

# Tax-Loss Carry Forwards and Returns\*

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## **Abstract**

Tax loss carry forward (TLCF), the accumulated corporate losses that can be applied to past or future taxable income, form an important asset in the corporate portfolio. In our sample (1964 - 2014) TLCF was on average equal to 17% of pretax income with considerable cross sectional variation. We show that a firm's TLCF is a complex contingent claim that has a significant non monotonic effect on the cash flow risk of assets in place. Consistent with this theoretical finding we show that TLCFs are highly significant in forecasting returns, volatility and market betas, even when a large number of controls are accounted for.

*Keywords:* tax-loss carry forward, equity returns.

# 1 Introduction

Corporate taxes are among the most studied of financial frictions. Taxes have been used to explain corporate decisions such as capital structure, dividend policy, real investment and risk management<sup>1</sup>. In contrast to the study of how they affect corporate decisions, however, much less is known about the relationship between taxes, corporate operations, and equity returns. This paper contributes to this understudied area by examining the importance of Net Operating Losses (NOLs) and, in cumulative form, Tax Loss Carry Forwards (TLCFs) to equity risk and return.

Tax codes do not allow firms to generally realize negative taxes, i.e. NOLs do not automatically generate payments from the government to the firm. Instead, tax codes only allow NOLs to generate immediate refunds if the firm can apply the losses to current or prior taxable income (Tax Loss Carry-backs). When this is not possible, NOLs must be carried forward and applied to future taxable income (Tax Loss Carry-forwards or TLCFs) within a particular time period (20 years in the U.S.A.), after which they expire. Since each NOL has a distinct maturity date, firms typically hold a portfolio of TLCFs. This portfolio introduces a convexity in the tax related cash outflows; taxes paid in any period are increasing in income above a threshold set by the existing TLCFs but are zero below this threshold. Indeed, the TLCF portfolio of a company is a valuable tax saving asset. Moreover, since future taxable income is risky, so is the potential tax savings.

Deriving a general relationship between TLCF and risk is made difficult by the fact that a firm's TLCF portfolio reflects a specific history of corporate operations and tax management. Non operating deductions such as depreciation and interest payments affect a firm's NOL in any one year and NOLs over the years accumulate through TLCF. When a firm generates taxable operating income it can reduce its tax payments by applying TLCF and/or using Investment Tax Credits (ITCs) to pay for the taxes owing. The resulting tax minimization is a complex problem. Each annual NOL has a different maturity so that at any time some historical NOL may be maturing. Firms can alter the size of their NOL by

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<sup>1</sup>See Graham (2006) for a comprehensive survey of the literature.

managing depreciation which can, for instance, be deferred to allow a soon to be lost TLCF to be used. ITCs also vary in maturity and size. All of these features imply a very complex optimization for the firm in using its portfolio of tax management assets.

The implications, therefore, of TLCF for firm value, risk and return are, to some extent, history dependent and idiosyncratic. Still some general features of the tax code emerge as important for the risk of the equity. Our focus is on the convexity of the tax schedule and its implication for the risk of the firm's equity. We follow Majd and Myers (1985) and Green and Talmor (1985) who recognize that a firm's equity can be seen as a claim to pretax operating cash flows minus a short position in a call option on corporate taxes. The implicit call held by the tax authorities is written on taxes that would be paid if TLCF were zero. The strike price of the option is the available non operating tax deductions which we will refer to collectively as TLCF. Green and Talmor recognized that being short a risky derivative would make the firm's equity safer. We add to this literature by showing that, while this is generally true, the risk reduction is non monotonic and, therefore, so is the relationship between TLCF and risk.

In our model the relationship between tax shields and risk decreases for low levels of tax shields and increases for high levels. Consider a firm with taxable income but zero TLCF. It pays taxes that are proportional to the pretax cash flows and hence its risk is equal to the risk of the cash flows. If we now add low levels of TLCF the tax shields will be used with certainty providing the firm with a certain tax saving, partially offsetting the cash flow risk. Eventually, however, the addition of more tax shields make the firm riskier as some of these tax shields are less likely to be used either in the current period or in the future. However, with large enough tax shields the firm will pay no taxes and the risk is again equal to the risk of cash flows.

In addition to being of theoretical interest, we are motivated by the large and growing importance of TLCFs. Auerbach (2006) shows that while the statutory corporate tax rate in the US has fallen from 46% in 1983 to 35% in 2003, the average tax rate – the ratio of taxes paid to corporate income – has *increased* dramatically from 27% to 45% over the same period. Auerbach shows that, by far, the largest contributor to the increase in the

average tax rate is the increase in NOLs over this period. Since, taxable income is taxed immediately but tax losses do not typically generate an immediate tax offset the average tax rate will be lower than the statutory tax rate. Moreover, as tax losses increase, the average tax paid increases relative to the statutory rate.

Accompanying the increase in NOLs and the average tax rate is an increase in TLCFs. Figure 1 presents the TLCF as a % of corporate assets over time. The increase in TLCF dominates cyclical factors and is not explained by two of the more significant legislated changes during this period; the Economic Recovery Act of 1981 that reduced some corporate taxes, and the Tax Reform Act of 1986 that reduced the level of depreciation allowances. It seems that the primary explanation of operating losses over this period is lower returns to assets.<sup>2</sup>

We first study TLCF in a simple one period binomial model where we show the sources of non monotonicity of risk in TLCF. We then examine a more complex single period and multiperiod model numerically. We are able to show that the relationship found in our simple model is also apparent in a more complete model of a firm that has depreciation, ITCs and TLCF.

Empirically we show that, consistent with our theoretical model, TLCF are able to forecast future returns, volatility, betas, and other factors even when we include a large number of known controls. Interestingly, we find that ITCs are also significantly but negatively related to returns, indicating that they reduce risk. This might be the case if ITCs are used first (since they would then be more likely to be used) which in turn suggests they may typically expire sooner than TLCFs.<sup>3</sup>

Our paper builds on the work of Green and Talmor (1985) who explicitly recognize the call option structure of the tax claim on the firm. They use this insight to study investment behavior by firms and the debt-equity conflict of interest. We instead look at the implication

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<sup>2</sup>See Altshuler, Auerbach, Cooper, and Knittel (2009) for an in depth discussion of the growth in corporate tax losses.

<sup>3</sup>Our findings that risk increases with TLCF contrasts with recent work of Schiller (2015), who finds that firms with low average tax rates are safer and have lower expected returns. In unreported regressions we find that the coefficient on average tax rate when added to our regressions is negative but that the significance of TLCF and ITC are little changed.

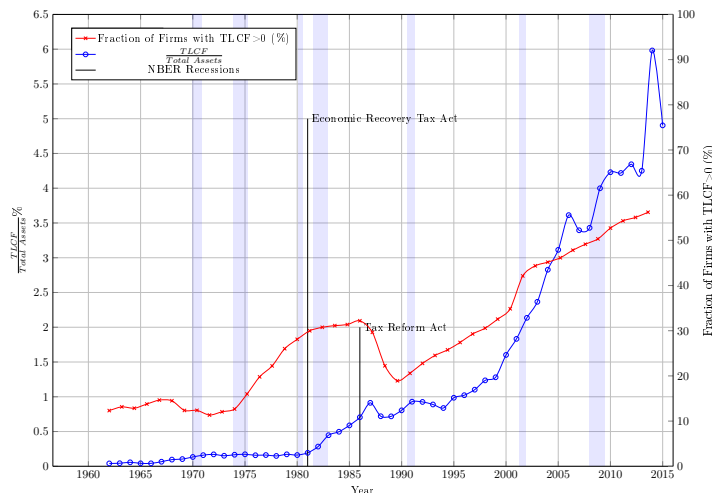


Figure 1: Tax Loss Carry Forward

on equity risk and return. We know of no other study that has directly looked at the relationship between TLCF and equity returns. Some have, however, indirectly looked at this relationship. Lev and Nissim (2004) consider the ratio of tax to book income as a measure of the quality of accounting information. They show that this ratio, which reflects tax deductions such as TLCF, forecasts firm growth but is not significant in forecasting returns.

The remainder of the paper is organized as follows. We present a theoretical analysis of the relationship of TLCF and risk in Section 2. In this section the main intuition of our analysis is illustrated in a simple Binomial model where we show that risk is non monotonic in TLCF. Section 3 provides empirical evidence on the TLC/risk relationship that is strongly supportive of the simple theory. We numerically explore a more realistic and complex setting in Section 4 which confirms the intuition conveyed in our simple model. Section 5 concludes the paper.

## 2 Simple Binomial Model

Consider an all equity firm that at  $t_0$  owns a future stochastic cash flow  $\Pi_1 \in \{\Pi^u, \Pi^d\}$ ,  $\Pi^u > \Pi^d$ . The corporate tax rate is  $\tau$ , and the firm is in possession of a non-cash tax deduction of  $\Phi$ .  $\Phi$  can be thought of as a tax-loss carry forward, depreciation, or any other non cash tax deduction<sup>4</sup>.

The value of the all equity firm at  $t_0$ ,  $V_E$ , is equal to the value of the expected pretax cash flows,  $V_\Pi$ , minus the value of expected taxes,  $V_T$ , i.e.

$$V_E = V_\Pi - V_T. \quad (1)$$

Accordingly, the risk of the equity,  $\beta_E$ , is given by

$$\beta_E = \frac{V_\Pi}{V_\Pi - V_T} \beta_\Pi - \frac{V_T}{V_\Pi - V_T} \beta_T, \quad (2)$$

where  $\beta_\Pi$  is the beta of the pre-tax cash flows and  $\beta_T$  is the beta of the tax payments.

Green and Talmor (1985) and Majd and Myers (1985) show that the expected tax payment is equivalent to a call option. The underlying asset is the tax payment with full tax offset,  $\tau\Pi$ , and the actual tax payments will be a call on this asset with an exercise price  $\tau\Phi$ , i.e. the tax payment will be

$$\max\{\tau(\Pi - \Phi), 0\}.$$

Since the firm is short the tax payment and, as we will show,  $\beta^T > 0$ , the risk of equity is lower than the risk of the pretax cash flows as long as  $\Phi > 0$ . Our theoretical contribution is to show that the risk reduction is non monotonic in  $\Phi$ . The relationship we will derive is graphically presented in Figure 2.

Three cases are apparent in Figure 2.: Case 1,  $0 \leq \Phi \leq \Pi^d$ ; Case 2,  $\Pi^d < \Phi < \Pi^u$ ; Case 3,  $\Phi \geq \Pi^u$ .

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<sup>4</sup>Investment tax credits (ITCs) would play a similar role.

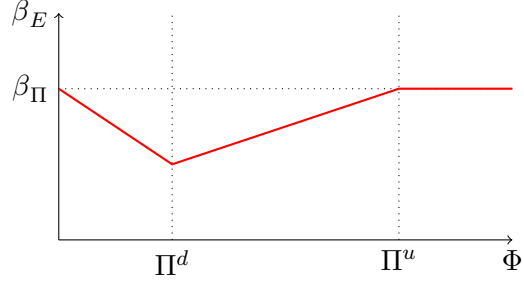


Figure 2: Firm Risk and Tax Loss Carry Forward

**2.0.1 Case 1:**  $0 \leq \Phi \leq \Pi^d$ . This case applies to firms that have taxable income but little or no tax deductions. As a result, the available tax shields  $\Phi$  are used with certainty making the tax savings risk-free. Hence, the value of the tax shield is

$$V_T = \tau V_\Pi - \tau V_\Phi. \quad (3)$$

Using (3) in the value of the equity claim (1) gives:

$$V_E = (1 - \tau)V_\Pi + \tau V_\Phi$$

In terms of the risk of the equity, the after tax cash flow and pretax cash flow have the same beta while the value of the tax shield from  $\Phi$  is riskless. That is, the firm has effectively sold an equity claim to the government but has received a risk free bond in return resulting in the following equity risk.

$$\beta_E = \frac{(1 - \tau)V_\Pi}{(1 - \tau)V_\Pi + \tau V_\Phi} \beta_\Pi. \quad (4)$$

As  $\Phi$  increases in this range, the value of the risk free bond,  $V_\Phi$ , increases and the overall equity risk decreases.



**2.0.2 Case 2:**  $\Pi^d \geq \Phi < \Pi^u$ . In this region the tax payment depends on the state.

$$T = \begin{cases} \tau (\Pi^u - \Phi) & \text{if } \Pi^u \\ 0 & \text{if } \Pi^d \end{cases} \quad (5)$$

The  $t_0$  value of the tax payment  $V^T$  is the value of the replicating portfolio, a levered long position in the underlying tax claim,  $\tau V^\Pi$ .

$$V^T = \Delta \tau V_\Pi - \Delta \tau \frac{\Pi^d}{(1 + r_f)},$$

where  $\Delta$  is

$$\Delta = \frac{\Pi^u - \Phi}{\Pi^u - \Pi^d} < 1. \quad (6)$$

Using (6) in (1) gives the equity value

$$V_E = (1 - \Delta \tau) V_\Pi + \frac{\Delta \tau \Pi^d}{(1 + r_f)}, \quad (7)$$

which implies that the firm risk will be

$$\beta_E = \frac{(1 - \Delta \tau) V_\Pi}{V_\Pi - V_\Phi} \beta_\Pi. \quad (8)$$

The tax deduction  $\Phi$  affects  $\beta_E$  through its impact on  $\Delta$  and  $V_E = V_\Pi - V_\Phi$ . The net result can be shown to be strictly increasing in  $\Phi$  in this range since

$$\frac{\partial \beta_E}{\partial \Phi} = \frac{\tau V_\Pi \beta_\Pi \Pi^d}{V_E^2 (\Pi^u - \Pi^d) (1 + r_f)} \quad (9)$$

is positive.

**2.0.3 Case 3:**  $\Phi \geq \Pi^u$ . Since deductions are larger than the maximum taxable income the firm will not pay taxes with certainty. Hence  $V_T = 0$  and

$$V_E = V_\Pi. \tag{10}$$

As a result,  $\beta_E = \beta_\Pi$  for any level of  $\Phi$  in this range.

This simple model demonstrates an important new insight to the literature. Prior studies have shown that firm risk is lower as a result of the asymmetric taxation of corporate earnings relative to losses. Essentially the government shares in the corporate losses by not collecting taxes when business is bad. To this we add an understanding of how this lower risk changes through the range of possible values of  $\Phi$  relative to taxable income. For low levels of  $\Phi$  risk is decreasing until  $\Pi^d$  at which point risk begins to increase up to a point where the firm pays no taxes after which firm risk is constant as  $\Phi$  increases.

In reality the relationship of risk with tax deductions is much more complex. A multi-period setting implies that tax deductions not used in one period can be carried forward. Tax-loss carry forwards compete with period deductions such as depreciation and interest as well as with investment tax credits. The tax loss carry-forward is made up of operating losses over various periods and each of these has a finite maturity. Insights from a more complete model are not analytically available but we do show that the relationship described in this section is evident in a more complete numerical model presented in section 4.

### 3 Empirical Results

In this section we present empirical evidence on the relationship between TLCF and equity returns. Table 1 reports summary statistics and correlations for some of the variables used in our analysis. In each period we compute each statistic for each firm, we then compute the equal weighted average, value weighted average, and standard deviation of the statistic for this period. We report the time-series average of each of these computations.

Our primary interest is with the relationship between TLCF and future equity returns,

Table 1: Summary statistics

Panel A: Summary statistics								
	$\frac{ME}{AvgME}$	$\frac{BE}{ME}$	$\frac{TLCF}{AT}$	$\frac{PROF}{AT}$	$\frac{EBITDA}{AT}$	$\frac{DEPR}{AT}$	$\frac{INT}{AT}$	$\frac{ITC}{AT}$
$E_{EW}[x]$	1.0000	0.8665	0.3217	0.0278	0.0938	0.0447	0.0208	0.0227
$E_{VW}[x]$	1.0000	0.6660	0.0339	0.1041	0.1627	0.0443	0.0198	0.0480
$\sigma[x]$	5.2724	1.9311	1.1712	0.2453	0.2113	0.0410	0.0233	0.0345
Panel B: Correlations								
	$\frac{BE}{ME}$	$\frac{TLCF}{AT}$	$\frac{PROF}{AT}$	$\frac{EBITDA}{AT}$	$\frac{DEPR}{AT}$	$\frac{INT}{AT}$	$\frac{ITC}{AT}$	
$\frac{ME}{AvgME}$	-0.0372	-0.0447	0.0560	0.0610	0.0076	-0.0085	0.1373	
$\frac{BE}{ME}$		-0.0909	-0.0872	-0.0778	0.0090	0.0747	0.0999	
$\frac{TLCF}{AT}$			-0.4931	-0.5083	0.1062	0.0973	-0.1423	
$\frac{PROF}{AT}$				0.9028	-0.1560	-0.2506	0.1172	
$\frac{EBITDA}{AT}$					0.0677	-0.1305	0.1429	
$\frac{DEPR}{AT}$						0.0967	0.1171	
$\frac{INT}{AT}$							0.0056	

volatility and betas. Our theory predicts a non-monotonic relationship between TLCF and beta. This implies a similar relationship with expected returns. Moreover, since the risk amplification is essentially due to option leverage a similar relationship should be present in volatility and other factor loadings.

We collected stock market data from CRSP and accounting data from Compustat. The sample includes firms observation from 1964 to 2014. Stock market data is measured at monthly frequency and accounting data at annual frequency. As in Fama and French (1993), we exclude firms in the financial sector, and firms with negative book equity and negative total assets.

For each firm-year observation in Compustat we computed the market betas, SMB betas, and HML betas, using Fama-MacBeth regressions. In some specifications we included the past stock return and past return volatility as controls. These controls were directly computed from CRSP's stock returns.

Table 3 reports the result of Fama MacBeth regressions of realized stock returns on past values of TLCF and various controls. Annual accounting variables are used to forecast the sum of log returns for the following 12 and 60 months separately. TLCF and ITC are standardized by total assets. Similar results were found (not reported) when we standardized TLCF by Size, book Asset, Book Debt plus size, and revenues.

TLCF enters significantly and positively in all models that predict future returns for both the 12 and 60 month horizons. The predictive power of TLCF is little changed when Size and Book to Market, both of which enter significantly, are also included. We note the relationship is monotonically positive, apparently inconsistent with the predictions of the basic model. This result may reflect the relatively large part of the sample that has TLCF=0 as these would be the riskier firms and this effect might swamp the risk reduction of relatively small levels of TLCF. Table 7 sheds light on this. Panel A of the table reports portfolio sorts where Portfolio 0 contains firms with TLCF=0 and portfolios 1, 2, and 3 contain equal numbers of firms with increasingly larger TLCF. We see the non monotonic relationship in the portfolio returns with a significant drop in return from Portfolio 0 to Portfolio 1. ITC, which is similar to TLCF is only significant in predicting 60 month returns but enters with a negative sign, indicating a risk reduction. To the extent that ITCs are more cash like (they substitute for cash in paying for taxes) they would not be subject to tax rate risk. In addition, ITCs would be safer if they were used before TLCF since that means the risk of their expiration is lower.

Table 4 reports the result of a similar exercise but where the dependent variable is the volatility of returns over 12 and 60 month horizons. We see again that TLCF enters positively and significantly for both the 12 and 60 month horizons. As with returns, size and book-to-market do not diminish the predictive power of TLCF. A significant difference from our return analysis is that volatility is strongly negatively related to ITC for both 12 and 60 month horizons. Moreover, the interaction between TLCF and ITC is positive and significant in predicting volatility.

The relationship between TLCF and future beta is not as clear in Table 5 as with returns and volatility. For a twelve month horizon the sign of the TLCF coefficient flips to be negative but over a 60 month horizon a weaker but positive relationship is evident. Moreover, the ITC and ITC interacted with TLCF are not significantly related to beta for the 12 month horizon but they are significant for the 60 month horizon.

Tables 6 and 7 examine SMB and HML betas. TLCF is positively and significantly related to SMB and HML betas for 60 month horizons but insignificantly positive for the

12 month horizon. ITC is significantly negatively related over both horizons while the interaction with TLCF is positive for both horizons but only significant over the 12 month horizon.

We also ran unreported regressions that included firm fixed effects, time fixed effects, and industry fixed effects. In all cases the tax loss carry forward coefficients have the same sign and significance as reported above, indicating that the forecasting power of the tax losses is not related to non observed firm, industry or time characteristics. Regressions including the Fama-French 5 Factor Model betas, and Hou-Xue-Zhang 4-factor q-factor model betas were also performed, but in all cases the reported results also hold.

Finally, a portfolio sort analysis was performed based on TLCF over Total Assets, and the firm's market equity (Table 8). In this analysis, all the firms with zero tax loss carry forwards were included in the first portfolio. All firms with positive tax losses are sorted into portfolios three portfolios, such that each portfolio contains 1/3 of positive tax losses.

In Panel B of Table 8 we report the measured alphas of the Fama and French 3 factors but replacing the size factor in the Fama and French 3-factor model by the tax factor, finding a positive and significant alpha for the adjusted model.

Overall, the empirical results lend support to the predictions of the model regarding TLCF for 60 month horizons while the support is slightly weaker for 12 month horizons. Our model does not provide separate predictions for ITC. Empirically, there is a strong negative relationship between ITCs and future return moments and a generally strong positive relationship between the interaction of ITC and TLCF and future return moments.

## 4 Numerical Model

While the empirical results support our simple model, the real world complexity of TLCFs suggests that other interactions may be at play and would be evident in a richer model. To examine this possibility we now numerically study a more realistic model.

Consider a multi-period, discrete time extension of our model. The firm owns capital  $K_t$  that produces EBITA of  $\Pi(K_t, A_t)$ , a function of the firm's capital and an exogenous

productivity shock  $A_t$ . The firm distributes all free cash flows to investors, hence dividends are equal to the pre-tax cash flow, minus its tax bill  $T_t$ , minus any capital expenditure costs that it incurs  $I_t$ , minus any capital adjustment costs  $\nu_t$ :

$$D_t = \Pi(K_t, A_t) - T_t - I_t - \nu_t$$

The firm makes no decisions and the firm's level of capital is fixed at  $K_t = 1$ . The firm pays a maintenance cost to replace depreciated capital, this cost is  $I_t = \delta^K K_t$ . The firm's value is equal to the present value of its dividends, discounted by an exogenously specified stochastic discount factor  $M_{t+1}$ .

The firm pays taxes at a rate  $\tau$  on taxable income  $\Pi(K_t, A_t)$  minus any tax shields  $\Phi_t$ . We also assume that the tax paid cannot be negative, thus the total tax paid is:

$$T_t = \tau \max(0, \Pi(K_t, A_t) - \Phi_t)$$

We assume that the firm has three types of tax-shields. First, non-depreciation and non-TLCF tax shields  $\Phi_t^0$ . The real world analog of  $\Phi_t^0$  are interest tax shields (although we abstract from financial leverage), R&D tax shields, and any other general tax-shields. Second, depreciation tax shields  $\Phi_t^\delta = \delta^K K_t$ . Third, tax-loss carry-forwards (TLCF)  $\Phi_t^{TLCF}$ , which will be described below. The firm's total tax shields are  $\Phi_t = \Phi_t^0 + \Phi_t^\delta + \Phi_t^{TLCF}$ .

We assume that the firm always uses as much TLCF as possible to reduce current tax liability. Define the firm's tax liability, before using the TLCF, as  $\tilde{T}_t = \Pi_t - \Phi^0 - \Phi_t^\delta$ . If  $\tilde{T}_t < 0$ , then the firm pays zero tax and no TLCF are used; furthermore, the stock of TLCF increases by  $-\tilde{T}_t$ . If  $0 < \tilde{T}_t < \Phi_t^{TLCF}$ , then TLCF fully reduce the firm's tax liability to zero, and the amount of TLCF remaining is  $\Phi_t^{TLCF} - \tilde{T}_t$ . If  $0 < \Phi_t^{TLCF} < \tilde{T}_t$ , then all of the TLCF are used and zero remain; in this case, the firm's tax liability is  $T_t = \tilde{T}_t - \Phi_t^{TLCF} > 0$ . We also assume that TLCF's expire at a rate  $\delta^\tau$  so that:

$$\Phi_{t+1}^{TLCF} = (1 - \delta^\tau) \max\left(0, \Phi_t^{TLCF} - (\Pi_t - \Phi^0 - \Phi_t^\delta)\right)$$

We can now formally describe the firm's value:

$$\begin{aligned}
V(A_t, \Phi_t^{TLCF}) &= D_t + E_t[M_{t+1}V(A_{t+1}, \Phi_{t+1}^{TLCF})] \text{ s.t.} \\
K_t &= 1 \\
D_t &= \Pi(A_t) - T_t - I_t - v_t \\
I_t &= \delta^K K_t \\
T_t &= \tau \max(0, \Pi(A_t) - (\Phi_t^0 + \Phi_t^\delta + \Phi_t^{TLCF})) \\
\Phi_t^\delta &= \delta^K K_t \\
\Phi_{t+1}^{TLCF} &= (1 - \delta^\tau) \max(0, \Phi_t^{TLCF} - (\Pi_t - \Phi_t^0 - \Phi_t^\delta))
\end{aligned} \tag{11}$$

This more realistic model preserves the basic insight of the simple binomial model. Figure 3 plots the expected return against the amount of TLCF implied by our numerical model when we restrict it to be a single period.<sup>5</sup> Note that if the firm has no pre-existing tax shields (solid line), then the expected return is non-monotonic in TLCF. The expected return first decreases, as additional tax shields imply a safe cash flow (tax refund) relative to a zero-tax shield firm. The expected return increases for high levels of TLCF because the TLCF will be used in the good state of the world, when cash flows are already high, but will be lost in the bad state of the world, when cash flows are low. On the other hand, when there are enough pre-existing tax shields (dashed line), then the expected return can be strictly increasing in TLCF.

#### 4.1 Calibrated multiperiod model

We assume that EBITDA is linear in a multiple of capital and productivity:  $\Pi(A_t) = \psi A_t K_t$  and we set  $K_t = 1$ .

The target moments, as well as some additional moments, for both model and data are presented in Panel A of Table 2. We first compute each moment, for each firm, using its time-series data. We then compute the average and median of each moment across all firms.

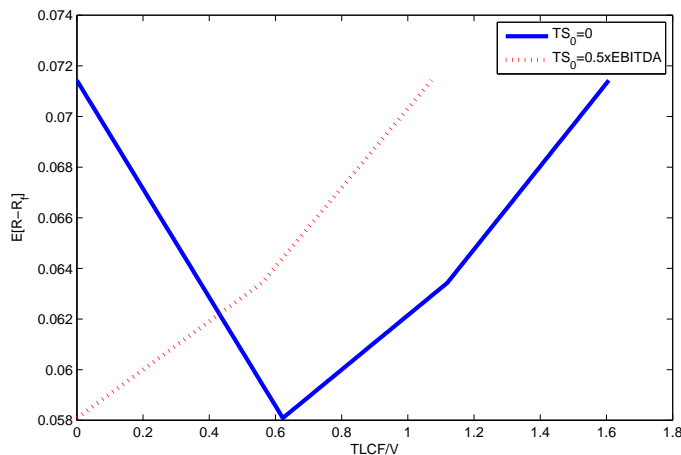
The productivity shock  $A_t = A_t^a A_t^i$  consists of an aggregate and an idiosyncratic com-

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<sup>5</sup>To create this figure, we assumed that there are three equally likely states. The stochastic discount factor is  $M_{t+1} = (1.2, 1.0, 0.8)$ , the pre-tax cash flow is  $\Pi(K_t, A_t) = (0.5, 1.0, 1.5)$ , and the tax rate is  $\tau = 0.3$ .

Figure 3: Expected return as a function of TLCF

This figure plots the expected return on the y-axis, against the amount of tax-loss carry forwards (TLCF) on the x-axis from the simple model. We compare a firm with no other tax shields (solid line) and existing tax shields (dashed line).



ponent, which are uncorrelated.  $A_t^a$  is a 3-state Markov chain with possible realizations (0.89, 1.00, 1.11) and an autocorrelation of 0.4.  $A_t^i$  is a 3-state Markov chain with possible realizations (0.40, 1.00, 1.60) and an autocorrelation of 0.75. We choose the volatilities of the aggregate and idiosyncratic components to match the volatilities of these components in the variation of the EBITDA-to-Total assets ratio.<sup>6</sup> We choose the persistence of the aggregate component to match the persistence of HP-filtered GDP. The persistence of the idiosyncratic component is somewhat higher than the analogous persistence in the data, 0.75 compared to 0.59. This is because the level of TLCF is too low relative to the data with a persistence of 0.59.<sup>7</sup>

We set  $\beta = 0.95$  and assume that the stochastic discount factor takes the form:  $M_{t+1} =$

<sup>6</sup>We use the EBITDA-to-Total assets ratio instead of just EBITDA because in the data EBITDA is non-stationary and takes on negative values, therefore we scale it by a non-negative, co-integrated series. Note that Total assets is slower moving than EBITDA, thus EBITDA-to-Total assets still captures the key variation in EBITDA.

<sup>7</sup>We separate the volatility of EBITDA-to-Total assets into aggregate and idiosyncratic components by the following procedure. We first regress it on HP-filtered GDP:  $\frac{EBITDA}{TotalAssets} = \gamma_0 + \gamma_{GDP}GDP + \epsilon$ . We then define the volatilities of the aggregate and idiosyncratic components, respectively, as  $\sigma(\gamma_{GDP}GDP)$  and  $\sigma(\epsilon)$ .



Table 2: Model results

This table reports results from the model. To compute the summary statistics in Panel A, we compute each statistic for each firm individually as a time-series average or standard deviation; we then report the average or median of each statistic across all firms. The reported statistics are: EBITDA as a share of total assets, depreciation, interest expenses, and TLCF all as a share of EBITDA, the volatility of the EBITDA to total assets ratio, the volatility of its systematic component, the volatility of its idiosyncratic component, the autocorrelation of the systematic component, the autocorrelation of the idiosyncratic component, the average excess stock return, and the volatility of the excess stock return. Panel B reports the results of Fama and MacBeth (1973) regressions of future realized stock returns on firm characteristics. The key characteristic in our results is the ratio of TLCF to total assets and each firm's size (market value) is used as a control. We report results for one period, and five period ahead returns. In Panels C and D we repeat the same exercise as in Panel B, but use volatility, and the asset's beta with the negative of the stochastic discount factor as variables to be explained.

Panel A: Model and data accounting moments

	$\frac{E}{TA}$	$\frac{DEPR}{E}$	$\frac{INT}{E}$	$\frac{TLCF}{E}$	$\sigma(\frac{E}{TA})$	$\sigma(\gamma_X X)$	$\sigma(\epsilon)$	$AC(X)$	$AC(\epsilon)$	$E[R^{i,e}]$	$\sigma[R^{i,e}]$
Data (Avg)	0.138	0.319	0.177	0.236	0.535	0.118	0.511	0.440	0.518	17.16	45.48
Data (Med)	0.140	0.298	0.133	0.087	0.427	0.072	0.407	0.440	0.588	16.92	42.16
Model	0.140	0.329	0.164	0.110	0.463	0.086	0.455	0.410	0.750	10.04	37.72

Panel B: TLMF and future return

	$k = 1y$		$k = 5y$	
$\frac{TLMF}{TA}$	0.0482	0.0024	0.1510	0.0057
$ME$		-0.0162		-0.0466

Panel C: TLMF and future volatility

	$k = 1y$		$k = 5y$	
$\frac{TLMF}{TA}$	0.2304	0.0343	0.1313	0.0212
$ME$		-0.0650		-0.0367

Panel D: TLMF and future beta

	$k = 1y$		$k = 5y$	
$\frac{TLMF}{TA}$	0.0483	0.0023	0.2171	0.0033
$ME$		-0.0146		-0.0759

$$\beta \left( \frac{A_{t+1}}{A_t} \right)^{-\gamma} \text{ where } \gamma = 5.$$

We set  $\psi = 0.14$  to match the average EBITDA-to-Total assets ratio,  $\delta^K = 0.046$  to match the average Depreciation-to-EBITDA ratio, and  $\Phi^0 = 0.023$  to match the average Interest-to-EBITDA ratio. We set the TLMF depreciation rate  $\delta^\tau = 0.05$  because the U.S. tax code allows a firm to keep TLMF for 20 years before they expire. Finally, as mentioned

above, we chose the persistence of the idiosyncratic shock to match the TLCF-to-EBITDA ratio.

## 5 Conclusion

This paper examines the implications of TLCF for equity return moments. Although it is known that the government's tax claim on the firm reduces a firm's risk, we add to this understanding by showing that the risk reduction is non monotonic. Risk decreases for low levels of TLCF but increasing as TLCF increases beyond a critical range.

Empirically, we show a clear relationship between TLCF and returns, volatility, and various betas. The relationship is generally positive for TLCF and negative for ITCs. This finding suggests that the ITCs may expire more quickly than TLCF and hence have less risk of redundancy.

Overall, our results suggest that TLCF and other tax management assets are important determinants of risk and return. A more complete understanding of the complex tax management task that firm's faces will be the subject of future research.

Table 3: TLCF and future return

This table reports the results of Fama MacBeth Fama and MacBeth (1973) regressions of future realized stock returns on firm characteristics. The key characteristic in our results is the ratio of TLCF to total assets. The controls are size, book-to-market, profitability, past market, SMB, and HML betas, past stock return, and past volatility. We use annual accounting variables from Compustat. Accounting variables in year  $t$  are used to forecast the sum of log monthly returns from January to December in year  $t + 1$  ( $k = 12$ ) or in years  $t + 1$  to  $t + 5$  ( $k = 60$ ). Backward looking variables are computed with the same  $k$  as the forward looking returns, i.e. when  $k = 12$ , then we use monthly returns in year  $t$  to compute the betas, average return, and volatility which are used to forecast returns for year  $t + 1$ .

	$k = 12$					$k = 60$							
$\frac{TLCF}{TA}$	0.06	0.05	0.07	0.07	0.08	0.07	0.22	0.18	0.23	0.21	0.16	0.17	0.14
t-stat	(2.47)	(2.23)	(2.95)	(2.96)	(3.49)	(3.33)	(3.03)	(2.66)	(3.32)	(3.27)	(3.34)	(3.66)	(3.32)
$\frac{ITC}{TA}$		-0.03				-0.04		-0.61					-0.19
t-stat		(-0.15)				(-0.36)		(-1.95)					(-0.81)
$\frac{ITC}{TA} \times \frac{TLCF}{TA}$		2.05				0.36		14.00					5.85
t-stat		(1.58)				(0.29)		(2.37)					(1.38)
$ME$			-0.01		-0.01	-0.01			-0.08			-0.05	-0.05
t-stat			(-0.72)		(-1.37)	(-1.47)			(-2.17)			(-1.83)	(-1.94)
$BE/ME$			0.06		0.05	0.05			0.16			0.12	0.11
t-stat			(5.25)		(4.92)	(4.57)			(8.70)			(7.37)	(6.73)
$PROF/ME$			-0.00		-0.00	-0.00			-0.01			-0.01	-0.01
t-stat			(-0.14)		(-0.61)	(-0.63)			(-2.31)			(-1.62)	(-1.72)
$INV/TA$			-0.13		-0.15	-0.14			-0.24			-0.24	-0.24
t-stat			(-1.88)		(-2.43)	(-2.36)			(-1.30)			(-1.49)	(-1.50)
$\beta_{MKT}^{t-k,t}$				-0.01	-0.01	-0.01				-0.03		-0.04	-0.04
t-stat				(-1.23)	(-1.58)	(-1.56)				(-1.62)		(-3.10)	(-3.13)
$\beta_{SMB}^{t-k,t}$				-0.00	-0.01	-0.01				0.02		0.00	0.00
t-stat				(-0.55)	(-0.87)	(-0.91)				(0.90)		(0.09)	(0.08)
$\beta_{HML}^{t-k,t}$				-0.00	-0.00	-0.00				0.00		-0.00	-0.00
t-stat				(-0.30)	(-0.43)	(-0.44)				(0.10)		(-0.15)	(-0.10)
$E[R_{t-k,t}]$					0.00	-0.00					-0.23	-0.18	-0.19
t-stat					(0.05)	(-0.05)					(-5.48)	(-5.26)	(-5.40)
$\sigma[R_{t-k,t}]$					-0.00	0.01					1.19	1.15	1.15
t-stat					(-0.03)	(0.10)					(3.66)	(3.94)	(4.15)
$R^2$	0.01	0.01	0.03	0.03	0.07	0.07	0.01	0.01	0.05	0.03	0.05	0.08	0.09

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Table 4: TLCF and future volatility

This table reports the results of Fama and MacBeth (1973) regressions of future realized stock return volatility on firm characteristics. The key characteristic in our results is the ratio of TLCF to total assets. The controls are size, book-to-market, profitability, past market, SMB, and HML betas, and HML betas, past stock return, and past volatility. We use annual accounting variables from Compustat. Accounting variables in year  $t$  are used to forecast the volatility of monthly returns from January to December in year  $t + 1$  ( $k = 12$ ) or in years  $t + 1$  to  $t + 5$  ( $k = 60$ ). Backward looking variables are computed with the same  $k$  as the forward looking returns, i.e. when  $k = 12$ , then we use monthly returns in year  $t$  to compute the betas, average return, and volatility which are used to forecast returns for year  $t + 1$ .

	$k = 12$					$k = 60$						
$\frac{TLCF}{TA}$	0.05	0.05	0.05	0.03	0.02	0.02	0.02	0.05	0.03	0.03	0.03	0.02
t-stat	(5.79)	(5.73)	(5.73)	(5.11)	(4.83)	(4.78)	(6.05)	(6.13)	(5.58)	(5.51)	(5.85)	(5.85)
$\frac{ITC}{TA}$	-0.46					-0.22	-0.51				-0.27	-0.27
t-stat	(-11.58)					(-9.92)	(-12.00)				(-10.72)	(-10.72)
$\frac{ITC}{TA} \times \frac{TLCF}{TA}$	1.45				0.60	0.60	2.16				1.18	1.18
t-stat	(4.68)				(3.32)	(3.32)	(4.19)				(3.22)	(3.22)
$ME$		-0.02			-0.01	-0.01		-0.02			-0.01	-0.01
t-stat		(-8.16)			(-7.96)	(-7.19)		(-8.78)			(-9.24)	(-8.37)
$BE/ME$		0.01			0.00	0.00		0.01			0.00	0.00
t-stat		(4.60)			(3.15)	(4.04)		(2.77)			(1.27)	(2.11)
$PROF/ME$		-0.00			-0.00	-0.00		-0.00			-0.00	-0.00
t-stat		(-5.08)			(-3.08)	(-3.09)		(-3.99)			(-2.90)	(-2.82)
$INV/TA$		0.02			0.01	0.02		0.02			0.01	0.03
t-stat		(1.75)			(0.91)	(3.13)		(2.57)			(1.86)	(5.16)
$\beta_{t-k,t}^{MKT}$			0.00		-0.00	-0.00		0.00			-0.00	-0.00
t-stat			(3.01)		(-4.50)	(-4.19)		(2.68)			(-4.56)	(-4.66)
$\beta_{t-k,t}^{SMB}$			0.00		-0.00	-0.00		0.01			0.00	0.00
t-stat			(1.46)		(-0.72)	(-0.76)		(3.11)			(0.31)	(0.22)
$\beta_{t-k,t}^{HML}$			-0.00		-0.00	-0.00		-0.00			-0.00	-0.00
t-stat			(-1.23)		(-1.15)	(-1.08)		(-1.14)			(-0.83)	(-0.72)
$E[R_{t-k,t}]$				-0.04	-0.04	-0.04					-0.04	-0.04
t-stat				(-11.54)	(-11.60)	(-11.57)					(-15.23)	(-15.85)
$\sigma[R_{t-k,t}]$				0.52	0.52	0.51					0.51	0.49
t-stat				(25.00)	(25.00)	(25.00)					(31.99)	(31.89)
$R^2$	0.08	0.11	0.11	0.13	0.30	0.33	0.08	0.15	0.32	0.35	0.35	0.36

Table 5: TLCHF and future market beta

This table reports the results of Fama and MacBeth (1973) regressions of future realized market beta on firm characteristics. The key characteristic in our results is the ratio of TLCHF to total assets. The controls are size, book-to-market, profitability, past market, SMB, and HML betas, past stock return, and past volatility. We use annual accounting variables from Compustat. Accounting variables in year  $t$  are used to forecast the market beta of monthly returns from January to December in year  $t + 1$  ( $k = 12$ ) or in years  $t + 1$  to  $t + 5$  ( $k = 60$ ). Backward looking variables are computed with the same  $k$  as the forward looking returns, i.e. when  $k = 12$ , then we use monthly returns in year  $t$  to compute the betas, average return, and volatility which are used to forecast returns for year  $t + 1$ .

	$k = 12$					$k = 60$								
$\frac{TLCHF}{TA}$	0.16	0.16	0.14	0.09	-0.00	0.02	0.04	0.19	0.15	0.16	0.15	0.08	0.07	0.05
t-stat	(3.99)	(3.86)	(3.33)	(2.43)	(-0.03)	(0.55)	(1.09)	(4.17)	(4.33)	(3.90)	(3.48)	(3.00)	(3.16)	(3.30)
$\frac{ITC}{TA}$		-1.77					-0.56		-1.67					-0.60
t-stat		(-4.06)					(-1.94)		(-5.00)					(-3.41)
$\frac{ITC}{TA} \times \frac{TLCHF}{TA}$		3.65					-0.10		13.83					10.29
t-stat		(0.75)					(-0.02)		(2.90)					(2.36)
$ME$			-0.07		-0.02					-0.06			-0.02	
t-stat			(-2.94)		(-1.32)		(-1.31)			(-3.64)			(-1.91)	(-1.84)
$BE/ME$			-0.10		-0.09		-0.09			-0.09			-0.08	-0.08
t-stat			(-3.89)		(-4.53)		(-4.47)			(-4.20)			(-4.57)	(-4.86)
$PROF/ME$			-0.01		-0.00		-0.00			-0.01			-0.00	-0.00
t-stat			(-1.35)		(-0.14)		(-0.13)			(-2.48)			(-1.39)	(-1.39)
$INV/TA$		0.06	(0.41)		0.10		0.16			0.06			0.09	0.12
t-stat					(0.85)		(1.52)			(0.74)			(1.22)	(1.84)
$\beta_{t-k,t}^{MKT}$				0.13			0.10				0.10		0.07	0.07
t-stat				(8.28)			(7.72)				(7.33)		(6.65)	(6.80)
$\beta_{t-k,t}^{SMB}$				0.04			0.03				0.04		0.02	0.02
t-stat				(2.04)			(2.51)				(3.22)		(3.27)	(3.20)
$\beta_{t-k,t}^{HML}$				-0.04		-0.03	-0.03				-0.04		-0.02	-0.02
t-stat				(-2.19)		(-2.43)	(-2.41)				(-3.06)		(-3.04)	(-3.09)
$E[R_{t-k,t}]$					-0.04		-0.06					-0.04	-0.07	-0.06
t-stat					(-0.78)		(-1.18)					(-1.27)	(-2.26)	(-2.19)
$\sigma[R_{t-k,t}]$					2.33		1.27					1.85	1.11	1.07
t-stat					(6.80)		(4.46)					(8.39)	(5.73)	(5.58)
$R^2$	0.01	0.01	0.02	0.05	0.06	0.09	0.09	0.01	0.02	0.04	0.09	0.09	0.14	0.14

Table 6: TLLCF and future SMB beta

This table reports the results of Fama and MacBeth (1973) regressions of future realized SMB beta on firm characteristics. The key characteristic in our results is the ratio of TLLCF to total assets. The controls are size, book-to-market, profitability, past market, SMB, and HML betas, past stock return, and past volatility. We use annual accounting variables from Compustat. Accounting variables in year  $t$  are used to forecast the SMB beta of monthly returns from January to December in year  $t + 1$  ( $k = 12$ ) or in years  $t + 1$  to  $t + 5$  ( $k = 60$ ). Backward looking variables are computed with the same  $k$  as the forward looking returns, i.e. when  $k = 12$ , then we use monthly returns in year  $t$  to compute the betas, average return, and volatility which are used to forecast returns for year  $t + 1$ .

	$k = 12$				$k = 60$			
$\frac{TLLCF}{TA}$	0.57	0.48	0.54	0.43	0.22	0.23	0.22	0.20
t-stat	(2.88)	(2.69)	(2.82)	(2.70)	(1.93)	(1.98)	(3.70)	(3.42)
$\frac{ITC}{TA}$		-5.30						-2.82
t-stat		(-9.40)						(-9.65)
$\frac{ITC}{TA} \times \frac{TLLCF}{TA}$		24.82						6.14
t-stat		(3.32)						(1.79)
$ME$			-0.37					-0.23
t-stat			(-7.15)					(-7.79)
$BE/ME$			0.03					-0.02
t-stat			(0.86)					(-1.14)
$PROF/ME$			-0.02					-0.01
t-stat			(-2.79)					(-1.86)
$INV/TA$			-0.33					-0.28
t-stat			(-1.43)					(-2.69)
$\beta_{MKT}^{t-k,t}$			0.09				0.03	0.04
t-stat			(2.44)				(2.01)	(2.42)
$\beta_{SMB}^{t-k,t}$			0.11				0.06	0.06
t-stat			(3.01)				(4.83)	(4.64)
$\beta_{HML}^{t-k,t}$			-0.03				-0.01	-0.01
t-stat			(-0.98)				(-1.14)	(-0.93)
$E[R_{t-k,t}]$					-0.32			-0.32
t-stat					(-3.79)			(-6.41)
$\sigma[R_{t-k,t}]$					5.11			4.79
t-stat					(10.55)			(12.05)
$R^2$	0.01	0.02	0.04	0.06	0.08	0.11	0.02	0.15
							0.10	0.18
								0.19

Table 7: TLCF and future HML beta

This table reports the results of Fama and MacBeth (1973) regressions of future realized HML beta on firm characteristics. The key characteristic in our results is the ratio of TLCF to total assets. The controls are size, book-to-market, profitability, past market, SMB, and HML betas, past stock return, and past volatility. We use annual accounting variables from Compustat. Accounting variables in year  $t$  are used to forecast the HML beta of monthly returns from January to December in year  $t + 1$  ( $k = 12$ ) or in years  $t + 1$  to  $t + 5$  ( $k = 60$ ). Backward looking variables are computed with the same  $k$  as the forward looking returns, i.e. when  $k = 12$ , then we use monthly returns in year  $t$  to compute the betas, average return, and volatility which are used to forecast returns for year  $t + 1$ .

	$k = 12$					$k = 60$							
$\frac{TLCF}{TA}$	-0.00	0.02	0.05	0.04	0.05	0.07	0.10	-0.03	-0.01	-0.03	-0.02	-0.01	0.03
t-stat	(-0.01)	(0.25)	(0.63)	(0.41)	(0.54)	(0.86)	(1.29)	(-0.58)	(-0.14)	(-0.78)	(-0.77)	(-0.60)	(0.79)
$\frac{ITC}{TA}$		2.86					1.84						1.52
t-stat		(4.11)					(5.00)						(4.61)
$\frac{ITC}{TA} \times \frac{TLCF}{TA}$		8.55					-2.81						-11.83
t-stat		(1.14)					(-0.38)						(-1.66)
$ME$			0.01			0.02	0.01					0.01	0.00
t-stat			(0.20)			(0.81)	(0.57)					(0.52)	(0.27)
$BE/ME$			0.40			0.33	0.31		0.29			0.24	0.23
t-stat			(9.24)			(7.53)	(7.22)		(13.67)			(10.56)	(11.18)
$PROF/ME$			0.00			0.00	0.00		0.00			0.01	0.01
t-stat			(0.21)			(0.46)	(0.30)		(0.56)			(1.66)	(1.66)
$INV/TA$			0.28			0.27	0.13		0.25			0.23	0.13
t-stat			(0.98)			(1.07)	(0.54)		(1.39)			(1.33)	(0.79)
$\beta_{t-k,t}^{MKT}$				-0.09		-0.07	-0.07			-0.06		-0.04	-0.04
t-stat				(-3.38)		(-2.34)	(-2.31)			(-3.16)		(-2.79)	(-2.73)
$\beta_{t-k,t}^{SMB}$				0.04		0.02	0.02		0.02			0.01	0.01
t-stat				(1.19)		(0.97)	(0.95)		(0.83)			(0.78)	(0.75)
$\beta_{t-k,t}^{HML}$				0.11		0.09	0.09		0.09			0.07	0.07
t-stat				(4.11)		(4.62)	(4.62)		(4.25)			(4.63)	(4.65)
$E[R_{t-k,t}]$					-0.27	-0.16	-0.16				-0.16	-0.08	-0.09
t-stat					(-3.09)	(-1.92)	(-2.04)				(-2.70)	(-1.46)	(-1.62)
$\sigma[R_{t-k,t}]$					-0.55	-0.21	-0.09				-0.58	-0.31	-0.18
t-stat					(-0.86)	(-0.38)	(-0.16)				(-1.17)	(-0.77)	(-0.48)
$R^2$	0.01	0.02	0.04	0.05	0.06	0.10	0.10	0.01	0.03	0.08	0.09	0.14	0.15



Table 8: TLCF factor

This table reports results using portfolio sorts based on TLCF/TA and ME. Stocks are sorted in the following way. For the univariate sort in Panel A, there are a total of four portfolios. Portfolio 0 contains all the firms with zero TLCF/TA; all other firms are sorted into portfolios 1,2, and 3 such that each portfolio contains 1/3 of positive TLCF/TA firms. For the double sort in Panel A, the breakpoints along the TLCF/SIZE dimension are formed exactly as in the univariate sort. At the same time, breakpoints along the SIZE dimension are formed independently of TLCF/TA breakpoints, so that 1/3 of all firms lies between each of the breakpoints. We then form 4x3=12 portfolios, with each containing all firms falling within the appropriate TLCF/TA and SIZE breakpoints. We compute a TLCF factor as univariate portfolio 4 minus portfolio 1. We regress the SMB factor, the TLCF factor, as well as the 25 ME and B/E double sorted portfolios, 10 profitability sorted portfolios, 10 investment sorted portfolios, 10 Earnings/Price sorted portfolios, and 49 industry portfolios provided on Ken French's website on the Fama and French 3-factor model. In the first row of the bottom panel, we report the alpha and t-statistic for the SMB and TLCF factors; for the portfolios, we report the root mean square error of the alphas, and of the t-statistics. In the second row of the bottom panel, we repeat exactly the same exercise but replace the ME factor in the Fama and French 3-factor model by the TLCF factor.

Panel A: Sort on TLCF

	Univariate sort			
	P0	P1	P2	P3
	1.33	1.19	1.36	1.63
	Bivariate sort			
	P0	P1	P2	P3
S1	1.64	1.58	1.73	1.90
S2	1.26	1.13	1.16	1.03
S3	1.15	0.96	0.96	1.29

Panel B:  $\alpha$  from two different 3-factor models

			<u>RMSE</u>				
	TLCF	SMB	FF25	PROF10	INV10	EP10	IND49
$\alpha_{FF3}$	-0.49		0.15	0.18	0.11	0.06	0.27
t-stat	1.17		1.77	2.25	1.50	0.85	1.47
$\alpha$		-0.03					
t-stat		-0.27					