The cost of stock trading to firms[*]

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Abstract

I study the effects of stock trading costs into a dynamic corporate finance model with financing frictions. When trading entails a cost (even small), the issuing firm needs to pay an illiquidity premium to shareholders, which increases the firm's cost of capital and the opportunity cost of cash. The illiquidity premium leads to a decrease in the firm's target cash, exacerbates financial constraints, increases default risk, and reduces firm value. This firm's response can feed back into trading costs and amplify their real effects. The model warns of some consequences of regulatory restrictions on trading in financial markets.

Keywords: Cash reserves, transaction costs, real effects of financial markets

JEL Classification Numbers: G32; G35

1 Introduction

Recently, there has been a reexamination of the determinants of corporate cash holdings. Benefit-based explanations motivated by fundamental (firm) characteristics have turned out to be unable to fully explain the patterns of cash reserves, which has led empirical studies to explore alternative explanatory variables (see Graham and Leary, 2015). Azar, Kagy, and Schmalz (2015) find that secular shifts in the cost of holding cash can explain secular trends in corporate cash holdings. Despite these empirical advancements, the theoretical literature takes the cost of cash as exogenous and has not, so far, provided a microfoundation. The current paper seeks to fill this gap. It does so by investigating how stock trading costs impact the policies of financially constrained firms.

Although secondary stock markets are exchanges where investors trade shares without the involvement of the issuing firm, a growing body of evidence documents that their characteristics and functioning *do* affect corporate outcomes.[1] Within this strand, several studies report that firms with more costly stocks are more constrained in their ability to raise fresh funds.[2] Other things being equal, precautionary concerns should prompt these firms to keep more cash than their more liquid peers. Yet, the available evidence shows the opposite: Firms with more costly stocks have smaller cash reserves (Nyborg and Wang, 2014; and Bakke, Jens, and Whited, 2012). The model presented in this paper can rationalize this evidence. The stark intuition is that by increasing a firm's cost of capital, trading costs make it more costly to keep cash inside the firm.

The paper makes three main contributions. First, it advances the theoretical cash holdings literature by focusing on a cost-based explanation that complements standard benefit-based explanations that rely on precautionary or transaction motives. Second, it analyzes how corporate policies respond to trading frictions, explaining documented facts and providing new testable predictions. Third, the paper delivers a framework to analyze

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[1]See Bond, Edmans, and Goldstein (2012) and Foucault, Pagano, and Röell (2014) for excellent surveys.

[2]E.g., Butler, Grullon, and Weston (2005); Stulz, Vagias, and VanDijk (2014); or Campello, Ribas, and Wang (2014).

the effects of policy measures that target stability in financial markets—which have been discussed since the 2007–2009 crisis—by focusing on the effects on the corporate sector.

To this end, I study a firm with assets in place that generate a stochastic flow of revenues. The firm is financially constrained in that it faces uncertainty regarding its ability to raise fresh funds, as in Hugonnier, Malamud, and Morellec (2015) or Bolton, Chen, and Wang (2013). The firm maximizes its value by deciding how much cash to retain, pay out, and raise from capital markets. As in the aforementioned contributions, the benefit from holding cash is driven by precautionary motivations. I depart from these contributions by relating the cost of cash to the cost of trading the firm's stocks.

To ease the analysis, I start by examining a limit case in which the cost of trading the stock is constant. When trading the stock is costly, I show that to invest in the stock, shareholders require an illiquidity premium to ex ante compensate for expected trading costs (as pioneered by Amihud and Mendelson, 1986). The illiquidity premium increases the firm's cost of capital and generates a wedge between the return on cash and the return required by the investors. That is, the illiquidity premium increases the opportunity cost of cash. The increase in the cost of cash calls for a decrease in the firm's target cash level and for a larger payout rate. As a result, financial constraints are exacerbated and the risk of inefficient liquidations increases. Furthermore, the illiquidity premium reduces the profitability of investment opportunities and leads to underinvestment at the firm level. As a result, firm value decreases.

In practice, shareholders' trading costs depend on the extent to which intermediaries provide immediacy in the market of the stock. When intermediaries face participation costs and competition, their liquidity provision in the market of a given stock (especially for those stocks that are less frequently traded) depends on the rents they can extract, which are related to the stock's attributes (e.g., market capitalization; see, among others, Anand, Irvine, Puckett, and Venkataraman, 2013). In this augmented version of the model, the illiquidity-driven drop in firm value reduces the expected rents to the intermediary sector and causes a decline in the participation of liquidity providers. As a result,

transacting the stock becomes more costly for shareholders. Firms then need to increase the illiquidity premium further, which leads to an additional decrease in firm value. A self-reinforcing relation ensues, which exacerbates the detrimental effects of trading costs on firm value.

The 2007–09 financial crisis (and the instability thereafter) has spurred regulatory proposals targeting financial markets, such as financial transaction taxes (FTT) or leverage constraints imposed on financial intermediaries. While academics and policymakers have focused on the effects on the efficiency and stability of financial markets (see, e.g., Adam, Beutel, Marcet, Merkel, 2015; or Colliard and Hoffman, 2016), this paper warns of some (unintended) effects on the corporate sector. It shows that FTT may exacerbate a firm's financial constraints and increase its liquidation risk. These effects should be stronger for those stocks that are fraught with larger trading costs. To limit these harmful effects, the model suggests that FTT should vary in the cross-section of stocks—i.e., be larger for cheaper stocks. Because cheaper stocks are also more traded, a stock-contingent FTT may optimize the amount raised through the tax. The model also suggests that intermediaries' leverage constraints can cause a larger-than-expected drop in participation because of their indirect effects on firm value.

Despite the secular decrease in trading costs, French (2008) reminds that "trading U.S. equities is not free" in recent years either.[3] Investors still face commissions, bid-ask spreads, and other costs for trading services. This is especially true for stocks that are traded less frequently. Appendix C takes a snapshot of stocks listed on the major exchanges (NYSE, AMEX, and NASDAQ) by quartiles of trading volume. In the highest quartile, trading volume is more than 200 times higher than in the lowest quartile, and bid-ask spreads are about one tenth than in the lowest quartile. The analysis provided in this paper accounts for this variation and suggests that the model predictions are quantitatively relevant for stocks traded on the major U.S. markets (which have relatively

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[3]French (2008) points out some factors that have contributed to this decrease in trading costs, such as the introduction of negotiated brokerage commissions in 1975, the development of electronic trading networks, the decimalization of stock prices in 2000 and 2001, and the U.S. Security and Exchange Commission's implementation of rules designed to increase market transparency and liquidity.

smaller trading costs) and increasingly larger across stocks sorted by decreasing trading volumes.[4] As documented by several contributions (see, e.g., Novy-Marx and Velikov, 2016; or Chordia, Roll, and Subrahmanyam, 2011), trading costs are larger for small caps. Because small caps are relatively young and more financially constrained firms, cash reserves are key for their survival and growth. This evidence exemplifies the economic relevance of the mechanism highlighted by the model.

This paper contributes to the literature studying the determinants of corporate cash reserves using dynamic models, including Décamps, Mariotti, Rochet, and Villeneuve (2011); Bolton, Chen, and Wang (2011, 2013); Anderson and Carverhill (2012); and Hugonnier, Malamud, and Morellec (henceforth HMM, 2015). These papers focus on a benefit-based explanation and show that financing frictions, such as costs or uncertainty in raising external funds, should increase a firm's propensity to keep precautionary reserves. Importantly, while these extant papers take the cost of holding cash as exogenous and impute it to a free-cash-flow problem, the current model endogenizes this cost via stock trading costs.

The paper also contributes to the literature studying the impact of stock-specific trading frictions on corporate outcomes. Butler, Grullon, and Weston (2005) find that stock market liquidity reduces equity issuance costs. Fang, Noe, and Tice (2009) show that firms with liquid stocks have better performance and operating profitability. Campello, Ribas, and Wang (2014) find that stock liquidity improves corporate profitability, investment, value, and productivity. Nyborg and Wang (2014) find a positive causal relation of stock liquidity on corporate cash holdings, which is robust to different measures of liquidity. Bakke, Jens, and Whited (2012) show that delisting (and the ensuing drop in liquidity) leads to a decrease in employment and, to a lower extent, investment and savings. Brogaard, Li, and Xia (2016) find that stock market liquidity reduces firms' bankruptcy risk. In this paper, I take a theoretical perspective and provide a rationale

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[4]E.g., stocks in the lower quartiles of trading volume of the major U.S. exchanges, as well as over-the-counter (OTC) stocks (e.g., traded on OTC Bulletin Board or OTC Link, which constitute roughly one-fifth of the stocks listed on the major markets; see Ang, Shtauber, and Tetlock, 2013).

for these findings as well as deliver new testable predictions.

Recently, there has been a surge of interest in the effects of bond market trading frictions on corporate outcomes. For instance, He and Xiong (2012) study how bond illiquidity affects shareholders' default decisions. Building on this effect, He and Milbradt (2014) find that there is a feedback effect from default decisions to bond illiquidity. Although bond markets are more illiquid on a relative scale, equity markets are not perfectly liquid either, as discussed earlier. This paper studies the effects of trading costs on corporate policies as well as the potential feedback effect that may arise for those stocks that are followed by fewer liquidity providers.

The paper proceeds as follows. Section 2 describes the model. Section 3 analyzes the effects of the costs of trading a firm' stock by assuming a constant trading cost. Section 4 relaxes this assumption and singles out the interconnection between firm value and trading costs. Section 5 examines some unintended effects of some regulatory proposals targeting the financial sector. Section 6 concludes. All the proofs, along with some motivating evidence, are in the Appendix.

2 The model

Time is continuous, and uncertainty is modeled by a probability space $(\Omega, \mathcal{F}, P)$ equipped with a filtration $(\mathcal{F}_t)_{t \geq 0}$. The filtration represents the information available at time t and satisfies the usual conditions (Protter, 1990). Agents are risk-neutral and discount cash flows at a constant rate, $\rho > 0$.

I first present the firm. I then describe the structure of transactions in the market of the stock. This modeling of secondary market transactions will be enriched in Section 4. Finally, I formulate the firm's optimization problem.

The firm I consider a firm owned by an atomless continuum of identical shareholders. The firm operates a set of assets in place that generate a continuous and stochastic flow

of revenues as long as the firm is in operation. The flow of revenues is modeled as an arithmetic Brownian motion, $(Y_t)_{t\geq 0}$, whose dynamics evolve as

$$dY_t = \mu dt + \sigma dZ_t. \tag{1}$$

In this equation, the parameters μ and σ are strictly positive and represent the mean and volatility of the firm's cash flows, respectively. $(Z_t)_{t\geq 0}$ is a standard Brownian motion that represents random shocks to corporate cash flows.

The cash flow process in equation (1) implies that the firm can make operating profits and losses. If capital supply was perfectly elastic at the correct price, as in the Modigliani-Miller benchmark, operating shortfalls could be covered by raising outside financing immediately and at no cost. In practice, firms often face financing frictions such as costs or uncertainty in their ability to raise funds. When this is the case, there is a benefit from retaining earnings in cash reserves.

I account for this realistic feature and assume that the firm's access to external financing is uncertain, as in HMM and Bolton, Chen, and Wang (2013). Specifically, I model capital supply uncertainty as HMM do and assume that it takes time to secure external financing. Upon searching, the firm raises new funds at the jump times of a Poisson process, $(N_t)_{t\geq 0}$, with intensity λ. That is, if the firm decides to raise outside funds, the expected financing lag is $1/\lambda$ periods. Bargaining between management and new investors over the terms of the issue simply results in a reduction of the effective rate of arrival of financing opportunities, as proved by HMM (Appendix A). I abstract from this feature, as it would only affect the interpretation of λ.

These assumptions regarding the firm's financing set are starker than they need to be. Financing frictions could also be modeled as costs of issuing securities. These costs could be time-varying, as in Bolton, Chen, and Wang (2013). In this case, the firm could find itself in one of two observable states of the world: a good state, in which external financing is accessible and affordable, or a bad state, in which external financing

is excessively expensive. The results in this environment would be qualitatively similar to those of the baseline model. I adopt the HMM setup only in the interest of parsimony and without loss of generality. Indeed, the focus of the paper is on the *cost* rather than the *benefit* of cash.

Besides cash reserves, firms often use credit lines to manage their liquidity needs (see Lins, Servaes, and Tufano, 2010). I extend the baseline model to credit line availability in Section 3.4. In this augmented environment, the model predictions are qualitatively the same. Moreover, the model setup can accommodate long-term debt. Following Bolton, Chen, and Wang (2014), the presence of long-term debt would reduce the cash flow drift by the coupon rate and trim shareholders' recovery in liquidation. This extension would not affect the model's main mechanism.

I denote by $(C_t)_{t \geq 0}$ the firm's cash reserves at any $t \geq 0$. Cash reserves earn a constant rate, $r \leq \rho$, inside the firm. Whenever $r < \rho$, keeping cash entails an opportunity cost. This cost can be interpreted as a free-cash flow problem à la Jensen (1986) or as tax disadvantages (see Graham, 2000). In contrast with extant cash holdings models—in which the strict inequality $r < \rho$ is needed to depart from the corner solution featuring firms piling infinite cash reserves—I do not need to impose this inequality and allow for the $r = \rho$ case. The cash reserves process satisfies

$$dC_t = rC_t dt + \mu dt + \sigma dZ_t - dD_t + f_t dN_t.$$

In this equation, $dD_t \geq 0$ represents the instantaneous flow of payouts at time t. The process $(D_t)_{t \geq 0}$ is non-decreasing, reflecting shareholders' limited liability. $f_t \geq 0$ denotes the instantaneous inflow of funds when financing opportunities arise, in which case management stores the proceeds in the cash reserves. This assumption is consistent with the strong, positive correlation between equity issues and cash accumulation documented by McLean (2011) or Eisfeldt and Muir (2015). The cash reserves increase with external financing, retained earnings, and the interest earned on cash, whereas they decrease with

payouts and operating losses. The controls D and f are endogenously characterized.

Management can distribute cash and liquidate the firm's assets at any time. Nonetheless, liquidation is inefficient, as the cash flow that shareholders recover in liquidation, denoted by ℓ, is smaller than the firm's first best, μ/ρ. The cost of liquidation may be due to the low marketability of intangible assets or to fire sale prices related to unfavorable market conditions. These costs erode a fraction, $1 - \phi \in [0, 1]$, of the firm's first best, so the liquidation value is $\ell = \phi\mu/\rho$. Because liquidation is inefficient, management shuts down operations when operating losses cannot be covered by drawing funds from cash reserves or by raising fresh equity. The time of liquidation is thus

$$\tau = \inf\{t \geq 0 : C_t \leq 0\}, \tag{2}$$

which represents the first time that the cash buffer is depleted.

Secondary market transactions   The key departure from previous dynamic corporate finance models with financing frictions is the introduction of costs faced by the firm's shareholders when trading the stock. Shareholders are ex-ante identical and infinitely lived. Each of them has measure zero and cannot short-sell.

Shareholders are exposed to private liquidity shocks that reduce their willingness to hold the asset. Specifically, shareholders keep their claim to cash flows for a random period that ends upon the arrival of liquidity shocks. These shocks are independent across investors and are described by Poisson shocks with intensity $\delta > 0$. In aggregate, this means that the firm has a fixed fraction, δdt, of shocked investors over any time interval $[t, t + dt]$. The mass of non-liquidity-shocked investors on the sideline is larger than the mass of liquidity-shocked shareholders.[5]

Liquidity shocks are idiosyncratic and short-lived. They can be interpreted as take-

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[5]He and Milbradt (2014) make a similar assumption and provide conditions under which it holds. It is the case when there is an almost negligible recovery intensity from a liquidity shock (as in the current model, as investors recover from liquidity shocks by bearing the holding cost or by selling the asset; see below) and a very large mass of investors.

it-or-leave-it (outside) investment opportunities, unpredictable financing needs, or un-foreseeable changes in hedging needs. When liquidity shocks hit, shareholders can sell the stock to intermediaries who make a market for the stock. Intermediaries should be interpreted as liquidity providers that are actively ready to trade—e.g., specialists, bro-kers, and market makers. Alternatively, shocked shareholders also have outside options: keeping the stock or selling it on alternative trading venues. In this way, I account for frag-mentation in trading venues in equity markets and the competition thereof, e.g., among listing exchanges, crossing networks, and electronic communication networks (ECN).[6] To economize on notation, I denote by χ the cost of the best outside option (i.e., the least expensive) available to shocked investors. The parameter χ thus represents the cost of selling on alternative trading venues or the opportunity cost of keeping the stock. The course of action ensuing a liquidity shock is illustrated in Diagram 1.

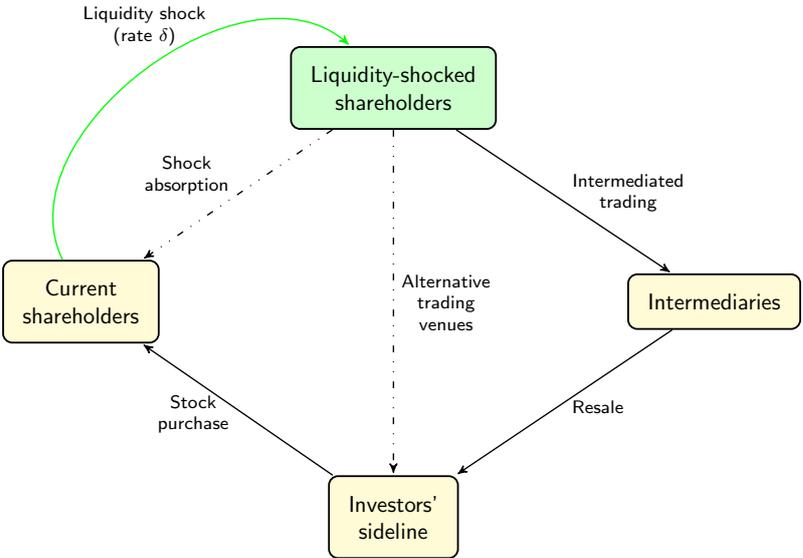DIAGRAM 1: The diagram shows the resolution of a liquidity shock—i.e., the course of action available to liquidity-shocked shareholders.

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[6]O'Hara and Ye (2011) report that, in 2008, off-exchange venues were executing almost 30% of all equity volume.

Constant presence in the market of the stock endows intermediaries with superior expertise in matching buy and sell orders. Intermediaries are active on the ask and bid sides of transactions. On the ask side, they trade with new investors on the sideline. These investors are not liquidity-shocked (i.e., they do not have an immediate need to trade) and, thus, are indifferent between staying out of the market or buying the stock at fair value, $\delta V(c)$. As a result, the intermediaries' gain on this side of the transaction is null.[7] On the bid side, intermediaries provide liquidity to shocked shareholders. These investors have a preference for liquidity, so they are willing to exploit the expertise of liquidity providers to save on the cost χ. While this expertise creates surplus for shocked shareholders, it enables intermediaries to extract rents from these shareholders.[8] Specifically, intermediaries extract a fixed surplus share, $\eta \in [0, 1]$, from shocked investors, which is motivated by the intermediaries' competitive advantage in providing liquidity. These trading costs pull together commissions—which are paid to the intermediary for accepting and routing the order as well as for clearing the trade—and trading losses—which arise because investors pay the bid-ask spread to suppliers of liquidity (see, e.g., Stoll (2006) for an excellent description of these trading costs).

This stark description of secondary market transactions deserves two remarks. First, the trading costs borne by selling investors are positive, whereas the trading costs of buying investors are zero. This modeling assumption is consistent with Brennan, Chordia, Subrahmanyam, and Tong (2010), who report that sell-order frictions are priced more strongly than buy-order ones. Second, while the modeling of the intermediaries sector is parsimonious, it is motivated by the goal (of the first part of the model) of understanding how shareholders' trading costs affect corporate policies. This crude description of the market for liquidity provision is enriched in Section 4, where participation costs and competition among liquidity providers are accounted for. These realistic features translate into time-varying intermediaries' participation as well as trading costs for

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[7]The measure of non-liquidity-shocked investors is larger than the measure of shocked shareholders, so Bertrand competition drives the surplus to intermediaries on this side of the transaction to zero.

[8]In the words of Stoll (2006), "trading remains a negotiation in which customers can find themselves at a disadvantage to dealers."

liquidity-shocked investors.

Corporate decision-making Throughout the paper, corporate management maximizes shareholder value. Namely, management solves

$$\sup_{(D,f)} \mathbb{E}\left[\int_0^\tau e^{-\rho^* t}\left(dD_t - f_t\, dN_t\right) + e^{-\rho^* \tau}\ell\right] \qquad (3)$$

by choosing cash retention, payout (D), and financing (f) policies. As is standard in dynamic cash management models, management maximizes the expected present value of net payouts plus the liquidation value of productive assets. Differently, I show that trading costs faced by firm shareholders lead to a change in the discount rate used by corporate management, which substantially affects corporate decision-making. This discount rate is denoted by $\rho^* \geq \rho$ and characterized in the following sections.

3 Trading costs and corporate policies

In this section, I start by studying the effects of constant trading costs on corporate policies. In this environment, I denote the value of the firm by $V(c)$.

3.1 Deriving firm value

A fraction δdt of shareholders is liquidity-shocked on any time interval. On aggregate, the reservation price of shocked shareholders for trading with the intermediaries is $\delta V(c)(1-\chi)$. Because intermediaries capture a fraction η of the surplus created, the transaction price of the aggregate claim of shocked shareholders is

$$\delta T(c) \equiv (1 - \chi\eta)\delta V(c) > \delta V(c)(1 - \chi). \qquad (4)$$

This price makes shareholders' outside options suboptimal. If outside options improve (lower χ) or intermediaries' rents decrease (lower η), the transaction price rises.

Consider the effect of trading costs on firm value. Management selects the firm's payout, retention, and financing policies to maximize firm value. The benefit of holding cash is decreasing in cash reserves, because precautionary needs are relaxed when cash reserves are large. The (opportunity) cost is the wedge between the return required by the investors and the return on cash. I conjecture and verify that there is a target cash level, C_V, above which it is optimal to distribute all excess cash to shareholders. Below C_V, management retains earnings in the cash reserves. By Itô's lemma, firm value satisfies the following ordinary differential equation (ODE):

$$\rho V(c) = (rc + \mu) V'(c) + \frac{\sigma^2}{2} V''(c) + \lambda \left[V(C_V) - V(c) - C_V + c \right] + \delta \left[T(c) - V(c) \right], \quad (5)$$

for any $c < C_V$. The left-hand side is the return that investors require to invest in the stock, irrespective of its liquidity. The first two terms on the right-hand side represent the effect of cash retention and cash flow volatility on firm value. The third term represents the effect of external financing on firm value. This term is the probability-weighted surplus accruing to incumbent shareholders when raising external funds. The last term reflects the effect of trading costs on firm value. Substituting the transaction price (4) into equation (5) gives

$$(\rho + \delta\chi\eta) V(c) = (rc + \mu) V'(c) + \frac{\sigma^2}{2} V''(c) + \lambda \left[V(C_V) - V(c) - C_V + c \right]. \quad (6)$$

The left-hand side of this equation reveals that shareholders' trading costs increase the firm's cost of capital up to $\rho^* = \rho + \delta\eta\chi$. The term $\delta\eta\chi \geq 0$ represents an illiquidiy premium, which is the ex-ante compensation that shareholders require to bear trading frictions (as in Amihud and Mendelson, 1986). Notably, the illiquidity premium increases the opportunity cost of cash from $\rho - r$ to $\rho + \delta\chi\eta - r$. In the case $r = \rho$, the illiquidity premium turns the opportunity cost from zero to positive and equal to $\delta\chi\eta$.

Firm value is solved subject to the following boundary conditions. Management stops operations when cash is exhausted after a series of negative operating shocks. Therefore,

the condition

$$V(0) = \ell$$

holds when the firm runs out of cash. Moreover, it is optimal to distribute cash reserves exceeding C_V with a specially designated dividend or a share repurchase. (As shown in Section 3.2, it is never optimal to buy back the shares of shocked investors when $c < C_V$.) Firm value is thus linear above C_V:

$$V(c) = V(C_V) + c - C_V \quad \forall c \in [C_V, \infty). \tag{7}$$

In this equation, $V(C_V)$ represents firm value at C_V (see Proposition 1 for closed-form expression of $V(C_V)$). Subtracting $V(c)$ from both sides of (7), dividing by $c - C_V$, and taking the limit as c tends to C_V gives

$$\lim_{c \uparrow C_V} V'(c) = 1. \tag{8}$$

That is, it is optimal to start paying out cash when the marginal value of one dollar inside the firm equals the value of a dollar paid out to shareholders. The target cash level that maximizes shareholder value is determined by the super-contact condition,

$$\lim_{c \uparrow C_V} V''(c) = 0; \tag{9}$$

see HMM. Solving for $V(c)$ leads to the following result.

Proposition 1 *When trading the stock is costly, firm value is given by*

$$V(c) = \begin{cases} \Pi_V(c) + \left[\ell - \Pi_V(0)\right] L_V(c) + \left[V(C_V) - \Pi_V(C_V)\right] H_V(c) & 0 \le c < C_V \\ c - C_V + V(C_V) & c \ge C_V \end{cases}$$

where the functions $L_V(c)$, $H_V(c)$, and $\Pi_V(c)$ are solved in closed form in Appendix A. Management liquidates the firm when cash reserves are depleted, raises cash reserves up

14

to the target C_V whenever financing opportunities arise, and pays out cash exceeding C_V. Firm value is concave for $c < C_V$ and, at the target level, satisfies

$$V(C_V) = \frac{rC_V + \mu}{\rho + \delta\chi\eta} .$$

(10)

Proposition 1 illustrates that when it is optimal to retain earnings ($0 \leq c < C_V$), firm value is the sum of three terms. The first term is the present value of the surplus created when financing opportunities arise. The second and third terms are the present value of payments to shareholders when cash reserves reach the liquidation threshold or the target cash level. The discount rate used to calculate these expressions embeds the illiquidity premium, which increases with shareholders' cost of trading. If there were no trading frictions ($\chi\eta = 0$), secondary market transactions would not impact corporate policies because shareholders could sell their claims costlessly.

3.2 Analyzing corporate policies

In this section, I investigate how the illiquidity premium affects corporate policies with respect to a counterfactual environment with no stock-specific trading costs.

Corporate cash holdings In a Modigliani-Miller world, cash reserves are irrelevant. A firm can raise funds whenever needed at any time and at no cost. In the presence of financing frictions (such as capital supply uncertainty or issuance costs), the benefit of cash stems from having a buffer to hedge against unexpected operating shocks, which provides the firm with operating flexibility. The benefit is decreasing in cash reserves, because the risk of forced liquidations or of incurring refinancing costs is relaxed when cash reserves are large. A firm's target level of cash reserves balances these benefits with the costs (see Figure 1). Previous dynamic cash management models have emphasized the benefit explanation while taking the cost as exogenous and constant.

<div style="text-align:center; border:1px solid; display:inline-block;">Insert Figure 1 Here</div>

If the marginal cost of cash were zero, a firm would have the incentive to pile infinite cash reserves. To rationalize the optimality of finite target cash levels, previous cash management models assume that the return on cash, r, is strictly lower than the discount rate, ρ. In these models, the opportunity cost of cash, $\rho - r$, is interpreted as a free cash flow cost à la Jensen (1986). This model's novel insight is that costs of trading the firm's stock generate a wedge between the return on cash and the return required by the investors; i.e., trading costs increase the opportunity cost of cash from $\rho - r$ to $\rho + \delta\eta\chi - r$. That is, trading costs shift the marginal cost curve upward, so the target cash level goes down (see Figure 1). This model then delivers finite target cash levels even when r and ρ coincide. In the next proposition, I denote by C^* the firm's target cash level absent trading costs (i.e., $\chi\eta = 0$) .

Proposition 2 *Capital supply uncertainty increases the firm's incentive to keep cash reserves; i.e., C_V (and C^*) decreases in λ. Costs of trading the firm's stock lead to an increase in the opportunity cost of cash and to a decrease in the target level; i.e., C_V decreases in η and χ. Then, the inequality $C_V \leq C^*$ holds.*

Proposition 2 highlights that, when making savings and payout decisions, management needs to balance precautionary concerns—driven by the firm's financial constraints—against the need to make the stock attractive to investors—driven by costs borne by shareholders when trading the stock. When trading is costly, management sets a smaller target level of cash reserves and increases the firm's payout rate. In so doing, it compensates ex ante the frictions that shareholders bear ex post when selling the stock.

The model then singles out a negative causal effect of trading costs on corporate cash reserves, which is consistent with Nyborg and Wang (2014) and Bakke, Jens, and Whited (2012). Remarkably, small caps—i.e., firms that are more financially constrained—are associated with larger trading costs (see, e.g., Novy-Marx and Velikov, 2016), so the illiquidity premium should be relatively larger and more harmful for these firms. By shifting the cost of cash upward, the model predicts that trading costs affect the availability of cash as a risk-management tool. This is detrimental for firm value, because risk

16

management improves financial flexibility and increases the value of constrained firms (see Bonaimé, Hanskin, and Harford, 2014; and Pérez-González and Yun, 2013). These aspects are investigated in the next paragraph.

Exacerbating financial constraints I now investigate the effects of stock trading costs on external financing decisions and on the firm's liquidation risk. Corollary 3 is a direct implication of Proposition 2.

Corollary 3 *Ceteris paribus, trading costs reduce the size of equity issues, as the inequality $C^* - c > C_V - c$ holds for any c.*

Corollary 3 illustrates that the reduction in target cash driven by the illiquidity premium leads to a decrease in the *size* of equity issues. This result is consistent with the evidence documenting a strong, positive correlation between equity issues and cash accumulation; see, e.g., McLean (2010), Eisfeldt and Muir (2015), and Warusawitharana and Whited (2015). To investigate the impact on the *frequency* of equity issues, I define the probability of raising external financing as

$$P^f(c, C_V) = E_c \left[1 - e^{-\lambda \tau(C_V)} \right].$$

Complementarily, I define the probability of foreclosing while searching for external funds as $P^l(c, C_V) = E_c \left[e^{-\lambda \tau(C_V)} \right]$. Trading costs enter these probabilities through $\tau(C_V)$, representing the first time that the cash process, reflected from above at C_V, is absorbed at zero. The following proposition studies these probabilities.

Proposition 4 *Trading costs decrease the probability of external financing, $P^f(c, C_V) < P^f(c, C^*)$, and increase the probability of liquidation, $P^l(c, C_V) > P^l(c, C^*)$.*

Note that stock-specific trading costs $(\chi \eta)$ do not change the parameter λ, which represents the frequency at which financing opportunities arise. Nonetheless, the model is able to capture the lower frequency of refinancing events displayed by less liquid firms

(see footnote 2 and the references therein). In fact, Proposition 4 highlights that trading costs affect the effective probability of exploiting these financing opportunities.

Overall, the model suggests that trading costs exacerbate firms' financial constraints. Specifically, it delivers three testable implications. First, trading costs lead firms to keep less precautionary cash. Second, they reduce the size of equity issues. Third, they increase the probability of inefficient liquidation, consistent with Brogaard, Li, and Xia (2016).

Providing liquidity via share repurchases   The analysis so far has shown that trading costs translate into an illiquidity premium component in the firm's cost of capital, which tightens the firm's financial constraints. A natural question that may arise is why the firm does not make a commitment to repurchasing the shares of shocked shareholders, thus acting as a liquidity provider. In so doing, the firm could decrease the illiquidity premium required by shareholders and, thus, its cost of capital.

Suppose that management follows this policy and repurchases the shares of shocked investors at their fair value. The firm would have a constant outflow equal to $\delta V(c)$ on any time interval. Firm value would satisfy

$$\rho V(c) = [rc + \mu - \delta V(c)] V'(c) + \frac{\sigma^2}{2} V''(c) + \lambda \left[V(C_V) - C_V + c - V(c) \right], \qquad (11)$$

subject to the same boundary conditions in Section 3.1. Equation (11) differs from (5), as there is no loss borne by liquidity-shocked investors as a result of the firm's commitment to buy back shares for $c < C_V$. However, comparing equation (11) with equation (6) shows that such a commitment would decrease firm value. In fact, the flow cost of committing to repurchasing shares of shocked shareholders, $\delta V(c) V'(c)$, is larger than the cost of investors' trading frictions on the firm's cost of capital, $\delta V(c) \chi \eta$, because the marginal value of cash is greater inside the firm than if paid out, for any $c < C_V$ (and so $V'(c) \geq 1 > \chi \eta > 0$; see Lemma 10 in Appendix A.1).

Alternatively, management could buy back shares at a price smaller than $\delta V(c)$. Yet this price cannot be smaller than $\delta T(c)$, otherwise shocked shareholders would choose to

trade with intermediaries. That is, management would have to buy back shares at a price at least equal to $\delta T(c)$. When following this policy, firm value would then satisfy

$$\rho V(c) = [rc + \mu - (1 - \eta\chi)\delta V(c)] \, V'(c) + \frac{\sigma^2}{2} V''(c) + \lambda \, [V(C_V) - C_V + c - V(c)]$$
$$+ (1 - \eta\chi - 1)\delta V(c), \tag{12}$$

where the last term on the right-hand side denotes the loss borne by shocked shareholders. A direct comparison of this equation with (6) highlights that this policy would decrease firm value too. In fact, the illiquidity premium paid by the firm, together with the flow cost of repurchasing shares, decreases firm value more than the illiquidity premium paid when following the policy in Section 3.1 (i.e., not committing to repurchase shares for $c < C_V$); i.e., the inequality $(1 - \eta\chi)\delta V(c)V'(c) + \eta\chi\delta V(c) > \eta\chi\delta V(c)$ holds. It is then easy to generalize that repurchasing shares at prices in the interval $[\delta T(c), \delta V(c)]$ is suboptimal whenever $c < C_V$.

These results imply that payouts in the form of share repurchases are suboptimal for levels of cash below the target, $c < C_V$. The reason is that cash is more valuable inside the firm ($V'(c) > 1$) than outside for any $c < C_V$. Instead of repurchasing shares for any c, the optimal policy in Section 3.1 implies that the firm provides liquidity to shareholders by reducing the target cash level with respect to the counterfactual with no trading costs ($C_V < C^*$ as from Proposition 2), meaning that the payout threshold can be hit more often.

Discouraging investment I next analyze the impact of the illiquidity premium on corporate investment. To keep the analysis tractable, I assume that the firm has monopolistic access to a growth option. Specifically, I assume that the firm can increase its income stream from dY_t to dY_t^+, defined as

$$dY_t^+ = dY_t + (\mu_+ - \mu)dt, \qquad \mu_+ > \mu,$$

by paying a lump sum $I > 0$. Using the results of Décamps and Villeneuve (2007) and HMM, I derive the zero-NPV cost—i.e., the maximum amount that the firm is willing to pay in order to exercise the option.

Proposition 5 *When trading the stock is costly, the zero-NPV cost is given by*

$$I_V = \frac{\mu_+ - \mu}{\rho + \delta\chi\eta} - (C_{V+} - C_V)\left[1 - \frac{r}{\rho + \delta\chi\eta}\right], \tag{13}$$

where C_{V+} denotes the target cash level after the growth option is exercised.

If instead trading the stock was costless, the zero-NPV cost would be

$$I^* = \frac{\mu_+ - \mu}{\rho} - (C_+^* - C^*)\left(1 - \frac{r}{\rho}\right), \tag{14}$$

with C_+^* denoting the post-investment target cash level. Comparing (13) and (14) highlights that trading costs decrease the investment reservation price. If the investment cost lies in the interval $[I_V, I^*]$, the growth option has negative NPV in the presence of trading costs but positive NPV if trading the stock is costless. That is, trading costs can lead to underinvestment. The severity of the underinvestment problem can be approximated as follows:[9]

$$\Delta I_V = I_V - I^* \approx -\frac{\mu^+ - \mu}{\rho}\frac{\delta\chi\eta}{\rho + \delta\chi\eta} < 0. \tag{15}$$

Because trading costs increase the firm's cost of capital, the growth option is less profitable and the zero-NPV cost decreases. The gap between I_V and I^* increases with η and χ; that is, the underinvestment problem is more severe when transacting the security is more costly. The model also suggests that a decrease in shareholders' horizon (a decrease in $1/\delta$) can effectively cause myopic decisions at the firm level if trading the stock is costly. In these cases, management foregoes profitable investment opportunities to favor payouts to current shareholders.

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[9]The difference between target cash levels (the second term in the expressions of I_V and I^*) plays a second-order effect, as in the model of HMM.

3.3 Quantitative analysis

In this section, I quantify the effects of shareholders' trading costs on corporate decisions. Table 1 reports the parameter values used in our numerical analysis. The parameters representing the firm's characteristics are the same as in HMM. In the comparative statics, I vary the magnitude of trading costs while keeping firm characteristics constant.

<div align="center">

Insert Table 1 Here

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Table 2 reports the effects of a wide spectrum of trading costs—ranging from 5 to 50 basis points—on a firm's cash accumulation, probability of liquidation, and investment decisions. The table shows that even small trading costs can significantly affect corporate policies. As an example, a trading cost equal to 20 basis points (which is consistent with the estimates for U.S. equities reported by Stoll, 2006; French, 2008; and Hameed, Kang, Viswanathan, 2010) leads to a 1.7% decrease in the target cash level with respect to the benchmark with no trading costs. It also leads to a 1.8% decrease in the probability of liquidation.[10]

<div align="center">

Insert Table 2 Here

</div>

Two implications follow. First, firms with illiquid stocks are less financially resilient because they hold less cash to absorb losses. Second, for a given cumulative shock, liquidation becomes more likely for firms with illiquid stocks. If the stock was traded at no cost, a series of shocks reducing the cash buffer from $C^*/2$ to $C^*/3$ would increase the probability of liquidation from 3.2% to 10%. When trading the stock entails a 20 basis points cost, the reduction from $C_V/2$ to $C_V/3$ would be caused by a cumulative loss 1.7% smaller than in the costless environment, but the probability of liquidation at $C_V/3$ would be 6% larger than at $C^*/3$.

Suppose that the firm has access to a growth option that, if exercised, leads to a 20%

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[10]I follow the same procedure as HMM and calculate the average probability of liquidation for a cross-section of firms with cash reserves uniformly distributed between 0 and C_V.

increase in operating profitability (i.e., $\mu_+ = 1.2\,\mu$). A 20 basis points trading cost leads to a 5% decrease in the zero-NPV cost (i.e., in the maximum cost that the firm is willing to spend to exercise the option). The larger the cost of trading the firm's stock, the smaller the firm's investment opportunity set. The reason is that the illiquidity premium erodes the profitability of investment opportunities by increasing the return required by shareholders.

The quantitative analysis in Table 2 illustrates that the effects of the illiquidity premium are increasing in the magnitude of trading costs. Other things being equal, a doubling of trading costs increases the effects on corporate policies by less than double. This means that while the model's predictions are quantitatively larger for more expensive stocks, the effects are also relevant for stocks traded with small bid-ask spreads and commissions, like those listed in the major U.S. equity markets. As mentioned, extensive literature (see, e.g., Novy-Marx and Velikov, 2016, or Brennan et al., 2010) reports that more expensive stocks are smaller caps. Because these firms are more constrained, cash reserves are extremely important for them. Thus, this evidence supports the relevance of the economic mechanism highlighted in the paper.

3.4 Bank credit as an alternative source of liquidity

The analysis so far has considered cash reserves as the only source of immediate financing available to firms. In practice, firms can also secure credit lines that may be used in times of need (see Sufi, 2009). I assess the model results in the presence of this additional source of liquidity. A credit line is a source of funding that the firm can access at any time up to a pre-established limit that I denote by L. Whenever the credit limit is finite ($L < \infty$), the firm has a positive demand for cash. As shown by Bolton, Chen, and Wang (2011), this is true for exogenous or endogenous (value-maximizing) L.[11]

I follow Bolton, Chen, and Wang (2011) by assuming that the firm pays a constant

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[11]Firms often face credit supply frictions that prevent them from taking the value-maximizing limit L. Endogenizing L is an interesting extension to understand the relation between stock liquidity and the firm's willingness to access bank credit, and I leave it for future research.

spread, β, over the risk-free rate on the amount of credit used. Because of this cost, it is optimal for the firm to tap the credit line only when cash reserves are exhausted. The firm then uses cash as the marginal source of financing if $c \in [0, C_V(L)]$ (the cash region), where $C_V(L)$ denotes the target cash level in this environment. Conversely, the firm draws funds from the credit line when $c \in [-L, 0]$ (the credit line region). Firm value satisfies (6) in the cash region, whereas it satisfies

$$(\rho + \delta\chi\eta) V(c) = [(\rho + \beta)c + \mu] V'(c) + \frac{\sigma^2}{2}V''(c) + \lambda [V(C_V) - V(c) - C_V + c] \quad (16)$$

in the credit-line region. On top of the smooth-pasting and super-contact conditions at $C_V(L)$ similar to (8) and (9), the system of ODEs (6)–(16) is solved subject to

$$V(-L) = \max [\ell - L, 0],$$

$$\lim_{c\uparrow 0} V(0) = \lim_{c\downarrow 0} V(0),$$

$$\lim_{c\uparrow 0} V'(0) = \lim_{c\downarrow 0} V'(0).$$

The first condition means that if $\ell \geq L$, the credit line is fully secured and shareholders are residual claimants in liquidation. The second and third conditions guarantee continuity and smoothness at the point where the cash and the credit line regions are pasted.

Insert Figure 2 Here

Figure 2 compares the impact of stock trading costs when a firm does and does not have access to bank credit. I use the same parametrization as before and additionally set $L = 0.05$ and $\beta = 1.5\%$ (in line with Sufi, 2009). The figure shows that the effects of trading costs on corporate policies are similar irrespective of the firm's access to bank credit. Access to credit relaxes the precautionary need to keep cash and leads to a decrease in the target cash level. However, trading costs reduce this target level below the counterfactual benchmark with no trading costs (in which the target cash level is

driven by precautionary motivations only). The probability of liquidation and investment decisions are almost the same irrespective of the firm's access to credit lines.

4 Liquidity provision, trading costs, and the firm

In this section, I introduce costly participation and competition in the market for liquidity provision and endogenize intermediaries' participation in the market of the stock. Specifically, I derive the measure of intermediaries actively following the stock. In Appendix B.4, I develop an alternative setup in which I endogenize the rents that intermediaries extract from shocked shareholders. In both cases, the trading costs faced by shareholders are time-varying and depend on intermediaries' participation, as is the firm's discount rate.[12] In this augmented setup, I denote firm value by $S(c)$.

4.1 Participation and firm value

Intermediaries acting as liquidity providers operate in a perfectly competitive environment. An intermediary pays a flow cost, γ, as long as it is active in the market of the stock. This participation fee pulls together the cost of monitoring market movements and handling the orders as well as the cost of intermediaries' funding.

Intermediaries are an infinite and atomless mass. Because of participation costs, however, only a finite measure, θ_t, is active.[13] The measure of intermediaries in the market of the stock determines the probability with which shocked shareholders trade with these liquidity providers rather than opting for their best outside option (keeping the asset or choosing alternative trading venues). I define this probability as

$$\pi_t \equiv \frac{\theta_t}{\alpha + \theta_t}, \tag{17}$$

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[12]Among others, Comerton-Forde et al. (2010), and Hameed, Kang, and Viswanathan (2010) report important constraints in the market-making sector even for U.S. equities, potentially translating into severe trading frictions for investors.

[13]In this context, see also Huang and Wang (2010).

where $\alpha > 0$ captures inefficiencies that reduce this probability (e.g., rigidities in trading protocols or technological deficiencies). This specification of π_t implies that shocked shareholders never opt for their outside option if θ_t tends to infinity, as in the environment analyzed in Section 3. This specification captures the notion of competition by order flow, as the probability with which intermediaries contact investors, $\frac{\pi_t}{\theta_t}$, decreases with θ_t.

Competition in the market for liquidity provision means that the active measure of intermediaries in the market of the stock is determined by the zero-profit condition:

$$\frac{\pi(\theta)}{\theta}\,\chi\eta\,\delta S(c) - \gamma = \; 0. \tag{18}$$

In this equation, the first term is the expected rent to an active intermediary. This term is the probability-weighted cost associated with the transaction price (4), and the second term is the cost of being active on the market. Plugging (17) into (18) gives

$$\theta(c) = \left(\frac{\delta\chi\eta}{\gamma}S(c) - \alpha\right)^+ \quad \text{and} \quad \pi(c) = \;\left(1 - \frac{\alpha\gamma}{\delta\chi\eta S(c)}\right)^+. \tag{19}$$

The measure of active intermediaries and the ensuing probability must be non-negative, which implies that intermediaries participate in the market for the stock if the value of equity is larger than a critical value defined by

$$\underline{S} \; = \; \frac{\alpha\gamma}{\chi\delta\eta}.$$

This critical value increases with the frictions faced by intermediaries, γ and α, and decreases with intermediaries' competitive advantage in providing liquidity (which allows them to extract more rents, η, from shocked shareholders), with the cost of investors' outside options (χ), or with liquidity demand (δ).

The active measure of liquidity providers affects the cost of shareholders' trading. This cost, in turn, affects firm value. Similar to Section 3, we conjecture the existence of a target cash level that I denote by C_S in this environment. The following result holds.

Proposition 6 *There is at most one threshold $\underline{C} \in [0, C_S]$ such that $S(\underline{C}) = \underline{S}$. For any $c > \underline{C}$, the measure of active intermediaries $\theta(c)$ (and the probability $\pi(c)$) is positive, non-decreasing, and concave in c.*

Proposition 6 highlights that competition and costly participation imply that the measure of active intermediaries varies with firm value. The model can depict the patterns of stocks characterized by trading costs of different magnitude. Liquid (exchange-listed) stocks are followed by a large number of intermediaries. For these stocks, $\theta(c)$ and $\pi(c)$ are positive and relatively large for any c (i.e., $\underline{C} \notin [0, C_S]$), and trading costs are small. Shareholders of more costly stocks (or, even more so, of OTC stocks) have fewer intermediaries they can trade with. For these stocks, $\theta(c)$ and $\pi(c)$ are relatively smaller, and the threshold $\underline{C}$ may lie in the interval $[0, C_S]$. In the following, I derive firm value in the general case $\underline{C} \in [0, C_S]$. The cases $\underline{C} \notin [0, C_S]$ are easily derived from this general case and, for completeness, are reported in Appendix B.2.

For any $c < \underline{C}$, shocked shareholders tap their best outside option (opt for alternative trading systems or keep the stock) and bear the cost $\delta \chi S(c)$ on aggregate. Firm value satisfies the following ODE:

$$(\rho + \delta\chi) S(c) = (rc + \mu) S'(c) + \frac{\sigma^2}{2} S''(c) + \lambda [S(C_S) - C_S + c - S(c)]. \qquad (20)$$

This equation admits an interpretation analogous to (6), but the return required by the investors (the left-hand side) is larger and equal to $\rho + \delta\chi$. The reason is that declining liquidity provision in the market of the stock increases trading costs to shocked shareholders, and so increases the illiquidity premium and the firm's cost of capital.

For any $c > \underline{C}$, the expected loss associated with a liquidity shock is

$$\left[\pi(c)\chi\eta\,\delta S(c) + \big(1 - \pi(c)\big)\chi\delta S(c)\right] dt = \chi\delta S(c)\left[1 - \pi(c)(1 - \eta)\right] dt.$$

The first term on the left-hand side represents the probability-weighted loss when selling the stock to an active intermediary. The second term represents the probability-weighted

loss associated with the best outside option. For any $c \geq \underline{C}$, firm value satisfies

$$(\rho + \delta\chi\eta)\, S(c) = (rc + \mu)\, S'(c) + \frac{\sigma^2}{2} S''(c) + \lambda\, [S(C_S) - C_S + c - S(c)] - \frac{\alpha\gamma(1-\eta)}{\eta}. \quad (21)$$

Equation (21) shows that the return required by the investors (the left-hand side) is the same as equation (6). Yet, the last term on the right-hand side reveals that intermediaries' participation frictions matter to shareholders. These frictions hamper liquidity provision in the market of the stock and so increase the expected trading costs incurred by shocked shareholders. This term is akin to a flow cost, which is larger if intermediaries' participation frictions, α and γ, increase.

Continuity and smoothness at $\underline{C}$ mean that the system of equations (20)–(21) is solved subject to the following conditions:

$$\lim_{c \uparrow \underline{C}} S(c) = \lim_{c \downarrow \underline{C}} S(c) \quad \text{and} \quad \lim_{c \uparrow \underline{C}} S'(c) = \lim_{c \downarrow \underline{C}} S'(c).$$

In addition, $S(c)$ satisfies

$$S(0) - \ell = \lim_{c \uparrow C_S} S'(c) - 1 = 0$$

at the liquidation threshold and at the target cash level, similar to Section 3.1. Lastly, the target cash level is identified by the super-contact condition, $\lim_{c \uparrow C_S} S''(c) = 0$. Solving for $S(c)$ leads to the following result.

Proposition 7 *When liquidity provision in the market of the firm's stock is endogenous and $\underline{C} \in (0, C_S)$, firm value satisfies*

$$S(c) = \begin{cases} \Pi_d(c) + \left[\ell - \Pi_d(0)\right] L_d(c) + \left[\underline{S} - \Pi_d(\underline{C})\right] H_d(c) & 0 \leq c < \underline{C} \\ \Pi_u(c) + \left[\underline{S} - \Pi_u(\underline{C})\right] L_u(c) + \left[S(C_S) - \Pi_u(C_S)\right] H_u(c) & \underline{C} \leq c < C_S \\ c - C_S + S(C_S) & c \geq C_S \end{cases}$$

The functions $L_i(c)$, $H_i(c)$, and $\Pi_i(c)$, $i \in \{d, u\}$, are defined in Appendix B, and firm

value at the target cash level is given by

$$S(C_S) = \frac{rC_S + \mu - \alpha\gamma(1-\eta)\eta^{-1}}{\rho + \delta\chi\eta}.$$

Insert Figure 3 Here

Figure 3 provides an illustration of the relation between firm value and liquidity provision in the market of the stock. Firm value depends on the discount factor used by corporate management. This discount factor reflects trading costs borne by shareholders when trading, which in turn depend on liquidity provision in the market of the stock. In the next section, I investigate the implications of this result.

4.2 Implications

Self-reinforcing effects. The analysis in Section 3 has shown that the costs of trading the firm's stock increase the return required by shareholders by an illiquidity premium. This premium constrains corporate policies—in particular, it leads to a decrease in target cash reserves and an increase in the firm's payout rate. As a result, the firm's financial constraints worsen and firm value decreases. When intermediaries face competition and participation costs, this illiquidity-driven drop in firm value shrinks the expected rents to intermediaries and affects their participation constraint via the free-entry condition. A self-reinforcing relation arises, as follows.

As the expected flow of rents to the intermediary sector decreases, some intermediaries stay away from the market of the stock. Liquidity provision becomes scarcer, and shareholders' expected cost of trading rises. The illiquidity premium increases, and so does the opportunity cost of cash. As a result, the target cash level decreases, financial constraints worsen, and firm value declines. This extra decrease in firm value feeds back again into the participation constraint of intermediaries, and so on. A self-reinforcing relation arises, which delivers a target cash level (C_S) that is suboptimal from a precautionary
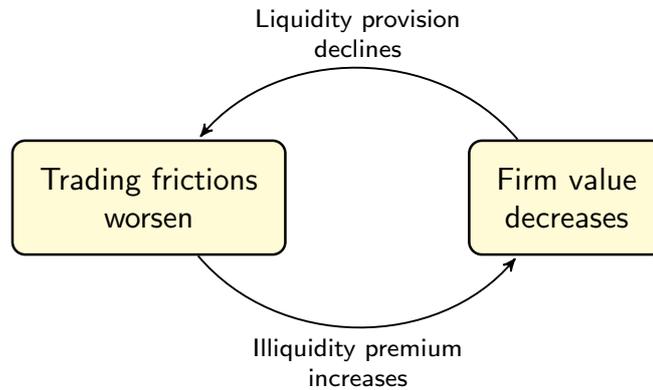
28

DIAGRAM 2: Self-reinforcing effects.

perspective and lowers firm value.

Proposition 8 *Frictions in the participation of liquidity providers increase investors'*
expected cost of trading and the illiquidity premium, which leads to an increase in the
opportunity cost of cash and to a decrease in the target cash level. Thus, the inequalities
$C_S \leq C_V \leq C^*$ *hold.*

In the next section, I illustrate the implications of this self-reinforcing relation.

The firm response to imperfect liquidity provision. The self-reinforcing relation
between participation and firm value magnifies the impact of trading costs on corporate
policies described in Section 3.2. Figure 4 shows the target cash level, the average proba-
bility of liquidation, and the zero-NPV cost in different environments: i.e., when trading
the stock is costless (solid line), when trading costs are constant (as described in Section
3.1; dashed line), and when trading costs depend on liquidity provision (as described in
Section 4.1; dotted line).

Insert Figure 4 Here

29

Figure 4 shows that the target cash level is lowest when considering the dynamic relation between trading costs and firm value via the illiquidity premium, in line with Proposition 8. The wedge between C_S and C^* (and between C_S and C_V) is larger when participation frictions are more severe. To counteract scarcer liquidity provision and larger costs of trading the stock, the firm needs to increase the illiquidity premium paid to shareholders. The opportunity cost of cash increases even further, so the firm's target cash level decreases. Yet this decrease comes at the cost of being more exposed to inefficient liquidations. In each graph of Figure 4, the dotted line becomes flat at the critical level of $\bar{\gamma}$ and $\bar{\alpha}$ where the inequality $\underline{S} \geq S(C_S)$ binds. For any $\gamma > \bar{\gamma}$ and $\alpha > \bar{\alpha}$, intermediaries stay away from the market of the stock for any c. Hence, an increase in α and γ above these critical levels has no effect on corporate decisions.

Endogenous participation can also exacerbate the underinvestment problem in Proposition 5. If shareholders always opt for their outside options, the firm promises the illiquidity premium $\delta\chi$ to shareholders. Thus, the zero-NPV cost is $I_S = \frac{\mu_+ - \mu}{\rho + \delta\chi} - (C_{S+} - C_S)\left[1 - \frac{r}{\rho + \delta\chi}\right]$.[14] The underinvestment problem worsens and is approximated by

$$\Delta I_S \approx -\frac{\mu^+ - \mu}{\rho} \frac{\delta\chi}{\rho + \delta\chi} < \Delta I_V < 0,$$

where $\Delta I_S = I_S - I^*$, and ΔI_V is defined in (15). However, investment may attract liquidity providers by increasing firm profitability. Indeed, liquidity providers may participate in the market for the stock *after* the firm has invested (corresponding to the interval in which the dotted line becomes steeper; i.e., $\bar{\alpha} < \alpha < \bar{\alpha}^+$ and $\bar{\gamma} < \gamma < \bar{\gamma}^+$, where $\bar{\gamma}^+$ and $\bar{\alpha}^+$ bind $\underline{S} = S(C^+)$).[15]

Insert Figure 5 Here

Overall, endogenous participation exacerbates the detrimental effect of trading costs

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[14]I derive this expression by using the same arguments as in Proposition 5.

[15]Note that, for $\alpha < \bar{\alpha}$ and $\gamma < \bar{\gamma}$ (corresponding to the interval where the dotted line gets flatter and closer to the dashed line), the severity of the underinvestment problem is close to ΔI_V because the illiquidity premium at the target cash level is $\delta\chi\eta$ both before and after the exercise of the growth option, and the difference in the target cash levels plays a second-order role.

on firm value (see Figure 5). Using the baseline parametrization in Table 1, a constant 20 basis points trading cost leads to a 4.8% drop in firm value at the target cash level (i.e., as assumed in Section 3). This loss is 3.4% larger when allowing for the dynamic adjustment of liquidity provision.

5 Regulatory proposals targeting financial markets: effects on firm value

This paper illustrates that costs borne by investors and liquidity providers in equity markets affect corporate outcomes. This relation is relevant in light of the recent regulatory proposals aimed at restoring and maintaining financial market stability after the 2007–2009 financial crisis. While abstracting from the direct effects on stock market volatility and short-term speculation—which have been the main focus of extant studies on this topic—this model can help to warn of some indirect, unintended effects on the corporate sector. Specifically, I discuss some effects of: (1) financial transaction taxes (FTT) (i.e., "Tobin tax"); and (2) leverage constraints imposed to financial intermediaries.

Financial transaction taxes (FTT).   The European commission has recently considered the introduction of a proportional FTT on round-trip transactions. France introduced a 0.20% FTT in 2012, followed by Italy introducing a 0.10% tax in 2013. Practically, the FTT is a surcharge on trading costs borne by the investors. The main goal of FTT is to prevent short-term speculation and limit volatility in financial markets.

The economic mechanism presented in this paper suggests that if investors face increased costs of trading equities, the issuing firms ultimately need to recognize a larger illiquidity premium to shareholders. A larger illiquidity premium strengthens the detrimental effects of stock trading costs on corporate outcomes. Consider the effects of a 0.20% FTT (like the French one) on two stocks with the same fundamental characteristics (as in Table 1) but different trading costs. I consider a stock A, which is associated

with a constant 20 basis points trading cost, and a stock B, associated with a constant 55 basis points trading cost. Absent FTT, trading costs imply a 6.1% increase in the probability of liquidation for stock A and a 17.2% increase for stock B (vis-à-vis a benchmark environment with no trading costs). A 0.20% FTT would increase the probability of liquidation by 12.6% for stock A and 23.6% for stock B (again, vis-à-vis the cost-free benchmark).

Notice that the numerical example assumes that the firm's fundamental characteristics are the same for the two stocks. As mentioned, however, the available evidence documents that stocks with larger trading costs are small caps and young firms. Small and young firms are less profitable, more volatile, and have more difficult access to the capital market. For these firms, cash reserves are vital, and the exacerbation of financial constraints driven by the illiquidity premium is more harmful. Thus, the reported numerical example is likely to understate the effect of FTT on default risk.

As mentioned, the analysis is silent on the desirability (the welfare gains) arising from this tax, in that it abstracts from a comprehensive general equilibrium analysis. Still, it seems useful to investigate the potential unintended effects on the corporate sector. The model suggests that if FTT are beneficial to warrant the stability of financial markets, it would be desirable to make them stock-contingent. That is, FTT should depend on a firm's market capitalization, which is negatively associated with the magnitude of stock trading costs (see Chordia, Roll, and Subrahmanyam, 2011). Imposing a greater tax rate on large caps might have two benefits: limiting the harmful effects of the illiquidity premium paid by small caps (as captured by the model) and increasing tax revenues (because large caps are more traded). While policy efforts tax only those firms with market capitalization above a given threshold (in France, above 1 billion euro), the model prompts a more articulated design such as a FTT increasing with firm's market capitalization.

Leverage constraints. The financial crisis also prompted regulators to tighten leverage constraints imposed to financial intermediaries. Leverage constraints are akin to margin

constraints (see also Buss, Dumas, Uppal, and Vilkov, 2013), which make participation more costly. In the model, an increase in participation costs is modeled as an increase in the parameter γ, which directly deters liquidity provision by making the free-entry condition binding with fewer intermediaries. This decrease in liquidity provision would be the only effect if firm value was independent of liquidity. When accounting for interdependence between firm value and participation, however, this decrease makes transactions, on average, more costly to shareholders. This is the channel through which shocks to liquidity providers hit corporations. Intermediaries' exits increase the illiquidity premium paid by firms, which decreases firm value. The free-entry condition becomes binding with fewer intermediaries. As a result, the decrease in the participation of intermediaries is amplified.

To disentangle and quantify the magnitude of the direct and indirect effects, I use a numerical example. Suppose that leverage constraints increase the intermediaries' participation costs by 8%. If this increase did not have bearings on shareholders' cost of trading, it would result in a 10.8% decrease in the measure of active intermediaries.[16] However, less liquidity provision increases the cost of trading to the investors. Thus, the firm needs to pay a larger illiquidity premium to shareholders, which eventually leads to a 11.3% decrease in the measure of active intermediaries. Clearly, these effects are most relevant for those stocks that are less frequently traded and followed by fewer intermediaries.

6   Concluding remarks

This paper investigates the effects of stock trading costs on the policies of the issuing firm. Trading costs increase the firm's cost of capital by an illiquidity premium. The illiquidity premium increases the firm's opportunity cost of cash, thereby decreasing the firm's target level of precautionary cash. While extant cash management models have mostly focused on the benefits of cash, this model shifts the attention to the cost. The model

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[16]Without loss of generality, I look again at the measure of intermediaries at the median cash level.

also illustrates that the illiquidity premium exacerbates financial constraints, increases default risk, deters investment, and decreases firm value. This decrease in firm value may reduce liquidity provision in the market of the stock, thus increasing trading cost and creating a self-reinforcing effect.

The paper supports the idea that the architecture of secondary market transactions is key to improving the efficiency of the corporate sector. Thus, it warns of some (unintended) real effects that restrictions on trading in financial markets may engender. Notice, however, that the analysis abstracts from asymmetric information or demand effects (i.e., informational cascades or panic selling). Although some trading patterns are surely fraught with asymmetric information, it seems useful to better understand the effects of stock illiquidity in the more parsimonious world of perfect information. By singling out a bare mechanism with risk-neutral agents and perfect information, the model opens the way to a number of levers that may be deepened by future research.

Appendices

A   Proof of the results in Section 3

A.1   Proof of Proposition 1

Throughout the Appendix, I define

$$\Phi \equiv \delta \chi \eta$$

to ease the notation. I consider the homogeneous ODE

$$(\rho + \lambda + \Phi) \, V(c) = V'(c) \, (rc + \mu) + \frac{\sigma^2}{2} V''(c) + \lambda \left[V(C_V) - C_V + c \right] \tag{22}$$

This equation is solvable in closed form by the change of variable $V(c) = g\left(-\frac{(rc+\mu)^2}{r\sigma^2} \right)$, that transforms the ODE into the following Kummer's equation with parameters

$$a = -\frac{\rho + \lambda + \Phi}{2r}, \quad b = \frac{1}{2}, \quad z = -\frac{(rc + \mu)^2}{r\sigma^2}.$$

The solution of the homogenous ODE can be found employing the two following linearly independent solutions (as in HMM),

$$F(c) = M(a, b, z) \quad \text{and} \quad G(c) = z^{1-b} M(1 + a - b, 2 - b, z)$$

where $M(.)$ is the confluent hypergeometric function. Therefore, the general solution takes the form

$$V(c) = a_1 F(c) + a_2 G(c) + \Pi_V(c) \tag{23}$$

where a_1 and a_2 are constants (derived in the following), while $\Pi_V(c)$ is the particular solution of the ODE. I conjecture $\Pi_V(c) = Bc + A$. By straightforward calculations, it follows that

$$\Pi_V(c) = \frac{\lambda}{\lambda + \rho + \Phi - r} c + \frac{\lambda}{\lambda + \rho + \Phi} \left[\frac{(r - \rho - \Phi)C_V + \mu}{\rho + \Phi} + \frac{\mu}{\rho + \lambda + \Phi - r} \right]. \tag{24}$$

Now, I turn to compute a_1 and a_2. I exploit the Abel's identity

$$F'(c)G(c) - F(c)G'(c) = -\frac{\sqrt{r}}{\sigma} e^{-(rc+\mu)^2 (r\sigma^2)^{-1}}.$$

Differentiating, I get

$$F''(c)G(c) - G''(c)F(c) = \frac{2}{\sigma^3} \sqrt{r} \, [rc + \mu] \, e^{-(rc+\mu)^2 (r\sigma^2)^{-1}}.$$

Finally, recalling that $F(c)$ and $G(c)$ are solution of the homogenous ODE, it follows that

$$F''(c)G'(c) - F'(c)G''(c) = \frac{2(\rho + \Phi + \lambda)}{\sigma^3}\sqrt{r}e^{-(rc+\mu)^2(r\sigma^2)^{-1}}$$

Using the three identity above, jointly with the smooth-pasting and super-contact conditions at C_V, I get by calculations

$$a_1 = -\frac{\sigma^3\ e^{(rC_V+\mu)^2(r\sigma^2)^{-1}}}{2\sqrt{r}(\rho + \lambda + \Phi)}\ \frac{\rho + \Phi - r}{\rho + \Phi + \lambda - r}\ G''(C_V),$$

$$a_2 = \frac{\sigma^3\ e^{(rC_V+\mu)^2(r\sigma^2)^{-1}}}{2\sqrt{r}(\rho + \lambda + \Phi)}\ \frac{\rho + \Phi - r}{\rho + \Phi + \lambda - r}\ F''(C_V).$$

Finally, to express the solution in the more intuitive guise involving the discount factors, I just need to define and solve the functions

$$L_V(c)\ =\ E\left[e^{-(\rho+\Phi+\lambda)\tau}1_{\tau \leq \tau_V}\right],$$

$$H_V(c)\ =\ E\left[e^{-(\rho+\Phi+\lambda)\tau_V}1_{\tau \geq \tau_V}\right],$$

where $\tau = \inf\{t \geq 0 : C_t \leq 0\}$, and $\tau_V = \inf\{t \geq 0 : C_t = C_V\}$. Then, the functions $L_V(c)$ and $H_V(c)$ denote respectively the first time that the cash reserve process hits zero or the payout boundary. The following result holds.

Lemma 9 *The functions $L_V(c)$ and $H_V(c)$ solve*

$$L_V(c)\ =\ \frac{G(C_V)F(c) - F(C_V)G(c)}{G(C_V)F(0) - F(C_V)G(0)},$$

$$H_V(c)\ =\ \frac{G(c)F(0) - F(c)G(0)}{G(C_V)F(0) - F(C_V)G(0)}$$

Proof. The functions $L_V(c)$ and $H_V(c)$ satisfy the homogeneous ODE associated to the dynamics of $V(c)$, so their general solution is $K_i\ F(c)\ +\ K_j\ G(c)$. In addition, they are solved subject to $L(0) = 1$, $L(C_V) = 0$, and $H(0) = 0$, $H(C_V) = 1$. Therefore, standard calculations deliver the result. ∎

I now prove that the value function $V(c)$ is increasing and concave for any $c < C_V$.

Lemma 10 $V'(c) > 1$ *and* $V''(c) < 0$ *for any* $c \in [0, C_V)$.

Proof. Simply differentiating equation (22), one gets

$$(\rho + \lambda + \Phi - r)\,V'(c) = V''(c)\,(rc + \mu) + \frac{\sigma^2}{2}V''(c) + \lambda.$$

By the conditions $V'(C_V) = 1$ and $V''(C_V) = 0$, it follows that $V'''(C_V) = \frac{2}{\sigma^2}(\rho + \Phi - r) > 0$ as $r < \rho$, meaning that there exists a left neighbourhood of C_V such that for any $c \in (C_V - \epsilon, C_V)$, with $\epsilon > 0$, the inequalities $V'(c) > 1$ and $V''(c) < 0$ hold. Toward a contradiction, I assume

that $V'(c) < 1$ for some $c \in [0, C_V - \epsilon]$. Then there exists a point $C_c \in [0, C_V - \epsilon]$ such that $V'(C_c) = 1$ and $V'(c) > 1$ over (C_c, C_V), so

$$V(C_V) - V(c) > C_V - c \tag{25}$$

for any $c \in (C_c, C_V)$. For any $c \in (C_c, C_V)$ it must be also that

$$V''(c) = \frac{2}{\sigma^2} \left\{ (\rho + \lambda + \Phi) V(c) - [rc + \mu]V'(c) - \lambda(V(C_V) + c - C_V) \right\}$$

Using (25), jointly with $V(C_V) = \frac{rC_V + \mu}{\rho + \Phi}$, it follows that

$$V''(c) < \frac{2}{\sigma^2} \left\{ (\rho + \Phi)(V(C_V) + c - C_V) - rc - \mu \right\} = \frac{2}{\sigma^2}(c - C_V)(\rho + \Phi - r) < 0.$$

This means that $V'(c)$ is decreasing for any $c \in (C_c, C_V)$, which contradicts $V'(C_c) = V'(C_V) = 1$. It follows that C_c cannot exists. So, $V'(c) > 1$ and $V''(c) < 0$ for any $c \in [0, C_V)$, and the claim follows. ∎

A.2 Proof of Proposition 2

In this section, I express the function $V(c)$ as a function of X, denoting the threshold satisfying $V'(X, X) - 1 = V''(X, X) = 0$. To prove the claim, I exploit the following auxiliary results.[17]

Lemma 11 *The function $V(c, X)$ is decreasing in the payout threshold X.*

    Proof. To prove the claim, I take $X_1 < X_2$, and I define the auxiliary function $k(c) = V(c, X_1) - V(c, X_2)$, that satisfies

$$(\rho + \Phi + \lambda)k(c) = (rc + \mu)k'(c) + 0.5\sigma^2 k''(c) + \lambda(X_1 - X_2)[r/(\rho + \Phi) - 1] \tag{26}$$

for any $c \in [0, X_1]$. By previous result and straightforward calculations, the function is positive at X_2 as $k(X_2) = (X_1 - X_2)[r/(\rho + \Phi) - 1] > 0$. By the definition of X_1 and X_2, the function $k(c)$ is decreasing and convex for $c \in [X_1, X_2)$. Therefore, $k(X_1) > 0$. Note that the function cannot have a negative local minimum on $[0, X_1]$ because the last term on the right hand side of (26) is positive. In addition, the function $k'(c)$ does not have neither a positive local maximum nor a negative local minimum, otherwise the equation $(\rho + \Phi + \lambda - r)k'(c) = (rc + \mu)k''(c) + 0.5\sigma^2 k'''(c)$ would not hold (respectively $k'(c) > 0 = k''(c) > k'''(c)$ and $k'(c) < 0 = k''(c) < k'''(c)$ at a positive maximum and at a negative minimum). As k is convex at X_1, this means that k' is increasing at X_1, and therefore it must be negative for any $c \in [0, X_1]$. Jointly with $k(X_1) > 0$, this means that $k(c) > 0$ for any $c \in [0, X_2]$. The claim follows. ∎

Lemma 12 *For a given payout threshold X and two given $\chi_1 > \chi_2$, $V(c, X, \chi_2) > V(c, X; \chi_1)$ for any $c \in [0, X]$.*

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[17]As specified in the main text, I set $\ell = \phi \frac{\lambda}{\lambda + 1} \frac{\mu}{\rho}$ to capture the idea that the liquidation value of the firm is a function of profitability and of supply frictions.

Proof. As in the previous section, I define $\Phi_i = \delta\chi_i\eta$ with $i = 1, 2$, and the auxiliary function $h(c) = V(c, X; \chi_2) - V(c, X; \chi_1)$. I need to prove that, for a given payout threshold X, $h(c) > 0$ for any $c \in [0, X]$. At X, the function is positive as

$$h(X) = (rX + \mu)\left(\frac{1}{\rho + \Phi_2} - \frac{1}{\rho + \Phi_1}\right) = (rX + \mu)\frac{\Phi_1 - \Phi_2}{(\rho + \Phi_1)(\rho + \Phi_2)} > 0,$$

because $\Phi_1 > \Phi_2$ as $\chi_1 > \chi_2$, and $h'(X) = h''(X) = 0$. In addition, the function evolves as

$$[rc + \mu]h'(c) + \frac{\sigma^2}{2}h''(c) - (\rho + \lambda + \Phi_2)h(c) + \lambda h(X) = (\Phi_2 - \Phi_1)V(c, X; \chi_1)$$

and the right hand side is negative. Differentiating, we have $[rc + \mu]h''(c) + \frac{\sigma^2}{2}h'''(c) - (\rho + \lambda + \Phi_2 - r)h'(c) = (\Phi_2 - \Phi_1)V_s'(c, X; \chi_1)$. At X, I get $\frac{\sigma^2}{2}h'''(X) = \Phi_2 - \Phi_1$, meaning that $h'''(X) < 0$. This means that the second derivative is decreasing in a neighbourhood of X, so one has $h''(c) > 0$ in a left neighbourhood of X. In turn, this means that $h'(c)$ is increasing in such a neighbourhood of X, then implying that $h'(c) < 0$ in a left neighbourhood of X. Now I need to prove that the function is decreasing for any c smaller that X. Note that, by the ODE above, $h'(c)$ cannot have a negative local minimum. As $h'(X) = 0$ and it is negative and increasing in a left neighbourhood of X, this means that $h'(c)$ should be negative for any $c < X$, so $h(c)$ is always decreasing. As it is positive at X, it means that it should be always positive, so $h(c) > h(X) > 0$ so it is positive for any $c < X$. $\blacksquare$

Exploiting the results above, I can prove the following lemma.

Lemma 13 *For any $\chi_1 > \chi_2$, $C_V(\chi_1) < C_V(\chi_2)$.*

Proof. The payout thresholds $C_V(\chi_1)$ and $C_V(\chi_2)$ are the unique solution to the boundary conditions $V(0, C_V(\chi_2); \chi_2) - \ell = 0 = V(0, C_V(\chi_1); \chi_1) - \ell$. Exploiting the result in Lemma 12, I now take, for instance, $X = C_V(\chi_1)$. It then follows that

$$V(0, C_V(\chi_1); \chi_2) - \ell > 0 = V(0, C_V(\chi_1); \chi_1) - \ell.$$

As V is decreasing in the payout threshold, this means that $C_V(\chi_1) < C_V(\chi_2)$ to get the equality $\ell - V(0, C_V(\chi_2); \chi_2) = 0$. The claim follows. $\blacksquare$

The next results stem from Lemma 13.

Corollary 14 *When trading the firm's stock is costly, the target cash level is lower than in the benchmark case with no trading costs, i.e. $C_V < C^*$.*

Corollary 15 *For any $\eta_1 > \eta_2$ and $\delta_1 > \delta_2$, $C_V(\delta_1) < C_V(\delta_2)$ and $C_V(\eta_1) < C_V(\eta_2)$.*

Finally, I prove the monotonicity of C_V with respect to the severity of primary market frictions, represented by the parameter λ.

Lemma 16 *C_V are monotone decreasing in λ.*

Proof. The monotonicity of C^* stems from Lemma B.10 in HMM. Thus, the result for C_V is a corollary of this Lemma, as the presence of stock illiquidity increases the required return from ρ to $\rho + \Phi$, preserving the monotonicity. $\blacksquare$

A.3 Proof of Proposition 4

I derive the results concerning the probability of liquidation $P_l(c, X)$, because the probability of external financing is just $P_f(c, X) = 1 - P_l(c, X)$. Using standard methods (see e.g., Dixit and Pyndick, 1994), the dynamics of $P_l(c, X)$ are given by

$$P_l'(c)(rc + \mu) + \frac{\sigma^2}{2} P_l''(c) - \lambda \, P_l(c) = 0$$

$$\text{s.t. } P_l(0) = 1 \tag{27}$$

$$P_l'(X) = 0, \tag{28}$$

where the first boundary condition is given by the definition of P_l, while the second boundary condition is due to reflection at the payout threshold.

Now I prove that the probability of liquidation is higher when the firm's stocks are illiquid. To do so, I first prove that the probabilities $P_l(c, C^*)$ and $P_l(c, C_V)$ are decreasing and convex in c. In the following, I employ the generic function $P_l(c, X) \equiv P_l(c)$.

Lemma 17 *The probability $P_l(c, X)$ is decreasing and convex for any $c \in [0, X]$.*

Proof. To prove the claim, I exploit arguments analogous to those of Lemma 10. As $P_l'(X) = 0$ and $P_l(X) \geq 0$, it must be that $P_l''(X) > 0$. Then, there exists a left neighbourhood of X, $[X - \epsilon, X]$ with $\epsilon > 0$, over which $P_l'(c) < 0$ and $P_l''(c) > 0$. Toward a contradiction, suppose that there exists some $c \in [0, X - \epsilon]$ where $P_l'(c) > 0$. Then, there should be a $\bar{C}$ such that $P_l'(\bar{C}) = 0$, while $P_l'(c) < 0$ for $c \in [\bar{C}, X]$. For any $c \in [\bar{C}, X]$ it must be that

$$P_l''(c) = \frac{2}{\sigma^2} \left[\lambda P_l(c) - P_l'(c)(rc + \mu) \right] > \frac{2}{\sigma^2} \lambda P_l(X) > 0.$$

Then, $P_l''(c) > 0$ for any $c \in [\bar{C}, X]$ means that $P_l'(c)$ is always increasing on $c \in [\bar{C}, X]$, contradicting $P_l'(\bar{C}) = P_l'(X) = 0$. The claim follows. ∎

Now I prove that $P_l(c, C_V) \geq P_l(c, C^*)$.

Lemma 18 *For any $\chi_1 > \chi_2$, $P_l(c, C_V(\chi_1)) \geq P_l(c, C_V(\chi_2))$.*

Proof. By Lemma 13, $C_V(\chi_1) < C_V(\chi_2)$. To ease the notation throughout the proof, I define $X_1 \equiv C_V(\chi_1)$ and $X_2 \equiv C_V(\chi_2)$. By Lemma 17, the functions $P_l(c, X_1)$ and $P_l(c, X_2)$ are positive, decreasing and convex over the interval of definition. I define the auxiliary function

$$h(c) = P_l(c, X_1) - P_l(c, X_2).$$

Note that $h(c)$ cannot have neither a positive local maximum ($h(c) > 0$, $h'(c) = 0$, $h''(c) < 0$) nor a negative local minimum ($h(c) < 0$, $h'(c) = 0$, $h''(c) > 0$) on $[0, X_1]$, as otherwise the equation $h''(c)\frac{\sigma^2}{2} + h'(c)[rc + \mu] - \lambda h(c) = 0$ would not hold. In addition, $h(0) = 0$, and $h'(X_1) = -P_l'(c, X_2) > 0$ because of the boundary conditions at zero and at X_1. This means that the function is null at the origin, and increasing at C_V. Toward a contradiction, assume that $h(X_1)$ is negative. This would imply the existence of a negative local minimum, given that the function is null at zero and it is increasing at X_1. This cannot be the case as argued above, contradicting that $h(X_1) < 0$. Therefore, the function must be always positive, and the claim follows. ∎

The result below is a straightforward consequence of Lemma 18 and the fact that, in the absence of trading costs, $\chi = 0$ or $\eta = 0$ (or $\delta = 0$).

Corollary 19 *When trading the firm's stock is costly, the probability of liquidation P_l is larger than in the benchmark case with no trading costs, i.e. $P_l(c, C^*) < P_l(c, C_V)$.*

These results can be extended for two parameters $\eta_1 > \eta_2$ or $\delta_1 > \delta_2$, as follows.

Corollary 20 *For any $\eta_1 > \eta_2$ or $\delta_1 > \delta_2$, $P_l(c, C_V(\eta_1)) \geq P_l(c, C_V(\eta_2))$ and $P_l(c, C_V(\delta_1)) \geq P_l(c, C_V(\delta_2))$.*

A.4   Proof of Proposition 5

I exploit the dynamic programming result in Décamps and Villeneuve (2007) and HMM, establishing that the growth option has a non-positive NPV if and only if $V(c) > V_+(c - I)$ for any $c \geq 0$, where I denote by $V_+(c - I)$ the value of the firm after investment. To prove the claim, I rely on the following lemma.

Lemma 21 *$V(c) \geq V_+(c - I)$ for any $c \geq I$ if and only if $I \geq I_V$, where I_V is defined as in Proposition 5.*

Proof. I define $\bar{c} = \max[C_V, I + C_{V+}]$. The inequality $V(c) \geq V_+(c - I)$ for $c > \bar{c}$ means that $c - C_V + V(C_V) \geq c - C_{V+} - I + V_+(C_{V+})$. Using the definition of I_V, the former inequality is equivalent to the inequality $I \geq I_V$, by straightforward calculations.

To prove the sufficient condition, I can just prove that $V(c) \geq V_+(c - I_V)$ for any $c \geq I_V$. I exploit the inequalities $C_V < C_{V+} + I_V$ and $\mu_+ - \mu - rI_V > 0$ (these inequalities stem from a slight modification of Lemma C.3 in HMM, so I omit the details). For $c \geq C_V$, the following inequality

$$V_+(c - I_V) \leq V_+(C_{V+}) + c - I_V - C_{V+} = c - C_V + V(C_V) = V(c)$$

holds. The first inequality is due to the concavity of V_+, the first equality is given by the definition of I_V, whereas the second equality is due to the linearity of V above C_V. I now need to prove the result for $c \in [I_V, C_V]$. To this end, I define the auxiliary function $u(c) = V(c) - V_+(c - I_V)$. The function $u(c)$ is positive at C_V as argued above, $u'(C_V) < 0$ and $u''(C_V) > 0$. On the interval of interest it satisfies

$$(\rho + \Phi + \lambda)u(c) = (rc + \mu)u'(c) + \frac{\sigma^2}{2}u''(c) + (\mu + rI_V - \mu_+)V_+'(c - I_V)$$
$$+ \lambda(V(C_V) - C_V - V_+(C_{V+}) + C_{V+} + I_V)$$

where the last term on the right hand side is zero by the definition of I_V, while the third term is negative. Then, the function cannot have a positive local maximum here, because otherwise $u(c) > 0$, $u''(c) < 0 = u'(c)$, and the ODE above would not hold. Jointly with the fact that $u(C_V)$ is positive, decreasing and convex means that the function is always decreasing on this interval. Then, $u(c)$ is also always positive, and the claim holds. ∎

B Proof of the results in Section 4.1

The proofs in this section are based on the general case in which $\underline{C} \in [0, C_S]$, according to which firm value satisfies the system of ODEs composed by (20) and (21). Note that the cases in which $\ell > \underline{S}$ (and therefore firm value satisfies (21) for any $c \in [0, C_S]$) or the $S(C_S) < \underline{S}$ (and therefore firm value satisfies (20) for any $c \in [0, C_S]$) are corner cases (as derived separately below).

B.1 Proof of Proposition 6

I start by showing that the value function is strictly monotone and concave over $0 \le c < C_S$.

Lemma 22 $S'(c) > 1$ and $S''(c) < 0$ for any $c \in [0, C_S)$.

    Proof. The first part of this proof follows the same arguments as Lemma 10. Accordingly, I differentiate equation (21) and get the following ODE

$$(\rho + \lambda + \Phi - r) S'(c) = S''(c)(rc + \mu) + \frac{\sigma^2}{2} S'''(c) + \lambda.$$

Jointly with the boundaries $S'(C_S) = 1$ and $S''(C_S) = 0$, this ODE implies that $S'''(C_S) > 0$, meaning that there exists a left neighbourhood of C_S such that for any $c \in (C_S - \epsilon, C_S)$, with $\epsilon > 0$, the inequalities $S'(c) > 1$ and $S''(c) < 0$ hold. Toward a contradiction, I assume that $S'(c) < 1$ for some $c \in [0, C_S - \epsilon]$. Then, there should be a point $C_c \in [0, C_S - \epsilon]$ such that $S'(C_c) = 1$ and $S'(Cc) > 1$ over (C_c, C_S), so $S(C_S) - S(c) > C_S - c$ for any $c \in (C_c, C_S)$. The point C_c could belong either in the interval $[0, \underline{C}]$ or in the interval $[\underline{C}, C_S]$. I now discriminate between these two cases. If $\underline{C} < C_c < C_S$, it must be that for any $c \in (C_c, C_S)$

$$S''(c) = \frac{2}{\sigma^2} \left\{ (\rho + \lambda + \Phi) S(c) - (rc + \mu)S'(c) + \frac{\alpha\gamma(1 - \eta)}{\eta} - \lambda[S(C_S) + c - C_S] \right\}$$

Using the fact that $S(C_S) - S(c) > C_S - c$, jointly with $S(C_S) = \frac{rC_S + \mu - \frac{\alpha\gamma(1-\eta)}{\eta}}{\rho + \Phi}$, it follows that

$$S''(c) < \frac{2}{\sigma^2} \left\{ (\rho + \Phi)(S(C_S) + c - C_S) - rc - \mu + \frac{\alpha\gamma(1 - \eta)}{\eta} \right\} = \frac{2}{\sigma^2}(c - C_S)(\rho + \Phi - r) < 0.$$

This means that $S'(c)$ is decreasing for any $c \in (C_c, C_S)$, which contradicts $S'(C_c) = S'(C_S) = 1$. So, $S'(c) > 1$ for any $c \in [\underline{C}, C_S)$, so such C_c does not exist on $[\underline{C}, C_S]$.

    I now consider the case $0 < C_c < \underline{C}$. Should such point C_c exist, the strict concavity of $S(c)$ over $[\underline{C}, C_S]$ means that there should be a maximum $C_m \in [C_c, \underline{C}]$ for the first derivative over the interval $(C_c, \underline{C})$, such that $S'(C_m) > 1$, $S''(C_m) = 0$ and $S'''(C_m) < 0$. Simply differentiating equation (20), I get that

$$S''(c)[rc + \mu] + S'''(c)\frac{\sigma^2}{2} - S'(c)(\rho + \delta\chi - r) + \lambda(1 - S'(c)) = 0.$$

Then, $S'''(C_m)\frac{\sigma^2}{2} = (\rho + \delta\chi - r)S'(C_m) + \lambda(S'(C_m) - 1) > 0$, which contradicts the existence of such a maximum C_m for $S'(c)$. It follows that C_c cannot exists, and the claim follows. ∎

The monotonicity of $S(c)$ implies that there exists at most one threshold $\underline{C}$ as established by Proposition 6. Moreover, the inequalities $\theta'(c) > 0 > \theta''(c)$ and $\pi'(c) > 0 > \pi''(c)$ on $c > \underline{C}$ follow by simply differentiating the functions $\theta(c)$ and $\pi(c)$ on $c \geq \underline{C}$, obtaining

$$\theta'(c) = \delta\eta\frac{\chi}{\gamma}S'(c) \geq 0, \qquad \theta''(c) = \delta\eta\frac{\chi}{\gamma}S''(c) \leq 0$$

$$\pi'(c) = \frac{\alpha\gamma}{\delta\chi\eta}\frac{S'(c)}{(S(c))^2} \geq 0, \qquad \pi''(c) = \frac{\alpha\gamma}{\delta\chi\eta}\frac{S(c)S''(c) - 2(S'(c))^2}{(S(c))^3} \leq 0$$

Therefore, the claim follows.

B.2   Proof of Proposition 7

The value function in the more general case with $\underline{C} \in [0, C_S]$ is defined over three intervals, namely $[0, \underline{C}]$, $[\underline{C}, C_S]$, and $[C_S, \infty]$.

On the interval $[0, \underline{C}]$, I define

$$a_d = -\frac{\rho + \lambda + \delta\chi}{2r}, \ b = \frac{1}{2}, \ z = -\frac{(rc+\mu)^2}{r\sigma^2}.$$

and the solutions of the homogeneous ODE is given by

$$F_d(c) = M(a_2, b, z) \quad \text{and} \quad G_d(c) = z^{1-b}M(1 + a_2 - b, 2 - b, z)$$

where $M(.)$ is the confluent hypergeometric function. The general solution on $[0, \underline{C}]$ is given by

$$w_1 F_d(c) + w_2 G_d(c) + \Pi_d(c)$$

with

$$\Pi_d(c) = \frac{\lambda}{\lambda + \rho + \delta\chi - r}c + \frac{\lambda}{\lambda + \rho + \delta\chi}\left[\frac{(r - \rho - \Phi)C_S + \mu - \frac{\alpha\gamma(1-\eta)}{\eta}}{\rho + \Phi} + \frac{\mu}{\rho + \lambda + \delta\chi - r}\right].$$

(29)

On the interval $[\underline{C}, C_S]$, the general solution is

$$w_3 F(c) + w_4 G(c) + \Pi_u(c)$$

where $F(c)$ and $G(c)$ are defined as in Appendix A.1, while the inhomogeneity $\Pi_u(c)$ is given by

$$\Pi_u(c) = \frac{\lambda}{\lambda + \rho + \Phi - r}c + \frac{\lambda}{\lambda + \rho + \Phi}\left[\frac{(r - \rho - \Phi)C_S + \mu}{\rho + \Phi} + \frac{\mu}{\rho + \lambda + \Phi - r}\right] - \frac{\alpha\gamma(1 - \eta)}{\eta(\rho + \Phi)}$$

Finally, on the interval $[C_S, \infty]$, the solution is linear and given by

$$S(C_S) + c - C_S.$$

The solution reported in the proposition exploits the following functions

$$L_d(c) = E\left[e^{-(\rho+\delta\chi+\lambda)\tau}1_{\tau\leq\bar{\tau}_S}\right], \qquad\qquad H_d(c) = E\left[e^{-(\rho+\delta\chi+\lambda)\bar{\tau}_S}1_{\tau\geq\bar{\tau}_S}\right]$$

$$L_u(c) = E\left[e^{-(\rho+\Phi+\lambda)\tau}1_{\bar{\tau}_S\leq\tau_S}\right], \qquad\qquad H_u(c) = E\left[e^{-(\rho+\Phi+\lambda)\tau_S}1_{\bar{\tau}_S\geq\tau_S}\right]$$

where τ is defined as in (2), while $\tau_S = \inf\{t\geq 0 : C_t = C_S\}$ and $\bar{\tau}_S = \inf\{t\geq 0 : C_t = \underline{C}\}$ are respectively the first time that the cash process hits the thresholds C_S and $\underline{C}$. Analogously to Lemma 9, we have the following results.

Lemma 23 *The functions* $L_d(c), L_u(c), H_d(c), H_u(c)$ *satisfy*

$$L_d(c) = \frac{G_d(\underline{C})F_d(c) - F_d(\underline{C})G_d(c)}{G_d(\underline{C})F_d(0) - F_d(\underline{C})G_d(0)}, \qquad H_d(c) = \frac{G_d(c)F_d(0) - F_d(c)G_d(0)}{G_d(\underline{C})F_d(0) - F_d(\underline{C})G_d(0)}$$

$$L_u(c) = \frac{G(C_S)F(c) - F(C_S)G(c)}{G(C_S)F(\underline{C}) - F(C_S)G(\underline{C})}, \qquad H_u(c) = \frac{G(c)F(0) - F(c)G(0)}{G(C_S)F(\underline{C}) - F(C_S)G(\underline{C})}$$

Proof. The result follows Lemma 9, so the proof is omitted. ■

As mentioned, two corner cases may arise. If $\ell > \underline{S}$, $\bar{C}\notin[0,C_S]$, firm value satisfies (21) for any $c\in[0,C_S]$. In this case, firm value is given by

$$\Pi_u(c) + \left[\ell - \Pi_u(0)\right]L_a(c) + \left[S(C_S) - \Pi_u(C_S)\right]H_a(c)$$

on $c\in[0,C_S]$ where, similarly to the previous cases, the function L_a (respectively H_a) denotes the first time that the cash process hits zero (the target cash level) before hitting the target cash level (zero). As before, $S(c) = S(C_S) - C_S + c$ for any $c > C_S$.

If instead $\underline{S} > S(C_S)$, firm value satisfies (20) for any $c\in[0,C_S]$. In this case, firm value is given by

$$\Pi_d(c) + \left[\ell - \Pi_d(0)\right]L_n(c) + \left[\frac{rC_S + \mu}{\rho + \delta\chi} - \Pi_d(C_S)\right]H_n(c)$$

on $c\in[0,C_S]$ and the functions L_n and H_n are defined similarly as before. Above $c > C_S$, firm value satisfies $S(c) = \frac{rC_S+\mu}{\rho+\delta\chi} - C_S + c$.

B.3   Proof of Proposition 8

To prove the claim, I exploit the following auxiliary result.

Lemma 24 $S(c)$ *is decreasing in* X.

Proof. As before, I consider two thresholds $X_1 < X_2$ satisfying the smooth-pasting and super-contact conditions $S'(c) - 1 = 0 = S''(c)$. I define the auxiliary function $k(c) = S(c, X_1) -$

$S(c, X_2)$. At X_2, $k(X_2) = (X_2 - X_1)\left(1 - \frac{r}{\rho + \Phi}\right) > 0$, and $k'(X_2) = k''(X_2) = 0$. At X_1,

$$\frac{rX_1 + \mu - \frac{\alpha\gamma(1-\eta)}{\eta}}{\rho + \Phi} - S(X_1, X_2) > \frac{rX_1 + \mu - \frac{\alpha\gamma(1-\eta)}{\eta}}{\rho + \Phi} - \left(\frac{rX_2 + \mu - \frac{\alpha\gamma(1-\eta)}{\eta}}{\rho + \Phi} - X_2 + X_1\right)$$

$$= (X_2 - X_1)\left(1 - \frac{r}{\rho + \Phi}\right) > 0$$

and $k'(X_1) < 0$ and $k''(X_1) > 0$, so the function is positive, decreasing and convex at X_1. Note that $k'(c) < 0 < k''(c)$ for any $[X_1, X_2]$. I conjecture (and verify later in this proof) that $\underline{C}(X_1) < \underline{C}(X_2)$. As $\underline{C}$ is uniquely identified by the level of the value function equal to $\frac{\alpha\gamma}{\chi\delta\eta}$, the strict monotonicity of S in c (see Lemma 22), jointly with $\underline{C}(X_1) < \underline{C}(X_2)$, means that $k(\underline{C}(X_1))$ and $k(\underline{C}(X_2))$ are non-negative.

On the interval $[\underline{C}(X_2), X_1]$, $k'(c)$ satisfies $(\rho + \Phi + \lambda - r)k'(c) = (rc + \mu)k''(c) + \frac{\sigma^2}{2}k'''(c)$ so $k'(c)$ cannot have neither a negative local minimum nor a positive local maximum on this interval. On the interval $[\underline{C}(X_1), \underline{C}(X_2)]$, $k'(c)$ evolves as

$$(\rho + \lambda + \Phi - r)k'(c) = [rc + \mu]k''(c) + \frac{\sigma^2}{2}k'''(c) + \delta\chi(1-\eta)S'(c, X_2)$$

where the last term is positive, meaning that $k'(c)$ cannot have a negative local minimum. Finally, on $[0, \underline{C}(X_1)]$, the first derivative satisfies $(\rho + \delta\chi + \lambda - r)k'(c) = [rc + \mu]k''(c) + \frac{\sigma^2}{2}k'''(c)$ so it cannot have neither a positive local maximum nor a negative local minimum. The fact that $k'(c)$ is negative and increasing over $[X_1, X_2)$, jointly with the fact that a negative local minimum can never occur, means that $k'(c)$ must be negative and increasing over all the interval of definition. Then, $k(c)$ is decreasing for any $c \in [0, X_2)$. Jointly with the fact that $k(X_2)$ is positive, this means that $k(c)$ is indeed positive for any $c \in [0, X_2)$.

Toward a contradiction, let us now assume that $\underline{C}(X_1) > \underline{C}(X_2)$. Given the monotonicity of S, the inequality $\underline{C}(X_1) > \underline{C}(X_2)$ would mean that $k(c) < 0$ at least on the interval $[\underline{C}(X_1), \underline{C}(X_2)]$. By previous results, $k(X_1)$ is positive, decreasing and convex, and also on the interval $[X_1, X_2)$. Then, if $k(c) < 0$ on the interval $[\underline{C}(X_1), \underline{C}(X_2)]$, there should exist a positive local maximum a for $k(c)$ on the interval $[\underline{C}(X_1), X_1]$. At a, the first derivative $k'(c)$ would go from positive to negative, and k'' would be negative here. Nevertheless, $k''(X_1) > 0$, and k' cannot have neither a negative local minimum on the interval $[\max(\underline{C}(X_2), \underline{C}(X_1)), X_1]$, contradicting the existence of a. In turn, this means that $\underline{C}(X_1) > \underline{C}(X_2)$ cannot hold. ∎

Lemma 25 *The inequality $C_V > C_S$ holds.*

Proof. To prove the claim, I first show that for a given payout threshold X, $V(c, X) > S(c, X)$ for any $c \in [0, X]$ (Consistently with the positiveness of the functions V and S, I am picking values of X such that $S(0)$ as well as $V(0)$ are non-negative). I define the auxiliary function $k(c) = V(c, X) - S(c, X)$. At X, the function is positive as

$$k(X) = \frac{\alpha\gamma(1 - \eta)}{\eta(\rho + \Phi)} > 0.$$

In addition, $k'(X) = k''(X) = 0$ because of smooth-pasting and super-contact at X. On the interval $[\underline{C}, X]$ the function satisfies

$$[rc + \mu]k'(c) + \frac{\sigma^2}{2}k''(c) - (\rho + \lambda + \Phi)k(c) + \lambda k(X) + \frac{\alpha\gamma(1 - \eta)}{\eta} = 0$$

Note that the function cannot have a negative local minimum, because the sum of the last two terms on the left hand side is positive. Differentiating the ODE above, it follows that k' satisfies

$$[rc + \mu]k''(c) + \frac{\sigma^2}{2}k'''(c) - (\rho + \lambda + \Phi - r)k'(c) = 0.$$

So, k' cannot have neither a positive local maximum nor a negative local minimum on $[\underline{C}, X]$. In addition, $\frac{\sigma^2}{2}k'''(X) = 0$. On the interval $[0, \underline{C}]$, $k(c)$ satisfies

$$[rc + \mu]k'(c) + \frac{\sigma^2}{2}k''(c) - (\rho + \lambda + \Phi)k(c) + \lambda k(X) = (\Phi - \delta\chi)S(c) = -\delta\chi(1 - \eta)S(c)$$

so that $k(c)$ cannot have a negative local minimum over this interval. In addition, $k'(c)$ cannot have a negative local minimum either on this interval, as its dynamics are given by

$$[rc + \mu]k''(c) + \frac{\sigma^2}{2}k'''(c) - (\rho + \Phi + \lambda - r)k'(c) = -\delta\chi(1 - \eta)S'(c) \tag{30}$$

as $S'(c) \geq 1$ by Lemma 22.

Now, S and S' are continuous because of the value-matching and smooth-pasting conditions at $\underline{C}$. Moreover, for $\underline{C}^- = \underline{C} - \epsilon$ (with $\epsilon > 0$ and small enough),

$$\frac{\sigma^2}{2}S''(\underline{C}^-) = (\rho + \lambda + \delta\chi)S(\underline{C}^-) - [r\underline{C}^- + \mu]S'(\underline{C}^-) - \lambda(S(X) - X + \underline{C}^-)$$

while for $\underline{C}^+ = \underline{C} + \epsilon$,

$$\frac{\sigma^2}{2}S''(\underline{C}^+) = (\rho + \lambda + \Phi)S(\underline{C}^+) - [r\underline{C}^+ + \mu]S'(\underline{C}^+) - \lambda(S(X) - X + \underline{C}^+) + \frac{\alpha\gamma(1 - \eta)}{\eta}$$

By simply subtracting the two equations above, taking the limit for $\epsilon \to 0$, and keeping in mind the continuity of $S(c)$ and $S'(c)$, I get

$$\lim_{\epsilon \to 0}\frac{\sigma^2}{2}\left(S''(\underline{C}^-) - S''(\underline{C}^+)\right) = (\delta\chi - \Phi)S(\underline{C}) - \frac{\alpha\gamma(1 - \eta)}{\eta} = 0$$

that establishes the continuity of the second derivative at $\underline{C}$. On the interval $[\underline{C}, X]$ the two functions V and S satisfy the same ODE but for the inhomogeneity term $-\frac{\alpha\gamma(1-\eta)}{\eta}(1 + \lambda/(\rho + \Phi))$ in the equation for S. Recall that we are just imposing the two boundaries at the payout threshold, i.e. X is exogenously taken in that the boundary at zero is lax at this stage of the proof. Then, $k'(\underline{C}^+) = k''(\underline{C}^+) = 0$, and this means that $k'(\underline{C}^-) = k''(\underline{C}^-) = 0$ because of continuity of the first and second derivative of V and S. By equation (30), it follows that $\frac{\sigma^2}{2}k'''(\underline{C}^-) = -\delta\chi(1 - \eta)S'(\underline{C}) < 0$. In turn, this means that $k'(\underline{C})$ is increasing in a left neighbourhood of $\underline{C}$, and given that $k'(\underline{C})$ is null at $\underline{C}$, it means that $k'(c)$ is negative in such

neighbourhood. Jointly with the fact that on $[0, \underline{C}]$ there cannot exist a negative local minimum for k', this means that $k'(c)$ is negative for any $[0, \underline{C}]$. Therefore, $k(c)$ is decreasing for any $c \in [0, \underline{C}]$. As $k(X)$ is positive, $k(c) > 0$ for any $c \in [0, X]$. This means that $V(c, X) > S(c, X)$. Now, taking $X = C_S$, it follows that the boundary condition at zero is satisfied for the function V if $C_V > C_S$, using arguments analogous to the proof of Lemma 13. ∎

Finally, I prove that C_S is decreasing in α e γ. Note that, when $\alpha = \gamma = 0$, it follows that $C_V = C_S$.

Lemma 26 *For any $\alpha_1 < \alpha_2$, $C_S(\alpha_1) > C_S(\alpha_2)$.*

Proof. First I prove that, for a given payout threshold X satisfying $S'(X) - 1 = 0 = S''(X)$ (but keeping the boundary at zero lax), the inequality $S(c, X; \alpha_1) > S(c, X; \alpha_2)$ holds. To this end, I define the auxiliary function $h(c) = S(c, X; \alpha_1) - S(c, X; \alpha_2)$. At X, the inequality

$$h(X) = \frac{\gamma(1-\eta)}{\eta(\rho + \Phi)} (\alpha_2 - \alpha_1) > 0$$

holds, and $h'(X) = 0 = h''(X)$. Over the interval $[\max(\underline{C}(X, \alpha_1), \underline{C}(X, \alpha_2)), X]$, $h(c)$ evolves as $(\rho + \lambda + \Phi)h(c) = [rc + \mu]h'(c) + \frac{\sigma^2}{2}h''(c) - \frac{\gamma(1-\eta)}{\eta}(\alpha_1 - \alpha_2) + \lambda h(X)$ and note that the sum of the last two terms is positive, meaning that the function cannot have a negative local minimum. Over the interval $[0, \min(\underline{C}(X, \alpha_1), \underline{C}(X, \alpha_2))]$, $h(c)$ satisfies $(\rho + \lambda + \delta\chi)h(c) = [rc + \mu]h'(c) + \frac{\sigma^2}{2}h''(c) + \lambda h(X)$, so there cannot be a negative local minimum either. Conjecturing (and verifying below in the proof) that $\underline{C}(X, \alpha_1) < \underline{C}(X, \alpha_2)$, it follows that $h(c)$ satisfies

$$(\rho + \lambda + \delta\chi)h(c) = (rc + \mu)h'(c) + \frac{\sigma^2}{2}h''(c) + (\delta\chi - \Phi) S(c, X; \alpha_1) - \frac{\alpha_1 \gamma(1-\eta)}{\eta} + \lambda h(X)$$

on $[\underline{C}(X, \alpha_1), \underline{C}(X, \alpha_2)]$. Since $(\delta\chi - \Phi) S(c, X; \alpha_1) = \delta\chi(1 - \eta)S(c, X, \alpha_1)$, the following inequality $(\delta\chi - \Phi) S(c, X; \alpha_1) - \frac{\alpha_1 \gamma(1-\eta)}{\eta} > 0$ holds on $[\underline{C}(X, \alpha_1), \underline{C}(X, \alpha_2)]$ by the definition of $S(\underline{C})$ and the monotonicity of S, so the function cannot have a negative local minimum because the sum of the last three terms in the equation above is positive. Simply differentiating, note also that $h'(c)$ cannot have a negative local minimum either. Exploiting arguments analogous to Lemma 25, $h'''(\underline{C}(X, \alpha_2))$ is negative and so $h(c)$ is positive for any c if $\underline{C}(X, \alpha_1) < \underline{C}(X, \alpha_2)$. Toward a contradiction, I assume that the inequality $\underline{C}(X, \alpha_1) > \underline{C}(X, \alpha_2)$ holds. If this were the case, $h(c)$ would be negative at least on the interval $[\underline{C}(X, \alpha_2), \underline{C}(X, \alpha_1)]$. However, $h(X) > 0 = h'(X)$. Exploiting arguments analogous to Lemma 25 regarding the dynamics of $S(c, X, \alpha_1)$ and $S(c, X, \alpha_2)$ on $[\max(\underline{C}(X, \alpha_1), \underline{C}(X, \alpha_2)), X]$, it follows that $h'(c)$ cannot be positive here. Then, $h(\underline{C}(X, \alpha_2))$ cannot be negative, contradicting that $\underline{C}(X, \alpha_1) > \underline{C}(X, \alpha_2)$.

Using arguments analogous to the proof of Lemma 13, the boundary condition at zero implies that $C_S(\alpha_1) > C_S(\alpha_2)$. Then, the claim follows. ∎

Following the same steps as for Lemma 26 but for γ, the result below holds.

Lemma 27 *For any $\gamma_1 < \gamma_2$, $C_S(\gamma_1) > C_S(\gamma_2)$.*

B.4 Endogenizing liquidity provision via $\eta(c)$

In this Section, I assess the robustness of the results in Section 4.1 to an alternative model specification. I maintain the assumptions that intermediaries' constant market presence entails a flow cost γ and that the market for liquidity provisions is perfectly competitive. Differently, I assume that the measure of active intermediaries is constant (I normalize it to one), and I endogenize the share of rents that intermediaries extract from shocked shareholders.

In this setting, the zero-profit condition determines the equilibrium $\eta(c)$, which satisfies

$$\chi \eta(c) \delta S(c) - \gamma = 0.$$

In this equation, the first term represents the expected rents accruing to the intermediary sector on any dt. The second term represents the instantaneous flow cost borne by active intermediaries. Therefore, the equilibrium share of rents extracted by the intermediaries is

$$\eta(c) = \min\left[\frac{\gamma}{\chi \delta S(c)}, 1\right].$$

This equation implies that when firm value decreases, intermediaries extract a larger share of rents from shocked shareholders. However, intermediaries' rents are capped by the condition $\eta \leq 1$. When $\eta = 1$, shocked shareholders are indifferent between trading with intermediaries or turning to their best outside option. Thus, there exists a critical value $\tilde{S} = \frac{\gamma}{\chi \delta}$ such that, for any $S(c) \leq \tilde{S}$, $\eta = 1$. As for the case analyzed in the main text, there exists at most one threshold $\tilde{C} \in [0, C_S]$ such that $S(\tilde{C}) = \tilde{S}$. For any $c \leq \tilde{C}$, the cost of a liquidity shock is the maximum for shareholders and given by $\delta \chi S(c)$. In this case, firm value satisfies

$$(\rho + \delta \chi) S(c) = (rc + \mu) S'(c) + \frac{\sigma^2}{2} S''(c) + \lambda \left[S(C_S) - C_S + c - S(c)\right],$$

which is identical to equation (20) in Section 4.1. For any $c \in [\tilde{C}, C_S]$, conversely, firm value satisfies the following ODE:

$$\rho S(c) = (rc + \mu) S'(c) + \frac{\sigma^2}{2} S''(c) + \lambda \left[S(C_S) - C_S + c - S(c)\right] - \gamma.$$

As in equation (21), the last term on the right-hand side implies that frictions that are borne by intermediaries also matter to investors. The structure of secondary market transactions and its relationship with firm value is as in Figure 3 (but replacing $\underline{S}$ and $\underline{C}$ with $\tilde{S}$ and $\tilde{C}$).

The self-reinforcing relation between participation and firm value arises in this alternative setting too. Consider the effect of an increase in the participation cost γ when $\tilde{S} < S(C_S)$. $\tilde{S}$ increases and so does the intermediaries' rents. As a result, trading the stock becomes more expensive on average. The resulting decrease in firm value leads to a further increase in intermediaries' rent share, and so on.

C   U.S. equity markets trading: Some evidence

In his presidential address, French (2008) noted that the cost of trading U.S. equities has decreased between 1975 and 2007 as a result of different factors, such as the introduction of negotiated brokerage commissions in 1975, the development of electronic trading networks, the decimalization in 2000 and 2001, and the SEC's implementation of rules designed to increase market transparency and liquidity. But he added: "Even at the end of the sample, trading is *not* free." Indeed, there is a large cross-sectional variation in the costs of trading U.S. equities. The purpose of this Appendix is to give a snapshot of this variation. I use CRSP data and focus on stocks exchanged on the NYSE, AMEX, and NASDAQ markets. I remove preferred stocks, stock rights and warrants, stock funds, and ADRs. I perform the same analysis over two non-consecutive years for robustness, and choose 2006 and 2012 to avoid abnormal patterns related to the 2007-2009 financial crisis. I remove observations with missing volume, where volume is defined as the number of firm shares sold in one day. The sample consists of 1,184,794 daily observations (936,195) in 2006 (in 2012), out of which 704,050 (546,801) are NASDAQ-traded stocks.

$$\boxed{\text{Insert Table 3 Here}}$$

I start by investigating the daily trading patterns for the year 2006. For each trading day, stocks are ranked by ascending trading volume. An average of 96% of stocks constitutes less than half of the daily volume. In turn, the least-traded 50% of the stocks only represents 2.3% of the daily volume.[18] The results reported in Table 3 for the year 2012 confirm these figures. These trading patterns are reinforced when performing the analysis on NASDAQ stocks only. The share of stocks accounting for half of the daily volume can be as high as 99.4%, whereas the least-traded 50% of stocks may account for less than 1.2% of the daily volume.

$$\boxed{\text{Insert Table 4 Here}}$$

Table 4 investigates the cross-sectional characteristics of listed stocks over a trading year. I split the sample into quartiles of annual trading volume, where the first quartile contains the least actively-traded stocks. The scale of trading volume differs substantially across quartiles: In the highest quartile, it is more than 200 times higher than in the lowest quartile. The same pattern holds for a normalized measure of volume, defined as the number of firm shares sold in one day normalized by the number of shares outstanding. Normalized trading volume is indeed ten times higher in the highest quartile than in the lowest quartile. The maximum number of days without trading also differs dramatically across the quartiles. In the lowest quartile, it peaks at 204 days (174) in the year 2006 (2012); this number decreases sharply and non-linearly for more liquid stocks. The frequency at which a stock is traded matters in as much as infrequently-traded stocks are subject to higher transaction costs (among others, Easley, Kiefer, O'Hara, and Paperman, 1996, Barclay and Hendershott, 2004). Table 4 indeed shows that bid-ask spreads considerably decrease as trading volume increases, being the highest for the least active stocks. The same holds true for half spreads.[19] This huge cross-sectional variation

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[18]This evidence recalls Easley, Kiefer, O'Hara, and Paperman (1996), reporting that "on the London Stock Exchange, 50 percent of listed stocks account for only 1.5 percent of trading volume, and over 1000 stocks average less than one trade a day. On the [...] NYSE, it is common for individual stocks not to trade for days or even weeks at a time, while one stock in London never traded in an eleven-year period."

[19]Half spreads measure the percentage trading cost incurred in a one-way trade; they are computed as

is supported by many empirical works. For instance, Brennan, Chordia, Subrahmanyam, and Tong (2010) confirm that liquidity problems are more important for smaller firms. By studying the cross-sectional and time-series variation in trading costs of the largest 2,000 firms, Novy-Marx and Velikov (2016) find that smaller caps stocks are far more expensive to trade. Because small firms are more constrained (firm size is a proxy for financial constraints), cash reserves are extremely important for them. Thus, this evidence supports the relevance of the economics underlying the model.

$HS = 100 * (A_{it} - B_{it})/(2M_{it})$, where A_{it} (B_{it}) is the closing ask (bid) price for security i at time t, and M_{it} is the quote midpoint, as Bessembinder and Kaufman (1997) and Huang and Stoll (1996).

References

Adam, K., J. Beutel, A. Marcet, and S. Merkel. 2015. "Can a Financial Transaction Tax Prevent Stock Price Booms?" *Journal of Monetary Economics* 76:590–609.

Amihud, Y., and H. Mendelson. 1986. "Asset Pricing and the Bid-Ask Spread." *Journal of Financial Economics* 17:223–249.

Anand, A., P. Irvine, A. Puckett, and K. Venkataraman. 2013. "Institutional Trading and Stock Resiliency: Evidence from the 2007-2009 Financial Crisis." *Journal of Financial Economics* 108:773–797.

Anderson, R., and A. Carverhill. 2012. "Corporate Liquidity and Capital Structure." *Review of Financial Studies* 25:797–837.

Ang, A., A. Shtauber, and P. Tetlock. 2013. "Asset Pricing in the Dark: The Cross Section of OTC Stocks." *Review of Financial Studies* 26:2985–3028.

Bakke, T.E., C. Jens, and T. Whited. 2012. "The Real Effects of Delisting: Evidence from a Regression Discontinuity Design." *Finance Research Letters* 9:183–193.

Barclay, M., and T. Hendershoot. 2004. "Liquidity Externalities and Adverse Selection: Evidence from Trading after Hours." *Journal of Finance* 54:682–710.

Bates, T., K. Kahle, and R. Stulz. 2009. "Why Do U.S. Firms Hold So Much More Cash Than They Used To?" *Journal of Finance* 64:1985–2021.

Bessembinder, H., and H. Kaufmann. 1997. "A Comparison for Trade Execution Costs for NYSE and NASDAQ-listed stocks." *Journal of Financial and Quantitative Analysis* 32:287–310.

Bolton, P., H. Chen, and N. Wang. 2014. "Debt, Taxes, and Liquidity." *Working paper*, pp. .

—. 2013. "Market Timing, Investment, and Risk Management." *Journal of Financial Economics* 109:40–62.

—. 2011. "A Unified Theory of Tobin's q, Corporate Investment, Financing and Risk Management." *Journal of Finance* 66:1545–1578.

Bonaimé, A.A., K.W. Hankins, and J. Harford. 2014. "Financial Flexibility, Risk Management, and Payout Choice." *Review of Financial Studies* 27:1074–1101.

Bond, P., A. Edmans, and I. Goldstein. 2012. "The Real Effects of Financial Markets." *Annual Review of Financial Economics* 4:339–360.

Brennan, M., T. Chordia, A. Subrahmanyam, and Q. Tong. 2010. "Sell-order Liquidity and the Cross-section of Expected Stock Returns." *Journal of Financial Economics* 105:523–541.

Brogaard, J., D. Li, and Y. Xia. 2016. "The Effect of Stock Liquidity on Default Risk." Working paper.

Buss, A., B. Dumas, R. Uppal, and G. Vilkov. 2013. "Comparing Different Regulatoy Measures to Control Stock Market Volatility: A General Equilibrium Analysis." Working Paper.

Butler, A., G. Grullon, and J. Weston. 2005. "Stock Market Liquidity and the Cost of Issuing Equity." *Journal of Financial and Quantitative Analysis* 40:331–348.

Campello, M., R. Ribas, and A. Wang. 2014. "Is the Stock Market Just a Side Show? Evidence from a Structural Reform." *Review of Corporate Finance Studies* 3:1–38.

Chordia, T., R. Roll, and A. Subrahmanyam. 2011. "Recent trends in trading activity and market quality." *Journal of Financial Economics* 101:243–263.

Colliard, J.E., and P. Hoffmann. 2015. "Financial Transaction Taxes, Market Composition, and Liquidity." Working paper.

Comerton-Forde, C., T. Hendershott, C. Jones, P. Moulton, and M. Seasholes. 2010. "Time Variation in Liquidity: The Role of Market-Maker Inventories and Revenues." *Journal of Finance* 65:295–331.

Décamps, J.P., T. Mariotti, J.C. Rochet, and S. Villeneuve. 2011. "Free Cash Flows, Issuance Costs and Stock Prices." *Journal of Finance* 66:1501–1544.

Décamps, J.P., and S. Villeneuve. 2007. "Optimal Dividend Policy and Growth Option." *Finance and Stochastics* 11:3–27.

Dixit, A., and R. Pindyck. 1994. *Investment Under Uncertainty*. Princeton, New Jersey: Princeton University Press.

Easley, D., N.M. Kiefer, M. O'Hara, and J.B. Paperman. 1996. "Liquidity, Information, and Infrequently Traded Stocks." *Journal of Finance* 51:1405–1436.

Eisfeldt, A., and T. Muir. 2015. "Aggregate External Financing and Savings Waves." Working Paper.

Fang, V., T. Noe, and S. Tice. 2009. "Stock Market Liquidity and Firm Value." *Journal of Financial Economics* 94:150–169.

Foucault, T., M. Pagano, and A. Röell. 2014. *Market Liquidity: Theory, Evidence, and Policy*. Oxford: Oxford University Press.

French, K. 2008. "Presidential Address: The Cost of Active Investing." *Journal of finance* 63:1537–1573.

Graham, J. 2000. "How big are the tax benefits of debt?" *Journal of finance* 55:1901–1941.

Graham, J., and M. Leary. 2015. "The Evolution of Corporate Cash." Working paper.

Hameed, A., W. Kang, and S. Viswanathan. 2010. "Stock Market Declines and Liquidity." *Journal of Finance* 65:257–293.

He, Z., and K. Milbradt. 2014. "Endogenous Liquidity and Defaultable Bonds." *Econometrica* 82:1443–1508.

He, Z., and W. Xiong. 2012. "Rollover Risk and Credit Risk." *Journal of Finance* 67:391–429.

Huang, J., and J. Wang. 2010. "Market Liquidity, Asset Prices, and Welfare." *Journal of Financial Economics* 95:107–127.

Huang, R., and H. Stoll. 1996. "Dealer versus Auction Markets: A paired Comparison for Execution Costs on NASDAQ and the NYSE." *Journal of Financial Economics* 41:313–357.

Hugonnier, J., S. Malamud, and E. Morellec. 2015. "Capital Supply Uncertainty, Cash Holdings, and Investment." *Review of Financial Studies* 28:391–445.

Jensen, M. 1986. "Agency Costs of Free Cash Flow, Corporate Governance, and Takeovers." *American Economic Review* 76:323–329.

Lins, K., H. Servaes, and P. Tufano. 2010. "What Drives Corporate Liquidity? An International Survey of cash holdings and lines of credit." *Journal of Financial Economics* 98:160–176.

McLean, R. 2011. "Share Issuance and Cash Savings." *Journal of Financial Economics* 99:693–715.

Novy-Marx, R., and M. Vilkov. 2016. "A Taxonomy of Anomalies and Their Trading Costs." *Review of Financial Studies* 29:104–147.

Nyborg, K., and Z. Wang. 2014. "Stock Liquidity and Corporate Cash Holdings: Feedback and the Cash as Ammunition Hypothesis." Working paper.

O'Hara, M., and M. Ye. 2011. "Is Market Fragmentation Harming Market Quality?" *Journal of Financial Economics* 100:459–474.

Pérez-Gonzáles, F., and H. Yun. 2013. "Risk Management and Firm Value: Evidence from Weather Derivatives." *Journal of Finance* 58:2143–2176.

Protter, P. 1990. *Stochastic Integration and Differential Equations*. New York: Springer-Verlag.

Stoll, H.R. 2006. "Electronic Trading in Stock Markets." *Journal of Economic Perspectives* 20:153–174.

Stulz, R., D. Vagias, and M. vanDijk. 2014. "Do Firms Issue More Equity when Markets Are More Liquid?" Working Paper.

Sufi, A. 2009. "Bank Lines of Credit in Corporate Finance: An Empirical Analysis." *Review of Financial Studies* 22:1057–1088.

TABLE 1: BENCHMARK PARAMETERS.

Symbol	Description	Value
	FIRM PARAMETERS	
ρ	Discount rate	0.06
r	Return on cash	0.0055
μ	Cash flow drift	0.05
σ	Cash flow volatility	0.10
λ	Arrival rate of financing opportunities	1.5
ϕ	Asset tangibility	0.00
	SECONDARY MARKET PARAMETERS	
δ	Intensity of liquidity shocks	1.50
χ	Cost of best outside option	0.008
η	Intermediaries' bargaining power	0.250
γ	Intermediaries' participation cost	0.015
α	Market inefficiencies	0.040

Table 2: Effect of stock trading costs on corporate policies.

The table reports the change in the target cash level, in the probability of liquidation, and in the investment opportunity set when trading is costly with respect to a benchmark environment with no trading costs, as a function of the costs incurred in trading the firm's stock.

Trading cost (Basis points)	Target Cash ($\Delta\%$)	Prob. liquidation ($\Delta\%$)	Zero-NPV cost ($\Delta\%$)
5	-0.44%	0.45%	-1.29%
10	-0.88%	0.90%	-2.54%
15	-1.31%	1.34%	-3.77%
20	-1.73%	1.79%	-4.97%
25	-2.15%	2.23%	-6.14%
30	-2.57%	2.68%	-7.29%
35	-2.98%	3.12%	-8.41%
40	-3.38%	3.56%	-9.50%
45	-3.79%	4.00%	-10.6%
50	-4.18%	4.44%	-11.6%

TABLE 3: TRADING PATTERNS I

The table reports the daily trading patterns of active ordinary stocks in CRSP for the years 2006 and 2012. The columns headed by (W) refer to the whole sample (NYSE, AMEX and NASDAQ), while those headed by (N) refer to NASDAQ stocks only. The first pair of columns report the daily trading volume, defined as the number of shares sold in one day. The second pair of columns report the percentage of stocks representing 50% of the daily traded volume. The third pair reports the share of the daily volume accounted by 50% of the stocks (being the least traded when sorted by volume). Finally, the fourth pair reports the percentage of stocks that are not traded in one day.

	Daily volume (thousands)		Stock constituting 50% volume		Share of volume by 50% stocks		Share of non-traded stocks	
	(W)	(N)	(W)	(N)	(W)	(N)	(W)	(N)
Year 2006								
Average	3,803,795	1,767,535	96.8%	98.4%	2.30%	1.71%	2.65%	3.26%
Median	3,861,490	1,766,268	96.8%	98.4%	2.29%	1.69%	2.63%	3.25%
Minimum	1,257,030	583,961	94.8%	96.1%	1.71%	1.19%	1.20%	1.29%
Maximum	6,229,979	3,398,085	98.3%	99.4%	4.06%	3.46%	5.76%	6.83%
Std. Dev.	561,591.7	301,552	0.0043	0.0039	0.0032	0.0029	0.0055	0.0069
Year 2012								
Average	4,275,441	1,511,968	96.7%	97.8%	1.99%	1.63%	1.96%	2.57%
Median	4,289,221	1,510,397	96.7%	97.7%	1.97%	1.59%	1.98%	2.62%
Minimum	1,432,418	514,329	94.9%	95.6%	1.29%	0.90%	0.48%	0.22%
Maximum	7,505,599	3,212,558	98.0%	99.0%	3.54%	2.66%	4.41%	5.74%
Std. Dev.	672,877	250,053	0.004	0.0048	0.003	0.0029	0.005	0.0067

TABLE 4: TRADING PATTERNS II

The table reports some cross-sectional characteristics of listed stocks, when sorted into quartiles of annual trading volume. The columns headed by (W) refer to the whole sample (NYSE, AMEX and NASDAQ), while those headed by (N) refer to NASDAQ stocks only. The first and the second pair of columns report respectively the average daily volume and the normalized volume. The third pair of columns report the maximum number of days without trading over one trading year. Finally, the fourth and the fifth pairs of columns report respectively the average bid-ask spread and the half bid-ask spread.

| | Mean daily volume | | Mean daily volume (normalized) | | Maximum number of non-trading days | | Average bid-ask spread (%) | | Average half bid-ask spread (%) | |
	(W)	(N)	(W)	(N)	(W)	(N)	(W)	(N)	(W)	(N)
Year 2006										
All	805,808	630,124	0.77%	0.80%	204	204	0.65%	0.76%	0.33%	0.38%
Lowest	12,578	8,464	0.15%	0.13%	204	204	1.58%	1.76%	0.81%	0.90%
II	100,031	63,494	0.57%	0.46%	36	73	0.61%	0.72%	0.31%	0.36%
III	341,361	220,049	0.98%	0.90%	15	5	0.28%	0.36%	0.14%	0.18%
Highest	2,768,543	2,228,325	1.39%	1.70%	1	1	0.14%	0.19%	0.07%	0.10%
Year 2012										
All	114,115	691,014	0.76%	0.68%	174	174	0.91%	1.25%	0.47%	0.65%
Lowest	15,755	9,687	0.16%	0.13%	174	174	2.70%	3.41%	1.40%	1.78%
II	113,703	59,828	0.50%	0.36%	26	32	0.62%	0.99%	0.32%	0.51%
III	424,667	225,214	0.88%	0.76%	1	26	0.22%	0.40%	0.11%	0.20%
Highest	4,009,909	2,468,484	1.51%	1.49%	1	1	0.11%	0.20%	0.05%	0.10%
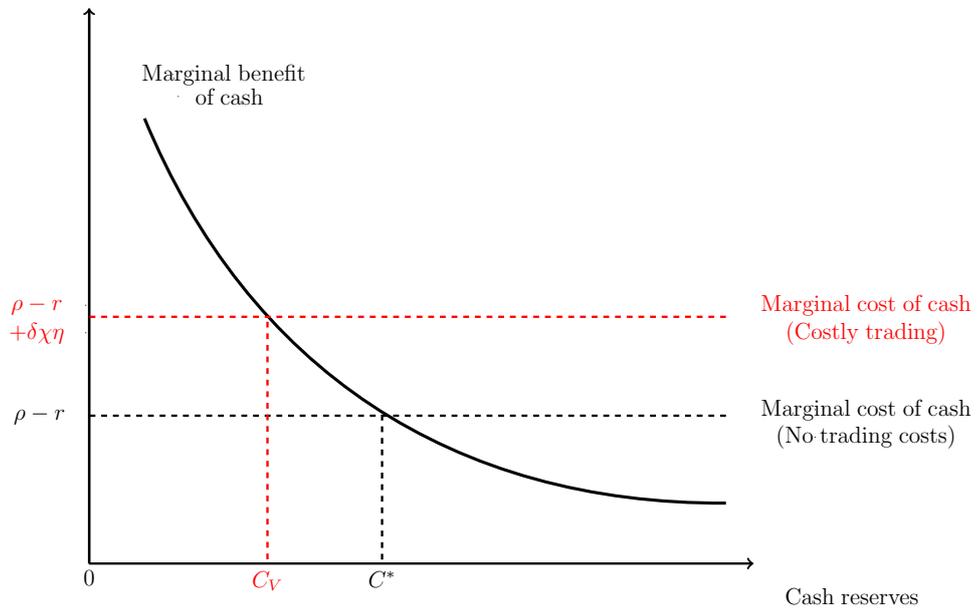
The target level of cash reserves is given by the intersection of the marginal benefit curve of holding cash and the marginal cost curve. The figure highlights that trading costs shift the marginal cost curve upwards, thereby leading to a decrease in the target level of cash from C^* to C_V.

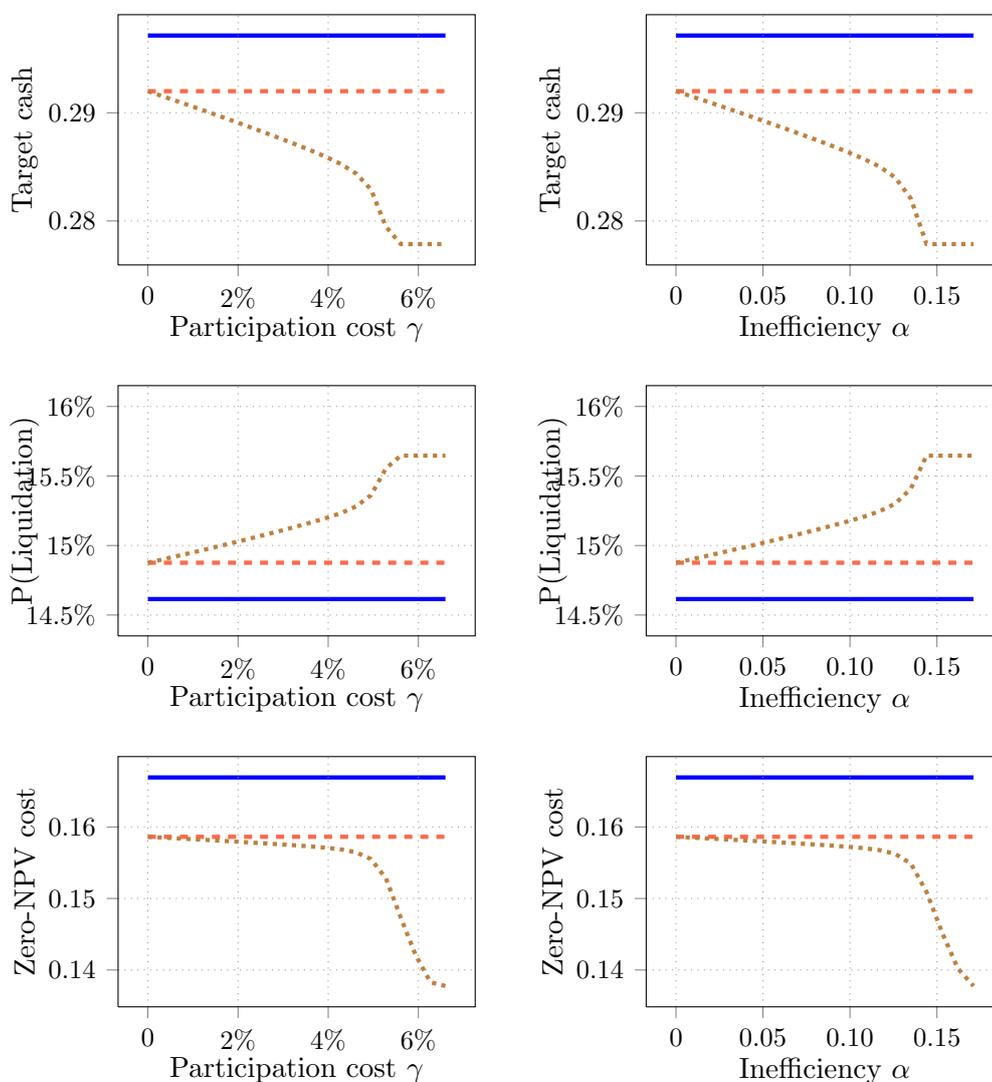FIGURE 2: STOCK ILLIQUIDITY AND CORPORATE POLICIES.
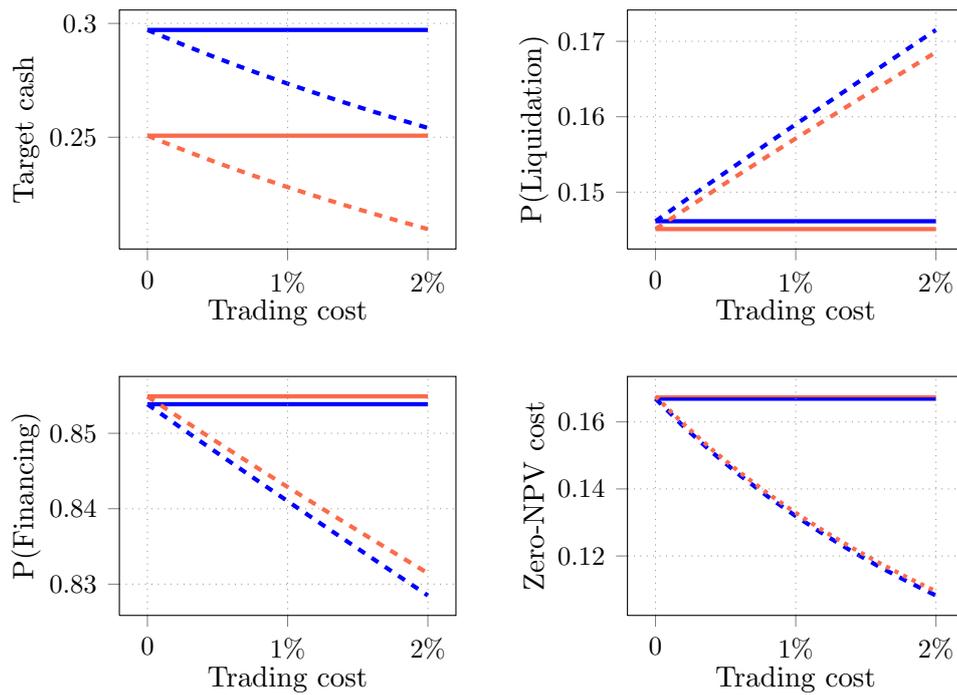
The figure shows the target level of cash holdings, the probability of liquidation, the probability of external financing, and the zero-NPV cost, as a function of the cost borne by firm's share-holders when trading the stock. The blue lines refer to a firm with no access to bank credit and whose stocks are costlessly traded (solid line) or trade at a cost (dashed line). The red lines refer to a firm having access to bank credit and whose stocks are costlessly traded (solid line) or trade at a cost (dashed line).

FIGURE 3: LIQUIDITY PROVISION, TRADING COSTS, AND FIRM VALUE.
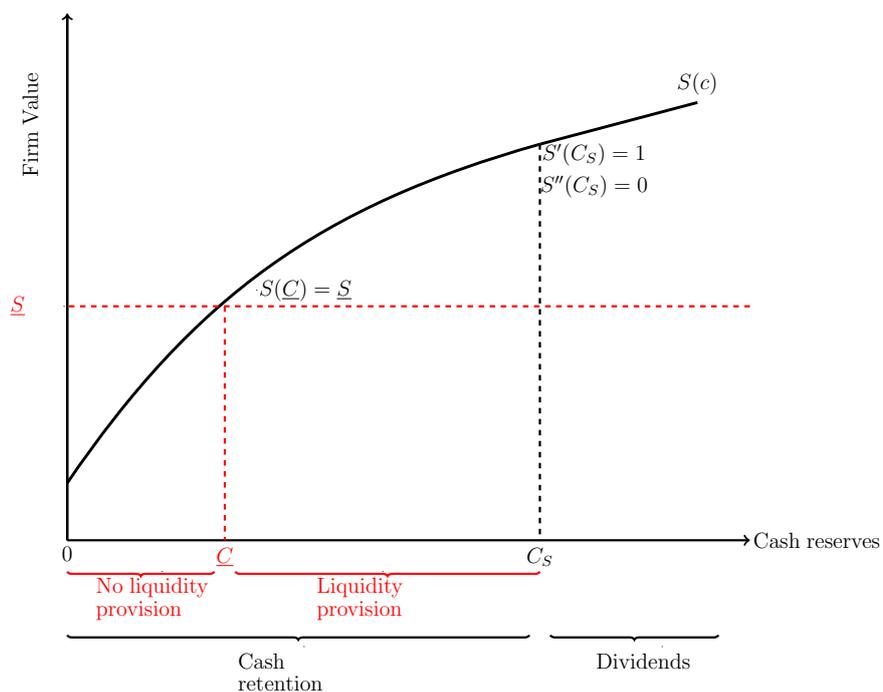
The figure represents the firm value $S(c)$ as a function of its cash reserves in relation to the provision of liquidity in the market of the stock. Above the target level of cash reserves C_S, the optimal policy is to pay out cash as dividends or share repurchases. Below C_S, the optimal policy is to retain earnings and raise external financing whenever financing opportunities arise. In this region, intermediaries provide liquidity in the market for the stock for any $c > \underline{C}$, while they do not provide liquidity for $c \leq \underline{C}$.

FIGURE 4: ENDOGENOUS LIQUIDITY PROVISION AND CORPORATE POLICIES.

The figure shows the target level of cash holdings, the probability of liquidation, and the zero-NPV cost, as a function of the participation cost faced by the intermediaries γ and market inefficiencies α. The solid blue line depicts the benchmark case with no stock-trading costs, the dashed red line depicts the case in which cost of trading is constant, and the dotted brown line depicts the environment in which cost of trading depends on intermediaries' liquidity provision.

FIGURE 5: ENDOGENOUS LIQUIDITY PROVISION AND FIRM VALUE.
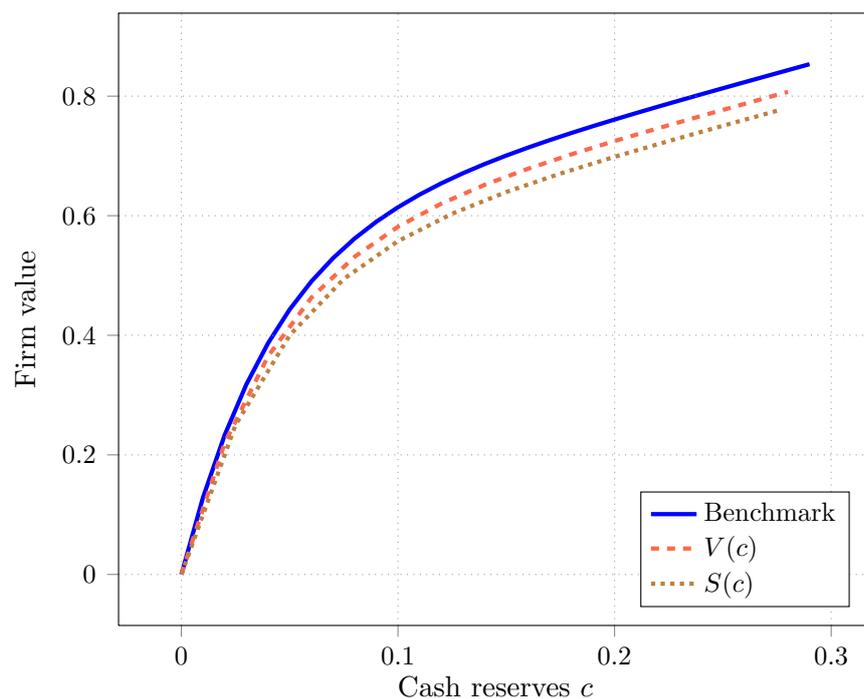
The figure shows firm value as a function of corporate cash reserves c. The solid blue line depicts the benchmark environment in which there are no costs of trading the stock, the dashed red line depicts the environment in which trading costs are constant (perfect), and the dotted brown line depicts the environment in which trading costs depend on liquidity provision.