risk free asset. This effectively increases the cost of funding for on balance sheet assets. By contrast, off balance sheet assets are not regulated but the source of funding is "smart money." As is usual in these OTC markets, we consider a simple bargaining game in which banks and financiers split the surplus from any project.

In equilibrium, banks choose to keep their high quality assets on balance sheet and move their lower quality assets off balance sheet. We note that the prevalence of AAA MBS does not imply that the quality of the pool was of particularly high quality, merely that a high quality tranche was sliced from a specific pool. In addition to banks choosing assets of varying quality, we also consider banks that are themselves of varying quality. Specifically, we consider good and bad banks: the latter are subject to idiosyncratic shocks that wipe out all their assets. We that, when there is a sufficient supply of high NPV projects, bad banks have relatively more of their assets off the balance sheet than good banks.

Because we relate the size of on an off balance sheet assets to the financing frictions that banks face, we can consider how regulatory changes affect the size of the shadow banking sector. The regulator does not know the state of each bank, but does know the macro state. We say the economy is in a high macro state if over half the banks are good banks, and in a low macro state otherwise. First, consider the effect of direct balance sheet regulation. The regulator in our model can require that banks hold a minimum proportion of their on-balance sheet assets in a safe liquid asset.² Banks that are required to hold more safe or liquid assets, *ceteris paribus* face higher costs of on balance sheet capital and therefore, as one might expect, move more assets off the balance sheet. More surprisingly, attempts to regulate the hedge fund sector may actually increase the shadow banking sector. The reason for this is simple: regulating hedge funds, *ceteris paribus* decreases the surplus that they get from any project. Thus, banks are able to extract more surplus from them and thus find moving assets off balance sheet marginally more attractive. Thus, regulating hedge funds can increase the size of the shadow banking sector.

²A liquidity requirement is a key component of the Basel III accord, which is due to be implemented in the next few years

A natural question in this framework is what optimal regulation might be. To answer this question, we consider a regulator who observes the macro state and then optimally sets her liquidity ratio. The regulator’s preferences are different from banks in the following ways. First, the regulator is worried about possible systemic risk in the high macro state, and so would like bad banks in the high state to under-invest relative to the investment that would obtain in a frictionless world. Second, the regulator has a mandate to boost employment, so would like to induce good banks in the low macro state to over-invest relative to the same benchmark. Finally, the regulator would like to minimize the size of the shadow banking sector, to minimize external threats to the banking system.

Increasing the liquidity requirement reduces on-balance-sheet investment, but may lead to more projects being off the balance sheet. Decreasing the liquidity requirement has the opposite effects. The liquidity requirement affects the size of the balance sheet, but, when shadow banking exists, it has no effect on the marginal project taken off the balance sheet. Thus, in the bad state, the optimal liquidity requirement is zero, implying in our model that shadow banking ceases to exist. In the good state, the optimal liquidity requirement is positive. Again, a suitably chosen regulation can prevent the existence of the shadow banking sector. However, the threshold regulation that achieves this purpose depends on the bank’s financing costs for on and off balance sheet activities. While our model is static, these costs in practice are determined by time-varying market prices, including the cost of shadow financing.

Our model therefore suggests that the prices of financial assets should be an important component of regulation. In practice, among other reasons, frictions in setting prices imply that a regulator will prefer to have regulation be constant over a large period of time rather than constantly vary. In such a setting, a constant regulation can depart quite widely from the optimal regulation in our model, leading to the emergence of the shadow banking sector, to take advantage of a form of regulatory arbitrage.

While the term “shadow banking” is somewhat imprecise, we note that it typically refers to banking activities that are unregulated.³ Specifically, financial institutions conduct maturity, credit, and liquidity transformation without government guarantees and free from regulation. As Figure 1 shows, the assets of the shadow banking sector started to grow at a faster speed than that of the traditional banking sector after 1988 when the central bank of the United States, together with other G-10 countries, adopted the Basel I Capital Accord. The introduction of more restrictive Basel II accord in 2004 witnessed a further growth in the shadow banking sector until the financial crisis.

The size of the shadow banking sector is a cause for concern if assets and liabilities are not truly

³Some jurisdictions (Spain, for instance) admit the regulation of “off balance sheet” assets.

bankruptcy remote. It would appear that frequently they are not, for various reasons. For example, instead of holding debt and loans on the balance sheet, Citigroup put them into Collateralized Debt Obligations (CDO) and Collateralized Loan Obligations (CLO). Although Citigroup sold the senior claims of these CDOs and CLOs to investors, it also provided liquidity puts, which brings in risk premium income but forces the bank to buy back the risky loans at face value when they deteriorate. Such securitization activities without risk transfer, as discussed in Acharya, Schnabl, and Suarez (2013). This increased the leverage of the bank. During the financial crisis, the liquidity puts required Citigroup to buy back \$25 billions of CDOs later valued at thirty-three cents per dollar.

Non-depository financial institutions such as investment banks and hedge funds also involved into the shadow banking sector. The 1999 Gramm-Leach-Bliley Act (GLBA) removed the barriers in the market among commercial banks, investment banks, and insurance firms. As a result, the non-depository financial institutions, insurance companies in particular, had more flexibility in engaging different types of businesses. For example, GLBA allowed insurance companies such as AIG to enter the derivative dealer business and sell CDS protection through its subsidiary, such as AIG Financial Products. By selling credit default swaps against the default of certain financial assets, AIG carried the credit risk of those assets without actually holding them on the balance sheet.

There are various ways in which off-balance-sheet credit transformation occurs. Poszar, et al. (2012) provide a detailed description of some of the off-balance sheets channels used by banks, which cover a wide range of securitization and secured funding devices. Stein (2010) provides the broader context for securitization and its role in the credit creation markets. We model shadow banking as a process which raises funds and keeps assets off the balance sheet, which could include secured funding and securitization through SPVs. We note that there is no centralized trading of off-balance-sheet products, and there are creation and search costs associated with assembling and then marketing these securities.

Adrian and Ashcraft (2012) present a comprehensive survey of the literature on shadow banking. Two recent papers provide insight as to when it might be efficient to reduce the regulatory burden on banks. Plantin (2014) presents a model in which bank insolvency is socially costly because agents use bank claims to settle transactions. Banks do not internalize the social cost of forgone transactions when they choose their leverage, and are thus subject to regulation. Making regulation more stringent can increase banks' use of the opaque shadow banking sector, which can improve overall welfare. Ordonez (2013) develops a framework in which regulation may inefficiently inhibit bank risk-taking, and regulated banks may be inferior to unregulated ones if the incentive of a bank to maintain a good reputation is sufficiently strong.

Gennaioli, Schleifer and Vishny (2012) present a model with idiosyncratic and systematic risk.

Banks securitize projects and the resulting portfolios are not exposed to idiosyncratic risk, but may have increased systematic risk. In a world in which investors may ignore tail risk, there may be over-investment and overpricing during the boom, followed by collapses during a bust. Of course, if all risks are priced correctly, the shadow banking sector improves welfare.

Our work is also related to Harris, Opp, and Opp (2015) who point out that the cross-sectional distribution of firms requiring credit is important to understand the effects of regulation. For firms that are dependent on bank financing, increasing capital requirements may reduce capital for firms that cannot access public debt markets. The net effect of more regulation trades off the benefit to firms who would otherwise not be funded against the reduction in overall bank risk-taking.

Various attempts have been made to estimate the size of the shadow banking sector. Using the concept of non-core liabilities developed in Shin and Shin (2010), Harutyunyan et al (2015) observe that noncore liabilities are procyclical. They also document that the size of the shadow banking sector is larger than that of the observed banking sector, and more volatile. The Financial Stability Board, in their 2015 global monitoring report, provide a somewhat smaller estimate of the size of the shadow banking sector, but note that it has a higher growth rate than traditional banks. They further note that a vast majority of shadow banking assets (80%) are concentrated in the advanced economies.

2 Model

An economy comprises a large but finite number of risk-neutral banks, N . At time 0, the bank invests in assets that pay off at time 1. Each bank has access to two kinds of assets: a safe asset that offers a zero return, and a pool of risky projects of heterogeneous quality.

Each risky project requires an investment of \$1 at time 0. At time $t = 1$, a risky project it generates a stochastic cash flow that may be either $C > 0$ or 0. The probability of receiving C depends on two sources of risk: one that is specific to the project, and one that depends on a local economic state faced by the bank. We denote the project-specific component by x , and refer to it as the productivity of the project. The bank has access to a continuum of projects that vary in their productivity. Specifically, $x \in [0, 1]$, with $f(x)$ denoting the density of projects with productivity x . Note that projects are therefore indexed by x . We assume that $f(\cdot)$ has support $[0, 1]$ with no mass points. Let F denote the associated distribution. Productivity-based success or failure is independent across risky projects.

Each bank faces a local economic state that affects the cash flows on all its projects. The local state is either good (g) or bad (b). In the bad state, all risky projects of the bank may fail. That is, with probability $\theta_b < 1$, each risky project pays off according to its own productivity. With probability $1 - \theta_b$, the risky projects are all insolvent and yield zero. The component θ_b is therefore

a bank-level shock, which affects all its projects in the same way. In the good local state, a risky project pays off according to its own productivity. Define $\theta_g = 1$. Then, in local state s , the expected cash flow on risky project x is $\theta_s x C$. There is no discounting, so the expected net surplus of loan x in a frictionless world is $\theta_s x C - 1$.

The local economic state is independent across banks. In the population, a proportion π of the banks have a good local state, while a proportion $(1 - \pi)$ have a bad local state.

On the financing side, each bank has equity capital E . We are interested in the asset portfolio chosen by a bank and in the breakdown between on- and off-balance-sheet financing, rather than its capital structure. We therefore assume that equity capital is limited relative to the pool of risky projects that the bank wishes to invest in. Further, raising additional equity capital is prohibitively expensive. Therefore, in order to fund projects, a bank has the following three choices: (i) deploy existing equity (ii) raise money from lenders by issuing debt, in which case both the debt and the assets it funds are on the balance sheet and (iii) raise money from hedge funds, which involves keeping the assets and the corresponding liabilities off the balance sheet.

Suppose the bank raises a debt amount D on its balance sheet. The debt is costly, in that \$1 of debt raised at time 0 requires the bank to repay $1 + r$ in expectation at time 1. As noted before, the safe (or risk-free) asset yields zero, whereas we assume that $r > 0$. That is, due to an unmodeled friction, bank debt earns more than the riskfree rate in expectation. We assume that lenders recognize the type of the bank, so that the premium is the same for a good and a bad bank. One way to motivate this premium is by appealing to the possibility of an economy-wide macro-economic shock, possibly originating in the financial sector, that leads to the possibility of a large-scale failure of the banking system.

Banks may also raise money in the shadow banking sector against assets that are held off the balance sheet. Financing in the shadow banking sector is available in a market opaque to regulators. In this market, banks negotiate with hedge funds or other “smart money.” In other words, this is an over-the-counter market. We therefore view this as a search market in which agents, upon meeting, bargain over the surplus. That is, we view smart money as being in limited supply, and therefore able to capture some of the surplus generated in the transaction with the bank. In contrast, lenders in the debt market capture none of the surplus beyond their desired return r . Another way of putting it is that while all investors know the type of the bank, and therefore the quality of its projects, only hedge funds understand the productivity of the specific project they are financing.

As banks are not the only suppliers of projects in the off-balance sheet sector, we assume that that financiers in the shadow banking sector have access to a distribution over arbitrage opportunities. Let $H(\cdot)$ be the distribution function of risk-adjusted excess returns available to a

hedge fund if it does not provide capital to the banking sector. Suppose that there are a number of hedge funds, which can be viewed as sampling $H(\cdot)$ sequentially, without recall. In other words, on finding an arbitrage opportunity, they remove it from the pool available to other funds. At the same time, due to market conditions, new investment opportunities arise. Searching for arbitrage opportunities costs ζ per draw from $H(\cdot)$. The presence of future arbitrage opportunities implies a minimum threshold return a hedge fund will require before it accepts an arbitrage opportunity. Let $\underline{\omega}$ denote this threshold return. Then, $\underline{\omega}$ is the opportunity cost of funds to a fund that provides a bank with funding for off-balance sheet assets.

When a bank approaches a hedge fund for financing, the hedge fund recognizes both the type of the bank and the productivity of the projects being financed. On each project, the bank and hedge fund bargain over the surplus being generated by the transaction. We adapt the bargaining protocol presented in Acharya, Gromb and Yorulmazer (2012) in which the bank offers a matched hedge fund a fraction of the surplus from the transaction. If the hedge fund does not accept, then with probability β the bargaining terminates. With probability $1 - \beta$, the process continues to the next round, the same hedge fund then makes an offer to the bank. The exogenous termination parameter, β is one way to capture the relative bargaining power of the hedge fund versus the bank. If $\beta = 1$, then the bank has all the bargaining power and makes a take-it-or-leave-it offer. By contrast, if $\beta = 0$, the hedge fund has all the bargaining power.

We model the bargaining game between the hedge fund and the bank in this manner as the outcome lends itself to the most transparent comparative statics, although we emphasize that our results are robust to the specific form of the bargaining protocol.⁴ We show the equilibrium outcome of the game in Lemma 1 below.

Each bank i knows its local state. At $t = 0$, it chooses A_i , the investment in the safe asset, R_i , the amount invested in projects held on the regulated balance sheet, and U_i , the amount invested in projects held off the balance sheet (OBS), and therefore unregulated. The balance-sheet identity $A_i + R_i = D_i + E$ implies that the amount of debt raised is $D_i = A_i + R_i - E$. The quantity U_i determines the amount of OBS financing raised. Embedded in the choice of R_i and U_i is a decision on each invested project — the bank may choose to hold any project either on or off the balance sheet. The bank’s objective is to maximize the value to the equity-holders.

Finally, a regulator can attempt to control the size and composition of a bank’s balance sheet. In particular, the regulator can impose a liquidity requirement on a bank. A liquidity requirement is an important feature introduced by the recent Basel III accord for bank regulation. The regulator specifies that a proportion α of the total assets on the balance sheet be safe assets. That is, the regulator requires that, for each bank i , $A_i \geq \alpha(A_i + R_i)$, or that $A_i \geq \frac{\alpha}{1-\alpha}R_i$. The regulator has

⁴Binmore, Rubinstein and Wolinsky (1986) clarify the relationship between Nash Bargaining, Rubinstein Bargaining and protocols with exogenous termination.

no direct control over the assets off the balance sheet. However, as we show, the regulator's choice of the liquidity requirement α influences the size of the shadow banking sector.

An alternative way to control the size of the balance sheet would be to impose a capital requirement κ , so that $E \geq \kappa(D_i + E)$. In this model, if the capital requirement were binding on both good and bad banks, they would have the same size of balance sheet. Alternatively, if the requirement were binding on good banks but not on bad banks, the former would have a larger balance sheet. As our focus is on the assets the bank invests in, we directly work with a regulation that operates on the asset side of the balance sheet.

3 Banking versus Shadow Banking

We first consider a bank's choice of projects and where to hold them. Central to this choice, is the relative cost of holding assets on or off balance sheet. A bank faces financing frictions, both in the public debt market or in the OTC market. Each bank faces a tradeoff between financing projects with debt, at a cost of r , and being subject to this regulatory requirement, or getting funding from hedge funds and moving assets off balance sheet

The regulatory requirement means that the effective cost of financing projects on balance sheet is higher than r . Suppose that the regulator has imposed a liquidity requirement, α . When the bank issues a loan for \$1 to fund a project, it needs to raise an additional $\frac{\alpha}{1-\alpha}$ to put into safe assets. The total increase in debt is therefore $1 + \frac{\alpha}{1-\alpha} = \frac{1}{1-\alpha}$. The total amount repaid in expectation to debt holders is $\frac{1}{1-\alpha}(1+r) = \frac{1+r}{1-\alpha}$. Let r_E denote the effective required rate of return to fund on balance sheet activities, so $r_E = \frac{r}{1-\alpha}$.

The payoff to the bank from making a loan on the balance sheet is therefore

$$x\theta_i C + \frac{\alpha}{1-\alpha} - \frac{1+r}{1-\alpha} = (x\theta_i C - 1) - r_E. \quad (1)$$

This is depicted in Figure 2 below.

Next, consider the surplus accruing to a bank that holds assets off balance sheet. Recall, the bank negotiates over the fraction of the surplus it retains, in addition the hedge fund sector must get at least its opportunity cost of funds. Implicitly, both the hedge fund and bank are small relative to the shadow banking sector.

UR: Comment on surplus from project

Lemma 1 *In the subgame-perfect equilibrium of the bargaining game with the hedge fund:*

- (i) *If $\theta_i x C \geq 1 + \underline{\omega}$ the bank's offer is accepted at stage 1. Given project x and bank type i , the payoff to the bank is $\beta(\theta_i x C - (1 + \underline{\omega}))$, and the payoff to the hedge fund is $(1 - \beta)(\theta_i x C - (1 + \underline{\omega}))$.*

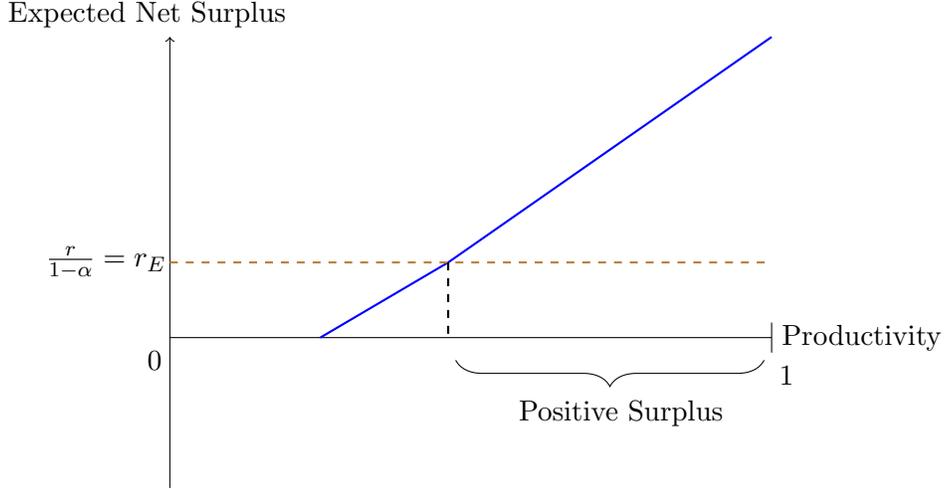


Figure 2: **Surplus to the bank from retaining an asset on Balance Sheet**

(ii) If $\theta_i x C < 1 + \underline{\omega}$, the game terminates without agreement being reached, and each party earns its reservation payoff. The bank obtains zero and the hedge fund obtains $\underline{\omega}$.

Notice that the bank receives β of the surplus from the project. Effectively, it has a cost of funds equal to $\beta \underline{\omega}$, which depends on β , it's on bargaining power, and on $\underline{\omega}$, the return on the outside option for the hedge funds. If β is high, the bank receives a larger portion of the surplus. Offsetting this is the fixed cost of moving assets off balance sheet that is also higher (for a fixed $\underline{\omega}$). The payoffs to the bank from funding a project off-balance sheet are depicted in Figure 3 below.

Note that, under Nash Bargaining, the bank's payoff would be $\frac{1}{2} (x \theta_i C - 1 - \underline{\omega})$ whenever project x would not be financed on the balance sheet. That is, in this case β is normalized to a half. To see this, let t denote the return at which the hedge fund will fund a given project x . The hedge fund's opportunity cost of funds is $\underline{\omega}$, so that its payoff if agreement is reached is $(t - \underline{\omega})$. The bank's outside option is zero if the project is not financed, so that its payoff is $(\theta_i x C - 1 - t)$. Multiplying the two and solving for t , we obtain $t = \frac{\theta_i x C - 1 + \underline{\omega}}{2}$, so that each party receives half of the surplus.

Examining Figures 2 and 3 makes the tradeoff clear between holding assets on and off balance sheet. Because the cost of debt financing in the public market is constant, the bank retains all the surplus from projects financed on balance sheet. By contrast, if a bank moves an asset off balance sheet, it has to (effectively) pay a fixed cost — the hedge fund's reservation payoff. In addition, the bank shares the surplus with the the hedge fund. We note in passing, that any bargaining protocol in which hedge funds are not held down to their reservation will have a “share the surplus” feature.

Notice also from Figures 2 and 3 that the expected payoff to the bank, both on and off the balance sheet is increasing in the productivity of the project, x . Based on this observation, we

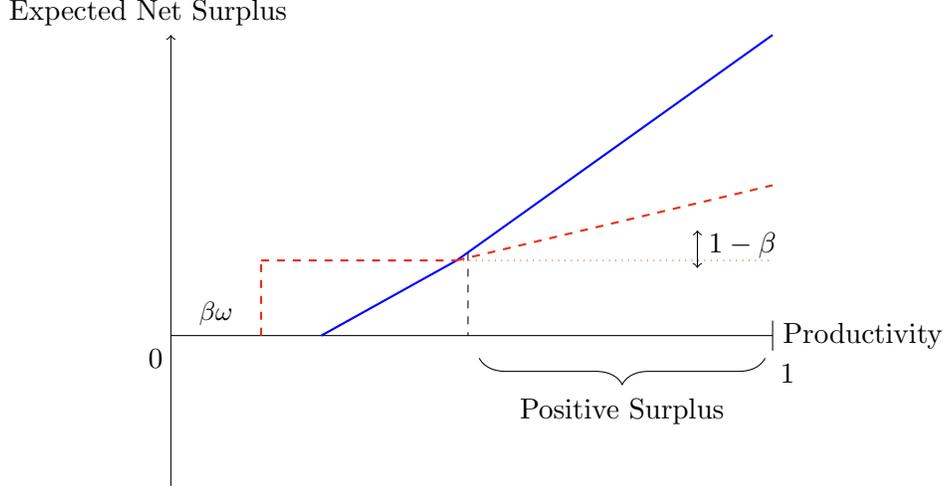


Figure 3: Surplus accruing to a bank funding a project off balance sheet

proceed by defining various thresholds:

$$\underline{x}_i^\omega = \frac{1 + \underline{\omega}}{\theta_i C} \quad (2)$$

$$\underline{x}_i^d = \frac{1}{\theta_i C} (1 + r_E) \quad (3)$$

$$\tilde{x}_i = \frac{1}{\theta_i C} \left(1 + \frac{r_E - \beta \omega}{1 - \beta} \right). \quad (4)$$

The thresholds help determine which projects are held on the balance sheet and which are retained off the balance sheet.

Lemma 2 *Suppose a bank faces a liquidity requirement α , cost of debt of r , and hedge fund sector with outside option $\underline{\omega}$. Then,*

- (i) *All projects with productivity $x > \underline{x}_i^\omega$ yield a strictly positive payoff if they are kept off balance sheet.*
- (ii) *All projects with productivity $x > \underline{x}_i^d$ yield a strictly positive payoff if they are kept on the bank's balance sheet.*
- (iii) *All projects with productivity $x > \tilde{x}_i$ are more profitable funded on the balance sheet, while those with productivity $x < \tilde{x}_i$ are more profitable off balance sheet.*

That is, \underline{x}_i^ω is the zero-payoff project facing bank i if it plans to keep the asset off balance sheet, while \underline{x}_i^d is the zero-payoff project facing the bank if it retains it on balance sheet. If $\underline{x}_i^d < \underline{x}_i^\omega$, then the cost of obtaining financing in the public market is lower than going to the opaque market.

Finally, \tilde{x}_i denotes the project for which the bank is indifferent between retaining on and off balance sheet.

We now show in Proposition 1 that higher quality assets are held on the balance sheet and, if holding an asset off balance sheet is attractive, the bank will move lower quality assets off balance sheet. In the context of our model, lower quality means a higher probability of failure (payoff of zero). The condition $\underline{\omega} \geq r_E$ comes from comparing the funding costs of the opaque market versus the effective cost of issuing regulated debt.

Proposition 1 *Consider a bank that faces a liquidity requirement α , and an opaque market in which financiers have an outside option $\underline{\omega}$. Then*

- (i) *If $\underline{\omega} \geq r_E$, bank i invests in projects in the interval $[\underline{x}_i^d, 1]$ and retains all its projects on the balance sheet.*
- (ii) *If $\underline{\omega} < r_E$, bank i invests in projects in the interval $[\underline{x}_i^\omega, \tilde{x}_i)$ and retains them off the balance sheet, and invests in projects in the interval $[\tilde{x}_i, 1]$ and places them on the balance sheet.*

4 The size and quality of on balance sheet and off balance sheet assets

Given Proposition 1, we can define the size of any bank's investment in on-balance-sheet risky assets as:

$$R_i = \begin{cases} 1 - F(\underline{x}_i) & \text{if } \underline{\omega} \geq r_E \\ 1 - F(\tilde{x}_i) & \text{otherwise.} \end{cases} \quad (5)$$

The off balance sheet (unregulated) assets are

$$U_i = \begin{cases} 0 & \text{if } \underline{\omega} \geq r_E \\ F(\tilde{x}_i) - F(\underline{x}_i^\omega) & \text{otherwise.} \end{cases} \quad (6)$$

It is worthwhile emphasizing the implications of Proposition 1 for the quality of assets off balance sheet for any type of bank. Intuitively, because of the nature of financing available for off balance sheet financing, the bank retains only a portion (β) of the surplus. By contrast, if it puts assets on the balance sheet, it obtains the full surplus. It is therefore optimal to put the higher payoff assets on the balance sheet, even though the effective interest rate (r_E) is relatively high. Figure 4 illustrates the choice of on or off balance sheet assets.

It is immediate, given the bank's investment strategy that

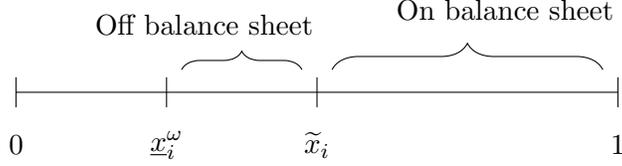


Figure 4: **Illustration of Proposition 1 when $\underline{\omega} < r_E$**

Corollary 1.1 *Suppose that $\underline{\omega} < r_E$ so that an off-balance sheet sector exists. Then,*

- (i) *The average return of assets held off balance sheet is lower than assets held on balance sheet.*
- (ii) *The average probability of failure is higher for off-balance sheet assets rather than on-balance sheet assets.*

In our framework, some banks are inherently more risky than others (bad banks). Such banks are subject to catastrophic failure. Do such banks hold relatively more or fewer assets off balance sheet than good banks? The aggregate size of banks' investment depends on the distribution of projects that are available. We define a notion, project sufficiency, satisfying which requires that the density function over possible projects does not decrease too rapidly in project quality x . We allow there to be “enough” good projects.

Definition 1 *A density function $f(\cdot)$ satisfies project sufficiency if $f(kx) > \frac{1}{k}f(x)$ for all x and all $k \in (1, \frac{1}{x}]$.*

The condition is equivalent to requiring that the function $h(x) = xf(x)$ is strictly increasing. Essentially, it requires that the density function should not decline too rapidly as project quality improves. Distributions that satisfy this condition include the generalized uniform distribution, $F(x) = x^a$ for $a > 0$. The standard uniform distribution, $a = 1$, is a special case of this class.

This condition allows us to compare the relative size of the off-balance-sheet investment of a good bank and a bad bank.

Proposition 2 *Suppose that (i) $\underline{\omega} \leq r_E$ so banks hold assets off balance sheet (ii) project sufficiency holds. Then, the relative size of off-balance-sheet investment is higher for a bad bank than a good bank. That is, $\frac{U_b}{R_b} > \frac{U_g}{R_g}$, and similarly $\frac{U_b}{R_b+U_b} > \frac{U_g}{R_g+U_g}$.*

4.1 Changes in Funding Costs and off and on balance sheet assets

In our parsimonious framework, banks are behaving optimally given financing frictions. That is, they take the funding costs as given and allocate projects accordingly. Empirically, we observe

changes in funding costs and so it is natural to consider comparative statics. We consider both on and off balance sheet financing costs in turn.

First, suppose that the regulator increases the liquidity requirement on the banks; that is, the regulator increases α . It is immediate that (holding everything else fixed) if a shadow banking sector exists, then the quantity of assets held off the balance sheet grows, whereas the total quantity of funded projects remains the same as before. More surprisingly we identify a condition such that the effect of a change in the liquidity requirements is greater for a bad bank than a good bank.

Proposition 3 *Suppose that $r_E \geq \underline{\omega}$. Then:*

- (i) *For each $i = g, b$, an increase in the regulatory requirement, α , results in an increase in off-balance-sheet assets U_i , leaving the total quantity of assets $R_i + U_i$ unchanged.*
- (ii) *Further, if the density function $f(\cdot)$ satisfies project sufficiency, then $\frac{\partial U_b}{\partial \alpha} > \frac{\partial U_g}{\partial \alpha}$. That is, a change in the liquidity requirement α has a greater impact on the off-balance sheet assets of the bad bank compared to the good bank.*

This result is intuitive. If the regulatory requirement, α is higher, then the effective cost of funding assets in the public debt market and keeping them on balance sheet is higher. (Recall, $r_E = \frac{r}{1-\alpha}$.) Therefore, holding the cost of opaque financing fixed, it is optimal to move some of the projects off balance sheet and use the opaque market funding. Further, under project sufficiency, the increase in off balance sheet assets is larger for a bad bank than a good bank. This is potentially a cause for concern: Given the chance of catastrophic loss for assets held by the bad bank, an increase in the regulatory requirement leads to a relative increase in shadow banking in the economy at large. We note, however, that if $r_E \geq \underline{\omega}$, the total assets invested in the banking sector are not affected, but rather the unobserved portion part of the assets are.

Proposition 4 *Suppose that $\underline{\omega} \leq r_E$. Consider a regulation that decreases $\underline{\omega}$, the outside option for hedge funds. Then:*

- (i) *For each $i = g, b$, there is an increase in U_i , the investment in off-balance sheet assets, and in $R_i + U_i$, the total investment.*
- (ii) *Further, if the project density function $f(\cdot)$ satisfies project sufficiency, the effect is greater for the bad bank than the good bank.*

Recall that $\underline{\omega}$ is the outside option of the capital suppliers in the opaque funding market. Any regulation that is imposed that makes it more difficult or costly for such capital to find investible projects decreases this outside option. (In the context of the model, this can be interpreted as an

increase in the ζ parameter. It is immediate that this will lead ceteris paribus to a lower $\underline{\omega}$.) Banks will be the beneficiaries of such a change because it will make it cheaper for them to obtain financing for their off balance sheet assets. They will therefore increase their investment in off balance sheet assets. They do this in two ways. First, they move projects off their balance sheet. This also reduces the size of the regulated balance sheet. Second, they accept projects that previously had been rejected. We note that under project sufficiency, this effect is more pronounced for bad banks than good banks.

Notice now that the effect of regulating hedge funds is to increase the total size of the assets in the banking sector. Further, the effect is more pronounced for bad banks than good banks. This implies that total book value of assets at risk for catastrophic failure is higher after this regulatory attempt. Of course, this is a partial equilibrium point of view in that it only considers the effect on the banking sector and not the other shadow banking sectors in which hedge funds might be participating. However, it is worth noting.

Another way in which the relative cost can change is in β , the exogenous probability that bargaining breaks down. Recall, that is $\beta = 1$, the bank effectively makes a take-it-or-leave-it offer to the hedge fund and can drive them down to their reservation value. Conversely, if $\beta = 0$, then the hedge fund effectively makes a take-it-or-leave-it offer. We are only focussing on a sectoral analysis, however it is reasonable to consider that β is larger if the macro state is good. In this case, projects are scarce relative to funding sources and so banks retain more of the surplus from the shadow banking sector. The implication of this is that banks increase the size of their off balance sheet positions.

Proposition 5 *Suppose that $\underline{\omega} \leq r_E$. Then, an increase in β , the share of the surplus captured by the bank under off-balance-sheet financing, leads to an increase in off-balance-sheet investment, and total investment of the bank remains unchanged.*

4.2 Banks' Tobin's q

Assume that $\underline{\omega} \leq r_E$. Tobin's q for a bank with type θ_i may be determined as follows. Total investment in regulated projects is R_i . Of this, an amount $\frac{\alpha}{1-\alpha}R_i$ is invested in the safe asset. Therefore, the total debt is $D_i = \frac{R_i}{1-\alpha} - E$. Note that because debtholders are competitive, the market value of debt also equals D_i . At time 1, debtholders must be paid $(\frac{R_i}{1-\alpha} - E)(1+r)$.

The expected cash flow from on-balance-sheet assets is

$$\theta_i C \int_{\tilde{x}_i}^1 x f(x) dx + \frac{\alpha}{1-\alpha} R_i. \quad (7)$$

The expected cash flow to equity-holders from off-balance-sheet assets is

$$\beta \left(\theta_i C \int_{\underline{x}_i^\omega}^{\tilde{x}_i} x f(x) dx - (1 + \omega) U_i \right). \quad (8)$$

Therefore, we can write the value of equity as

$$V_i = \theta_i C \int_{\tilde{x}_i}^1 x f(x) dx + \frac{\alpha}{1 - \alpha} R_i - \left(\frac{R_i}{1 - \alpha} - E \right) (1 + r) + \beta \left(\theta_i C \int_{\underline{x}_i^\omega}^{\tilde{x}_i} x f(x) dx - (1 + \omega) U_i \right). \quad (9)$$

Tobin's q is now defined as $q = \frac{\text{Mkt value of equity} + \text{Mkt value of debt}}{\text{Book value of equity} + \text{Book value of debt}}$. When $\omega < r_E$, carrying out the calculation and simplifying yields:

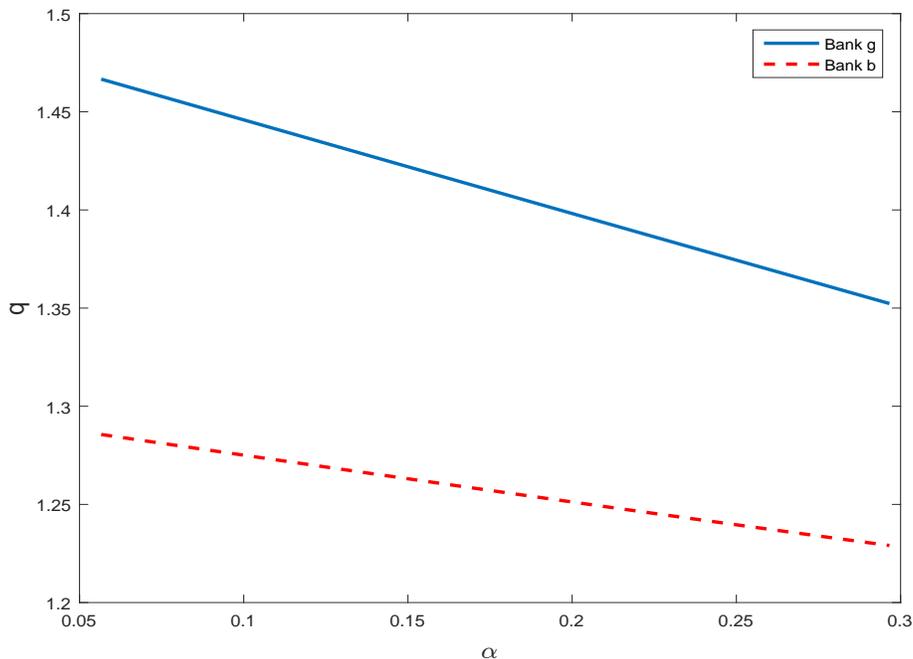
$$\begin{aligned} q_i = & \alpha + (1 - \alpha) \left(\frac{\theta_i C \left(\int_{\tilde{x}_i}^1 x f(x) dx + \beta \int_{\underline{x}_i^\omega}^{\tilde{x}_i} x f(x) dx \right)}{R_i} - \beta (1 + \omega) \frac{U_i}{R_i} \right) \\ & - \left(1 - \frac{(1 - \alpha) E}{R_i} \right) r. \end{aligned} \quad (10)$$

To understand how q varies with regulation, we compute an example in which $F(x)$ is the uniform distribution. We set $r_d = 5\%$, $\omega = 5.2\%$, $E = 0.2$, and $\beta = 0.6$. Figure 5 displays the Tobin's q value for both good and bad banks as α varies above $1 - \frac{r}{\omega}$, the threshold α at which $r_E = \omega$. As can be seen from the figure, q decreases with α , and the q is higher for a good bank than a bad bank. Both these features are expected. Note also that, as expected from a static model, the q exceeds 1 for both types of bank. Numerically, we find similar results (i) using the generalized uniform distribution $F(x) = x^a$, both for values of $a < 1$ and values $a > 1$ and (ii) varying the other parameters.

5 Implications for the Macro Economy

Collectively, the banks' local states determine the macro state. We simplify our discussion of macro states in a few ways. First, because banks are ex ante symmetric, without loss of generality, the macro state depends only on the number of banks in a good state rather than their specific identity. Second, we have in mind a situation in which N is large. Thus, although in principle there are $N + 1$ macro states, when N is large, market participants (including the regulator) are unlikely to change their actions if the number of banks in a good local state changes by a small amount. Therefore, we think of macro states in terms of the range of banks that are in a good local state. Finally, to simplify our discussion, we reduce the number of macro states to two, which we label as high (h) and low (ℓ).

Formally, we define the macro state to be high if at least half the banks are in a good local



This figure shows the variation in Tobin's q for good and bad banks as the regulatory environment varies. To generate the figure, we set $F(x) = x$ (i.e., the uniform distribution), $r_d = 0.05$, $\underline{\omega} = 0.053$, and $E = 0.2$.

Figure 5: Tobin's q as α varies

state. Otherwise, the macro state is low. Let $M = \frac{N}{2}$ if N is even and $\frac{N+1}{2}$ if N is odd. Then, the probability of a high macro state is $\sum_{k=M}^N \frac{N!}{k!(N-k)!} \pi^k (1-\pi)^{N-k} \stackrel{d}{=} \phi$.

We begin with a preliminary result, Lemma 3. Consider two situations in which the number of banks in a good local state differs. Then, under the project sufficiency condition, the shadow banking sector has a greater relative size in the economy when there are fewer banks in a good state.

Lemma 3 *Consider two situations with k_1 and k_2 good banks in the economy respectively, where $k_1 < k_2$. Then, $\frac{R_{k_1}}{U_{k_1}} > \frac{R_{k_2}}{U_{k_2}}$. That is, the relative size of the shadow banking sector is greater when there are fewer banks in the good local state.*

Next, we consider the sizes of the on- and off-balance-sheet sectors in the high and low macro states. For any variable Z , denote Z^h as the conditional expectation of the variable in the high macro state, and Z^ℓ as the conditional expectation in the low macro state. Let Z^k indicate the

value of the variable when exactly k banks are in the good local state. Then,

$$Z^h = \frac{\sum_{k=M}^N \frac{N!}{k!(N-k)!} \pi^k (1-\pi)^{N-k} Z^k}{\phi} \quad (11)$$

$$Z^\ell = \frac{\sum_{k=1}^{M-1} \frac{N!}{k!(N-k)!} \pi^k (1-\pi)^{N-k} Z^k}{1-\phi} \quad (12)$$

The shadow-banking sector has a greater proportional size in the low macro state, as compared to the high macro state.

Proposition 6 *The relative size of the shadow banking sector is greater in the high state; that is, $E[U/R | \ell] > E[U/R | h]$.*

That is, when the shadow banking sector exists, it is relatively larger in the bad macro state.

6 Optimal MacroPrudential Regulation

We now turn to the optimal policy of the regulator. Recall that in our model the regulator has a single policy variable, α , which determines the proportion of total balance sheet assets that must be held in the safe asset. The Liquidity Coverage Ratio is a key component of the Basel III accord, which is currently scheduled to be implemented in 2019.

There are two components to the regulator's preferences. First, the regulator cares about total investment in the economy. At the single-bank level, suppose that a bank is in the local state θ . Consider a frictionless economy in which $r = 0$. In such an economy, there is no shadow-banking sector (as $\omega \geq 0$). Let $n_\theta = \frac{1}{\theta_i C}$ be the zero-NPV project the bank would undertake in this economy, so that the total investment by the bank is $1 - F(n_\theta)$. In the model, the actual investment undertaken by the bank (given the frictions present) is $R_\theta + U_\theta$. Here, investment by the bank may be taken as a proxy for the employment generated by the funded projects.

When either (i) the macro state is high and the local state is good or (ii) the macro state is low and the local state is bad, the regulator would like the investment by the bank to be as close to $1 - F(n_\theta)$ as possible. However, there is a conflict in the preferences of the regulator and the bank in the other two cases. In the high macro state, the regulator is worried about systemic risk engineered by the default of bad banks. Specifically, the regulator suffers a disutility δ_1 from the default of a bank. Therefore, the regulator's utility from a project at a bad bank is $\theta x C - 1 - (1 - \theta)\delta_1$. Conversely, in the bad macro state, the regulator wishes to boost employment in the economy, and obtains an additional benefit δ_2 from a project undertaken by a good bank. Therefore, the regulator's utility from such a project is $x C - 1 + \delta_2$.

Let $\gamma_{m\theta}$ denote the regulator's marginal preferred project undertaken by a bank when the macro state is m and the bank's local state is θ . The corresponding total investment desired by the regulator in each pair of macro and local state is $\Gamma_{m\theta}$.

Second, the regulator cares about the size of the shadow-banking sector. Recall that a bank with local type θ has unregulated assets in the amount U_θ . We treat the size of the shadow-banking sector as a proxy for the notion that instability can arise within the financial system outside the regulated sector. While we do not formally model this, what we have in mind is that the hedge fund can withdraw its financing before the project is completed, leading to costly liquidation and potential bank distress. A larger shadow-banking sector therefore represents a greater potential for threats to the overall system.

Putting these two components together, we write a disutility function for the regulator as follows. Consider a bank i with type $\theta_i \in \{g, b\}$. The disutility of the regulator when faced with this bank is:

$$\Sigma_i(\alpha \mid \theta, m) = (\Gamma_{mi} - (R_i + U_i))^2 + \lambda U_i, \quad (13)$$

where $\lambda > 0$ is a parameter determining the relative disutility of the two components of the regulator's preferences. The regulator's overall disutility may then be written as follows. Let $\psi_m = E[\# \text{ of good banks} \mid m]$. Note that ψ_m is function only of N and π . Then, $\Sigma(\alpha \mid m) = \sum_{i=1}^N \binom{N}{i} \pi^i (1 - \pi)^{N-i} [i \Sigma(\alpha \mid g, m) + (N - i) \Sigma(\alpha \mid b, m)] = \psi_m \Sigma(\alpha \mid g, m) + (1 - \psi_m) \Sigma(\alpha \mid b, m)$. The regulator chooses the optimal macro-state dependent policy to minimize $\Sigma(\alpha \mid m)$.

We begin with the simple observation that the liquidity requirement can have no effect on banks unless external capital is costly; that is, $r > 0$. The argument is straightforward—if the cost of debt to the bank is the same as the return on the safe asset, the bank can just add as much in safe assets as it needs in order to satisfy the liquidity requirement, leaving its investment in risky projects untouched.

Lemma 4 *A change in the liquidity requirement α affects R_θ or U_θ if and only if $r > 0$.*

We assume that $r < \underline{\omega}$, as lending to a bank through the debt market is one of the investment options available to a hedge fund. Notice that when $r < \underline{\omega}$, a regulatory policy that sets $\alpha \leq 1 - \frac{r}{\underline{\omega}}$ eliminates shadow banking, but creates investment distortions in both the high and low macro states. In the low macro state, the regulator wants good banks to over-invest in projects less profitable than the frictionless zero-NPV project, whereas it wants bad banks to invest up to the frictionless zero-NPV project. When $\underline{\omega} > r > 0$, both good and bad banks under-invest relative to the regulator's desired level. Therefore, it is optimal to set the liquidity requirement to $\alpha = 0$.

In the high macro state, there may be a trade-off in setting the optimal regulatory policy. As

before, reducing α reduces the size of the shadow banking sector, and if α falls below $1 - \frac{r}{\bar{\omega}}$, the shadow banking sector ceases to exist. Further, reducing α increases investment by good banks, bringing their investment level closer to the regulator's desired level. However, reducing α also increases investment by bad banks. Thus, if δ_1 (the regulator's disutility from potential failure of a bad bank) is sufficiently high, a regulator will prefer to keep α high, to prevent over-investment by bad banks.

We formalize these notions in the next proposition.

Proposition 7 (i) *In the low macro state, the optimal policy is to set $\alpha = 0$.*

(ii) *In the high macro state, if $\delta_1 > \frac{\bar{\omega}}{1-\theta}$ and π is sufficiently low, the optimal policy sets some strictly positive $\alpha \leq 1 - \frac{r}{\bar{\omega}}$.*

Therefore, in the model as specified, given the optimal regulation, there is no shadow banking in either the high or the low macro states. In the high macro state, there is under-investment by good banks and over-investment by bad banks. In the low macro state, there is under-investment by good banks relative to the regulator's desired state.

In the high macro state, if either δ_1 is sufficiently low or π is sufficiently high, again it is optimal to set $\alpha = 0$.

7 Conclusion

Following the basic logic of Modigliani-Miller, financing sources affect firm (or bank) value only in so far as there are financing frictions. In this paper, we have presented a parsimonious model of banking in which banks either move assets off balance sheet or keep them on balance sheet with an eye to different financing frictions.

A Appendix

Proof of Lemma 1

We first establish an hedge fund's search rule and then determine the bank's payoff. As is standard in search models, each hedge fund will follow a threshold strategy and accept arbitrage opportunities that offer a minimum surplus. Specifically, they will accept any project that satisfies $\omega > \underline{\omega}$, where

$$\underline{\omega} = E \max[\omega, V], \quad (14)$$

where V is the value of continuing the search. Notice that

$$V = (V - \zeta) \int_0^{\underline{\omega}} dH() + \int_{\underline{\omega}}^{\infty} (\omega) dH() \quad (15)$$

$$= \frac{\int_{\underline{\omega}}^{\infty} (\omega) dH - \zeta H(\underline{\omega})}{(1 - H(\underline{\omega}))} \quad (16)$$

Taking the first order condition of V with respect to $\underline{\omega}$, yields

$$\begin{aligned} \frac{dV}{d\underline{\omega}} &= \frac{-(\underline{\omega})h(\underline{\omega}) - \zeta h(\underline{\omega})}{1 - H(\underline{\omega})} + h(\underline{\omega}) \frac{\int_{\underline{\omega}}^{\infty} \omega dH - \zeta H(\underline{\omega})}{[1 - H(\underline{\omega})]^2} \\ &= \frac{h(\underline{\omega})}{[1 - H(\underline{\omega})]^2} \left(\int_{\underline{\omega}}^{\infty} \omega dH - \underline{\omega}(1 - H(\underline{\omega})) - \zeta \right) \end{aligned}$$

This implies that

$$\int_{\underline{\omega}}^{\infty} \omega dH + \underline{\omega} H(\underline{\omega}) - \underline{\omega} - \zeta = 0 \quad (17)$$

$$\text{Thus, } \underline{\omega} = E \max[\omega, \underline{\omega}] - \zeta \quad (18)$$

Now consider a hedge fund that approaches a bank with this search rule. The hedge fund's outside option is $\underline{\omega}$, while if the bank does raise funding, it receives zero. The surplus from any project is of the form $S = \theta_i x C - 1$. If the bargaining game ends without agreement between the bank and the hedge fund, the hedge fund obtains its outside option, which earns a payoff $\underline{\omega}$, and the bank obtains zero.

We proceed by backward induction. At time 2, the hedge fund makes the last offer to the bank. Thus, it makes a take-it-or-leave-it offer ψ_2 to the bank, where ψ_2 represents the share of the surplus that accrues to the bank. Therefore, if the bank accepts, it obtains the amount $\psi_2(x\theta_i C - 1)$. If it rejects, it obtains 0. Therefore, it accepts whenever either (i) $\psi_2 > 0$ and $x\theta_i C \geq 1$, or (ii) $\psi_2 = 0$.

The optimal offer is $\psi_2 = 0$. The hedge fund obtains $x\theta_i C - 1$, which is individually rational as long as $x\theta_i C - 1 \geq \underline{\omega}$.

That is, at stage 2 the hedge fund offers $\psi_2 = 0$ as long as $x\theta_i C \geq 1$. If $x\theta_i C < 1$, the hedge fund chooses to make no offer and the game terminates.

At period 1 suppose the bank makes an offer $1 - \psi_1$ to the hedge fund, where $1 - \psi_1$ represents the share of the surplus accruing to the hedge fund. If the hedge fund accepts, it receives the payoff $(1 - \psi_1)(x\theta_i C - 1)$. If it rejects, the following are the possibilities:

1. With probability β the game ends and the hedge fund gets $\underline{\omega}$.
2. With probability $1 - \beta$, we go to a second period of bargaining. Bargaining periods are short relative to the periods in which transitions between states occur, so the state remains the same. In the second bargaining period, the hedge fund gets to make a take-it-or-leave-it offer, and (as above) obtains a payoff $x\theta_i C - 1$.

Therefore, the expected payoff to rejecting is $\beta\underline{\omega} + (1 - \beta)(x\theta_i C - 1 - \underline{\omega})$. The hedge fund will accept an offer of $1 - \psi_1$ if

$$\begin{aligned} (1 - \psi_1)(x\theta_i C - 1) &\geq \beta\underline{\omega} + (1 - \beta)(x\theta_i C - 1) \\ 1 - \psi_1 &\geq \frac{\beta\underline{\omega}}{(x\theta_i C - 1)} + 1 - \beta \\ \psi_1 &\leq \beta \left(1 - \frac{\underline{\omega}}{x\theta_i C - 1} \right). \end{aligned} \tag{19}$$

The bank will set ψ_1 to be as high as possible subject to the last inequality. That is, $\psi_1 = \beta \left(1 - \frac{\underline{\omega}}{(x\theta_i C - 1)} \right)$. Therefore, the bank's payoff is

$$\beta(x\theta_i C - (1 + \underline{\omega})). \tag{20}$$

■

Proof of Lemma 2

- (i) The payoff to the bank from making a loan and funding it through the hedge fund sector is

$$\beta(\theta_i x C - 1 - \underline{\omega}). \tag{21}$$

The marginal loan is therefore

$$\underline{x}_i^\omega = \frac{1 + \underline{\omega}}{\theta_i C} \tag{22}$$

Now, all loans have the same investment size of \$1, and the payoff to a loan is strictly increasing in its productivity. Therefore, every loan with $x > \underline{x}_i^\omega$ is strictly profitable if funded in the hedge fund sector.

(ii) The payoff to the bank from making the loan and holding it on balance sheet is

$$x\theta_i C + \frac{\alpha}{1-\alpha} - \frac{1+r}{1-\alpha} = x\theta_i C - 1 - r_E. \quad (23)$$

The marginal loan \underline{x}_i^d is therefore given by

$$\underline{x}_i^d = \frac{1}{\theta_i C} (1 + r_E). \quad (24)$$

Now, all loans have the same investment size of \$1, and the payoff to a loan is strictly increasing in its productivity. Therefore, every loan with $x > \underline{x}_i^d$ is strictly profitable if funded on the balance sheet.

(iii) Consider the loan \tilde{x}_i such that the bank is indifferent between holding it on or off the balance sheet. This loan satisfies:

$$x\theta_i C - 1 - r_E = \beta(x\theta_i C - 1 - \underline{\omega}) \quad (25)$$

$$(1 - \beta)x\theta_i C = 1 + r_E - \beta(1 + \underline{\omega}) \quad (26)$$

$$\tilde{x}_i = \frac{1}{\theta_i C} \left(1 + \frac{r_E - \beta\underline{\omega}}{1 - \beta} \right). \quad (27)$$

Finally, observe that the LHS of equation (25) is increasing in productivity at a rate $\theta_i C$, while the RHS is increasing at a lower rate $\beta\theta_i C$. Therefore, for higher productivities $x > \tilde{x}_i$, the bank prefers to fund them on the balance sheet. Conversely, when $x < \tilde{x}_i$, the bank prefers to fund the loan in the hedge fund sector. ■

Proof of Proposition 1

(i) Suppose that $\underline{x}_i^d \leq \underline{x}_i^\omega$. From the definitions of the two variables, this condition corresponds to $r_E \leq \underline{\omega}$. In this case, the lowest productivity project that is still profitable to fund off balance sheet is lower than the lowest productivity project that is profitable to fund in the hedge fund sector. Therefore, all projects will be funded on balance sheet, and the bank invests in projects with $x \in [\underline{x}_i^d, 1]$.

(ii) Conversely, suppose that $\underline{x}_i^\omega < \underline{x}_i^d$, which corresponds to $r_E > \underline{\omega}$. Then, projects in the range $(\underline{x}_i^\omega, \underline{x}_i^d)$ are strictly profitable if funded in the hedge fund sector, but not if funded on the balance

sheet. Recall that the bank is indifferent about holding project $\tilde{x}_i \in (\underline{x}_i^\omega, \underline{x}_i^d)$ on or off the balance sheet. Therefore, it funds all projects with $x \in [\underline{x}_i^\omega, 1]$, and holds projects $[\underline{x}_i^\omega, \tilde{x}_i)$ off the balance sheet and projects $[\tilde{x}_i, 1]$ on the balance sheet. ■

Proof of Proposition 3

(i) From Proposition 1, when $r_E \geq \underline{\omega}$, projects $[\underline{x}_i^\omega, \tilde{x}_i)$ are funded off the balance sheet and projects $[\tilde{x}_i, 1]$ are funded on the balance sheet. By inspection, \underline{x}_i is invariant to α and \tilde{x}_i strictly increasing in α . Therefore, U_i increases for each i , whereas $R_i + U_i$ is unchanged.

(ii) We have

$$\frac{\partial U_i}{\partial \alpha} = f(\tilde{x}_i) \frac{\partial \tilde{x}_i}{\partial \alpha} = f(\tilde{x}_i) \frac{1}{(1-\beta)\theta_i C} \frac{r}{(1-\alpha)^2}. \quad (28)$$

Therefore, $\frac{\partial U_g}{\partial \alpha} < \frac{\partial U_b}{\partial \alpha}$ if and only if $f(\tilde{x}_g) \frac{1}{(1-\beta)C} \frac{r}{(1-\alpha)^2} < f(\tilde{x}_b) \frac{1}{(1-\beta)\theta_b C} \frac{r}{(1-\alpha)^2}$, or $f(\tilde{x}_g) < \frac{f(\tilde{x}_b)}{\theta_b}$. Further, by definition, $\tilde{x}_b = \frac{\tilde{x}_g}{\theta_b}$. Therefore, the required condition reduces to

$$\frac{1}{\theta_b} f\left(\frac{\tilde{x}_g}{\theta_b}\right) > f(\tilde{x}_g). \quad (29)$$

Set $k = \frac{1}{\theta_b} > 1$, and set $x = \tilde{x}_g$. Then, project sufficiency directly implies that inequality (29) holds. ■

Proof of Proposition 4

(i) Recall that when $r_E \geq \underline{\omega}$, we have $U_i = F(\tilde{x}_i) - F(\underline{x}_i^\omega)$ and $R_i + U_i = 1 - F(\underline{x}_i^\omega)$. Suppose that, due to a regulatory change on hedge funds, there is a decrease in ω . By inspection, \tilde{x}_i increases and \underline{x}_i^ω decreases. It is immediate that U_i and $R_i + U_i$ both increase.

(ii) Consider a change in ω . We have

$$\frac{\partial Y_i}{\partial \omega} = -f(\tilde{x}_i) \frac{\beta}{(1-\beta)\theta_i C} - f(\underline{x}_i^\omega) \frac{1}{\theta_i C} = -\frac{1}{\theta_i C} \left(\frac{\beta}{1-\beta} f(\tilde{x}_i) + f(\underline{x}_i^\omega) \right). \quad (30)$$

Now, the effect on the bad bank is greater than the effect on the good bank if $\frac{\partial U_b}{\partial \omega} < \frac{\partial U_g}{\partial \omega} < 0$. The first inequality may be written as:

$$\frac{1}{\theta_b} \left(\frac{\beta}{1-\beta} f(\tilde{x}_b) + f(\underline{x}_b^\omega) \right) > \frac{\beta}{1-\beta} f(\tilde{x}_g) + f(\underline{x}_g^\omega). \quad (31)$$

Now, $\tilde{x}_b = \frac{1}{\theta_b} \tilde{x}_g$, and $\underline{x}_b^\omega = \frac{1}{\theta_b} \underline{x}_g^\omega$. Substituting into the last inequality, we obtain the condition

$$\frac{1}{\theta_b} \left(\frac{\beta}{1-\beta} f\left(\frac{\tilde{x}_g}{\theta_b}\right) + f\left(\frac{\underline{x}_g^\omega}{\theta_b}\right) \right) > \frac{\beta}{1-\beta} f(\tilde{x}_g) + f(\underline{x}_g^\omega), \quad (32)$$

or

$$\frac{\beta}{1-\beta} \left(\frac{1}{\theta_b} f\left(\frac{\tilde{x}_g}{\theta_b}\right) - f(\tilde{x}_g) \right) > f(\underline{x}_g^\omega) - \frac{1}{\theta_b} f\left(\frac{\underline{x}_g^\omega}{\theta_b}\right). \quad (33)$$

Under project sufficiency, the LHS is strictly positive and the RHS is strictly negative, so the inequality holds. ■

Proof of Proposition 5

Note that we can write \tilde{x}_i as follows:

$$\tilde{x}_i = \frac{1}{\theta_i C} \times \frac{1 + r_E - \beta(1 + \underline{\omega})}{1 - \beta}. \quad (34)$$

Therefore,

$$\begin{aligned} \frac{\partial \tilde{x}_i}{\partial \beta} &= \frac{1}{\theta_i C} \left[\frac{-(1-\beta)(1+\underline{\omega}) - (1+r_E - \beta(1+\underline{\omega}))(-1)}{(1-\beta)^2} \right] \\ &= \frac{1}{\theta_i C(1-\beta)^2} [-(1-\beta)(1+\underline{\omega}) + 1 + r_E - \beta(1+\underline{\omega})] \\ &= \frac{1}{\theta_i C(1-\beta)^2} (r_E - \underline{\omega}). \end{aligned} \quad (35)$$

Therefore, whenever $r_E > \underline{\omega}$ (which is the condition under which the shadow-banking sector exists), it follows that $\frac{\partial \tilde{x}_i}{\partial \beta} > 0$.

Now, an increase in \tilde{x}_i immediately implies an increase in Y_i and a decrease in X_i . Note that \underline{x}_i^ω is unchanged as β changes, so total investment is unchanged. ■

Proof of Proposition 2

(i) Recall that, when $\underline{\omega} < r_E$, we have $X_i = F(\tilde{x}_i) - F(\underline{x}_i^\omega)$, and $Y_i = 1 - F(\tilde{x}_i)$. Therefore,

$$\frac{R_b}{U_b} > \frac{R_g}{U_g} \iff \frac{F(\tilde{x}_b) - F(\underline{x}_b^\omega)}{1 - F(\tilde{x}_b)} > \frac{F(\tilde{x}_g) - F(\underline{x}_g^\omega)}{1 - F(\tilde{x}_g)}. \quad (36)$$

As $\tilde{x}_b = \frac{\tilde{x}_g}{\theta}$ and $\underline{x}_b^\omega = \frac{\underline{x}_g^\omega}{\theta}$, the last inequality may be written as

$$\frac{F(\tilde{x}_g/\theta) - F(\underline{x}_g^\omega/\theta)}{1 - F(\tilde{x}_g/\theta)} > \frac{F(\tilde{x}_g) - F(\underline{x}_g^\omega)}{1 - F(\tilde{x}_g)} \quad (37)$$

Now, $F(\tilde{x}_g/\theta) - F(\underline{x}_g^\omega/\theta) = \int_{\underline{x}_g^\omega/\theta}^{\tilde{x}_g/\theta} f(s) ds$. Set $s = t/\theta$, so that $ds = \frac{1}{\theta} dt$. Then,

$$F(\tilde{x}_g/\theta) - F(\underline{x}_g^\omega/\theta) = \int_{\underline{x}_g^\omega/\theta}^{\tilde{x}_g/\theta} f(s) ds = \int_{\underline{x}_g^\omega}^{\tilde{x}_g} \frac{1}{\theta} f(t/\theta) dt. \quad (38)$$

project sufficiency implies that

$$\int_{\underline{x}_g^\omega}^{\tilde{x}_g} \frac{1}{\theta} f(t/\theta) dt > \int_{\underline{x}_g^\omega}^{\tilde{x}_g} f(t) dt, \quad (39)$$

or $F(\tilde{x}_g/\theta) - F(\underline{x}_g^\omega/\theta) > F(\tilde{x}_g) - F(\underline{x}_g^\omega)$.

Finally, observe that $F(\tilde{x}_g) < F(\tilde{x}_g/\theta)$, so that $1 - F(\tilde{x}_g) > 1 - F(\tilde{x}_g/\theta)$. Therefore, it follows that

$$\frac{F(\tilde{x}_b) - F(\underline{x}_b^\omega)}{1 - F(\tilde{x}_b)} > \frac{F(\tilde{x}_g) - F(\underline{x}_g^\omega)}{1 - F(\tilde{x}_g)}, \quad (40)$$

or $\frac{R_b}{U_b} > \frac{R_g}{U_g}$.

(ii) We can write $\frac{R_i}{R_i + U_i} = \frac{1}{1 + \frac{U_i}{R_i}}$. Now, $\frac{R_b}{U_b} > \frac{R_g}{U_g}$ implies that $\frac{U_b}{R_b} < \frac{U_g}{R_g}$, which immediately implies $\frac{R_b}{R_b + U_b} > \frac{R_g}{R_g + U_g}$. ■

Proof of Lemma 3

Consider two situations, one in which there are k_1 good banks and the other in which there are k_2 good banks. In situation s , the total quantity of shadow-banking assets in the economy is $U_s = (N - s)U_b + sU_g$. Similarly, the total quantity of on-balance sheet assets in the economy is $R_s = (N - s)R_b + sR_g$. Then,

$$\frac{X_{k_1}}{Y_{k_1}} > \frac{X_{k_2}}{Y_{k_2}} \iff \frac{NX_b + k_1(X_g - X_b)}{NY_b + k_1(Y_g - Y_b)} > \frac{NX_b + k_2(X_g - X_b)}{NY_b + k_2(Y_g - Y_b)}. \quad (41)$$

Cross-multiplying and simplifying, the last inequality is equivalent to

$$Nk_2(U_bR_g - U_gR_b) > Nk_1(U_bR_g - U_gR_b). \quad (42)$$

Now, from Proposition 2 (i), under project sufficiency $\frac{U_b}{R_g} > \frac{R_b}{R_g}$, so that $U_bR_g - U_gR_b > 0$. Therefore, inequality 42 holds whenever $k_2 > k_1$. ■

Proof of Proposition 6

We have $(R/U)^h = E[(R/U)^k \mid k \geq M]$, and $(R/U)^\ell = E[(R/U)^k \mid k < M]$, where $M = \frac{N}{2}$ when N is even and $\frac{N+1}{2}$ when N is odd. Now, consider any two cases as follows. Case 1 is in the high macro state, and has $j \geq M$ banks in the good local state. Case 2 is in the low macro state, and has $k < M$ banks in the good local state. It follows from Lemma 3 that $(R/U)^j < (R/U)^k$. Taking conditional expectations on either side of that inequality, we immediately have $(R/U)^h < (R/U)^\ell$. ■

Proof of Lemma 4

Suppose that $r = 0$, and consider bank i . Then, from the definition of \tilde{x}_i , we have $\tilde{x}_i < 0$. That is, for any project $x \geq 0$, it is more profitable to hold the project on the balance sheet than off the balance sheet, so that $U_\theta = 0$. Therefore, total investment is equal to $R_\theta = 1 - F(\underline{x}_i^d)$, where $\underline{x}_i^d = \frac{1}{\theta R}$. By inspection, a change in α has no effect on R_θ or U_θ .

Next, suppose that $r > 0$, and consider bank i . It is immediate to observe that both \underline{x}_i^d and \tilde{x}_i vary with α (recall that $r_E = \frac{r}{1-\alpha}$), but \underline{x}_i^ω does not. Now, either $r_E > \omega$, in which case $U_\theta = F(\tilde{x}_i) - F(\underline{x}_i^\omega)$ varies with α , or $r_E \leq \omega$, in which case $R_\theta = 1 - F(\underline{x}_i^d)$ varies with α . ■

Proof of Proposition 7

(i) Consider the low macro state first. Observe that $r_E \leq \underline{\omega}$ whenever $\alpha \leq 1 - \frac{r}{\underline{\omega}}$. Suppose that $\alpha \geq 1 - \frac{r}{\underline{\omega}}$. Then, recalling that $\psi_\ell = E[\# \text{ of good banks } | \ell]$, we can write

$$\Sigma(\alpha | \ell) = \psi_\ell[(\Gamma_{\ell g} - (R_g + U_g))^2 + \lambda U_g] + (1 - \psi_\ell)[(\Gamma_{\ell b} - (R_b + U_b))^2 + \lambda U_b]. \quad (43)$$

Here, $R_g + U_g = 1 - F(\underline{x}_g^\omega) = 1 - F(\frac{1+\omega}{C})$, and $R_b + U_b = 1 - F(\underline{x}_b^\omega) = 1 - F(\frac{1+\omega}{\theta C})$. Therefore, the only expressions on the RHS of the equation defining $\Sigma(\alpha | \ell)$ that change with α are $U_g = F(\underline{x}_g^\omega)$ and $U_b(\underline{x}_b^\omega)$. In particular, we have $U_i = F(\tilde{x}_i) - F(\underline{x}_i^\omega)$, where $\tilde{x}_i = \frac{1 + \frac{r/(1-\alpha) - \beta(1+\omega)}{1-\beta}}{\theta_i C}$. Note that \tilde{x}_i strictly increases in α . Therefore, $\Sigma(\alpha | \ell)$ strictly increases in α when $\alpha \geq 1 - \frac{r}{\underline{\omega}}$. Therefore, the optimal α in this range is $\alpha = 1 - \frac{r}{\underline{\omega}}$.

Next, suppose that $\alpha \leq 1 - \frac{r}{\underline{\omega}}$. Then, $r_E \leq \underline{\omega}$, so that $U_g = U_b = 0$. Here, $R_i = 1 - F(\underline{x}_i^d)$, where $\underline{x}_i^d = \frac{1+r/(1-\alpha)}{\theta C}$. Therefore, an increase in α decreases R_i and increases $(\Gamma_{\ell i} - R_i)^2$. Therefore, the optimal policy is to set $\alpha = 0$.

(ii) Now, consider the high macro state. We have

$$\Sigma(\alpha | h) = \psi_h[(\Gamma_{hg} - (R_g + U_g))^2 + \lambda U_g] + (1 - \psi_h)[(\Gamma_{hb} - (R_b + U_b))^2 + \lambda U_b]. \quad (44)$$

Again, when $\alpha \geq 1 - \frac{r}{\underline{\omega}}$, the expression $R_i + U_i$ is, for each θ_i , independent of α . Further, U_g and U_b are both increasing in α . Therefore, the optimal α in this range is $\alpha = 1 - \frac{r}{\underline{\omega}}$.

Now, consider $\alpha \leq 1 - \frac{r}{\underline{\omega}}$. Then, $U_g = U_b = 0$. Therefore, we can write

$$\Sigma(\alpha | h) = \psi_h(\Gamma_{hg} - R_g)^2 + (1 - \psi_h)(\Gamma_{hb} - R_b)^2. \quad (45)$$

Hence, the derivative with respect to α , $\frac{\partial \Sigma}{\partial \alpha}$, is

$$2\psi_h(\Gamma_{hg} - R_g)f(\tilde{x}_g)\frac{\partial \tilde{x}_g}{\partial \alpha} + 2(1 - \psi_h)(\Gamma_{hb} - R_b)f(\tilde{x}_b)\frac{\partial \tilde{x}_b}{\partial \alpha}. \quad (46)$$

Here, $R_i = 1 - F(\tilde{x}_i)$ for each θ_i . Further, $\Gamma_{hg} > R_g$ for all $\alpha \geq 0$, and $\Gamma_{hb} < R_b$ for all $\alpha \geq 0$.

Now, set $\alpha = 0$; then, $\tilde{x}_g = \frac{1+r}{C}$ and $\tilde{x}_b = \frac{\tilde{x}_g}{\theta}$. It follows that if ψ_h is sufficiently small, then the derivative is strictly negative at $\alpha = 0$; that is, increasing α reduces the disutility of the regulator. Recalling that $\psi_h = E[\# \text{ of good firms} \mid h]$, we require π_h to be sufficiently small for ψ_h , in turn, to be small. ■

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