FINANCIAL FRAGILITY AND OVER-THE-COUNTER MARKETS

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I propose a model to study the interaction between financial fragility and over-the-counter markets. In the model, the financial sector is composed of a large number of investors organized into coalitions, which are interpreted as financial institutions, and a large numbers of dealers. Coalitions and dealers trade assets in an over-the-counter market *à la* Duffie et al. (2005) and Lagos and Rocheteau (2009). Investors are subject to privately observed preference shocks, and coalitions use the balanced team mechanism, proposed by Athey and Segal (2013), to implement an efficient risk-sharing arrangement among its members. I provide conditions for the existence of a truth-telling equilibrium and its uniqueness. I then compute numerical examples that violate the uniqueness condition and produce run equilibria—where investors announce low valuation of assets because they believe everyone else in the coalition is doing the same. I use these numerical examples to investigate how asset price, trade volume, bid-ask spread and welfare, in an over-the-counter market, respond to runs on financial institutions.

KEYWORDS: Decentralized trade, search, trade volume, bid-ask spreads, liquidity, liquidity insurance, financial fragility, bank-run, dynamic mechanism design.

1. INTRODUCTION

IN DEVELOPED FINANCIAL SYSTEMS investors participate in asset markets as part of coalitions that trade assets (often over the counter) on their members' behalf and provide liquidity (withdrawal options) to the members. Examples of these coalitions are financial institutions such as money market mutual funds and bank conduits. An empirical literature suggests that large outflows from those type of institutions during the 2007-08 financial crisis were due to runs.[1] In particular, Schmidt et al. (2014) find evidence of runs within investors of money market mutual funds in the spirit of the equilibrium bank-run discussed in Diamond and Dybvig (1983), where strategic complementary plays an important role in withdrawals decisions. In this paper, I build a model connecting these runs to the over-the-counter market structure. I show that run equilibria are more likely to arise in environments with severe search frictions, and use numerical examples to study how market variables respond to runs on financial institutions.

The model builds on Lagos and Rocheteau (2009), hereafter LR,[2] and extends it in two ways: investors are organized in coalitions and have private information. The basic environment is the following. Time is infinite and there are two types of agents, investors and dealers. There is an asset which produces dividends that are non tradable. In order to consume the dividend the investor must hold the asset. Investors derive utility from consuming dividends but dealers do not. All agents can transfer utility using a linear technology. Investors belong to finite-size coalitions and, within the

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[1]See Kacperczyk and Schnabl (2010), Gorton and Metrick (2012), Schmidt et al. (2014), Kacperczyk and Schnabl (2013) and Covitz et al. (2013) for empirical evidence on runs during the 2007-08 financial crisis.

[2]Lagos and Rocheteau (2009) extends Duffie et al. (2005) by allowing investors to hold any positive amount of assets instead of only 0 or 1. However, it also simplified the model so there is no trade between investors.

same coalition, they can trade asset for utility in any given period. Coalitions do not trade with each other, only with dealers. Dealers, on the other hand, participate in a competitive inter-dealer market where they trade assets among themselves. Every period there is a random match between coalitions and dealers, and their terms of trade are established using Nash bargaining. Investors receive privately observed preference shocks over time following a Markov chain. Coalitions use the balanced team mechanism, proposed by Athey and Segal (2013), to implement an efficient risk-sharing arrangement against those shocks. The balanced team mechanism extends the AGV-Arrow mechanism to dynamic environments.[3]

I show that a truth-telling equilibrium exists if the bargaining power of dealers is sufficiently low. I impose this condition so investors have no incentives to misreport during the Nash bargaining with dealers. The advantage of using Nash bargaining is tractability, since it makes the payments in the bargaining linear in the gains from trade. It is worth mentioning that the truth-telling equilibrium generates the same outcome of a version of the model with complete information, and this outcome is constrained Pareto efficient in the limit case where dealer's bargaining power is zero.

Once I establish conditions for existence and efficiency of truth-telling equilibrium, I study conditions for its uniqueness. I show that the truth-telling equilibrium is the unique equilibrium if the coalitions' probability of meeting a dealer is high enough. The literature on over-the-counter markets associates this probability with the degree of search friction in the economy. When it is low, search frictions are severe and the equilibrium outcome is close to autarky, when it is high, search frictions are small and the equilibrium outcome is close to Pareto efficient. My result highlights another desirable feature of having high probability of meeting a dealer: it eliminates multiple equilibria, leading to a stable financial sector.

When the probability of meeting a dealer is not high, however, I show by computation that non-truth-telling equilibria also exists. I compute equilibria where the economy switches back and forth between a "run" and a "no-run" state following a Markov chain of sunspots. In the "run" state, investors in a subset of the coalition announce a low valuation of the asset dividends independently of their true preference. I verify numerically that in a large range of the parameter space this outcome is supported as a perfect Bayesian equilibrium of the game associated with the balanced team mechanism. I conclude then that multiplicity is not a particular outcome of some knife edge case of the parameter space, but rather a feature of the model with incomplete information.

But what generates incentives for a run in equilibrium? The answer lies in the liquidity insurance provided by the balanced team mechanism. This mechanism can be interpreted in the following way. Investors agree to buy/sell assets from other coalition members in case other members have the desire to sell/buy assets and it is costly (or unfeasible) to trade it with a dealer—a form of liquidity insurance that is also embedded in the contract offered by several real life financial institutions. The price an investor pays/receives for these assets is based on the expected welfare impact of his own announcement. The reason it must be based on the expected welfare impact and not on the realized welfare impact is to make the payments budget balanced (that is why it is called the *balanced* team mechanism). When investors truthfully announce their preference shocks, this liquidity insurance improves the allocation of assets in the coalition by equalizing the marginal utility of consumption of asset dividends. But this insurance also has another effect. The amount of assets purchased by a particular investor is increasing in the other investors' desire to sell the asset.

[3]The AGV-Arrow mechanism was initially proposed by Arrow (1979) and d'Aspremont and Gérard-Varet (1979). See Fudenberg and Tirole (1991) for details and results.

Hence, when an investor believes for sure other investors will announce they want to sell their assets, he expect to have a lot of assets and, since there is decreasing marginal utility from consuming asset dividends, he expects a low marginal utility from holding more of those assets. However, the price he pays does not react to this low marginal utility because it is based on the expected impact of his announcement, not the realized one. As a result, under this beliefs he may prefer to untruthfully announce a desire to sell his own assets as a way to avoid buying assets at a price that is higher than the marginal benefit he gets from buying those assets. And, of course, if every investors has this same belief they all prefer to announce the desire to sell their assets—confirming their initial beliefs in equilibrium and generating a self-fulfilling run on the coalition.

The runs have several implications for market outcomes. Asset price drops in the run states. Trade volume initially spikes, following a "fire sale" by the coalitions under a run, then collapses. The average bid-ask spread can go up or down with runs, depending on the fraction of coalitions under a run, where bid-ask spread in the model is the difference between the price in a coalition-dealer meeting and the price in the inter-dealer market. When about half of the coalitions are under a run, the average bid-ask spread increases in the run state. Otherwise, it decreases. I also investigate in the numerical examples how these effects change with changes in the probability of finding a dealer. I find that increasing this probability generates larger price decline, larger spike and faster collapse of trade volume, and stronger moves in bid-ask spreads.

I also use the numerical examples to study the welfare implications of the model. I find that welfare is decreasing in the fraction of coalitions under a run. This is somewhat expected. When there are more coalitions under a run, we have more investors misrepresenting their type and, therefore, a worse allocation of resources. Moreover, I find that in some regions of the parameter space welfare is also decreasing in the probability of finding a dealer. The intuition for this result is twofold. There is an effect on the extensive margin and one on the intensive margin. The coalitions under a run sell assets at a price below the average valuation of their members. They sell it because their investors are misreporting their preference shocks. If the probability of finding a dealer is higher, then in the same time interval more coalitions under a run have trade opportunity. So the fraction of coalitions inefficiently selling their assets is higher. This is the extensive margin effect. On the intensive margin, when the probability of finding a dealer is higher, coalitions chose to put more weight in their short term valuation when deciding how much to buy/sell of the asset. This effect has been extensively discussed in Lagos and Rocheteau (2009). And it implies that a coalition under a run inefficiently sells more of its assets if the probability of finding a dealer is higher.

The fact that increasing the probability of finding a dealer can decrease welfare in a run equilibrium creates a dilemma. As stated before, if this probability is high enough, the truth-telling equilibrium is the unique in the economy. Therefore, in one hand, implementing policies to increase this probability is desirable because it can eliminate all run equilibria. Which implements the equilibrium outcome associated with the highest welfare. On the other hand, if it does not do so, it can make agents in the economy worse off if they keep playing a run equilibrium.

This paper contributes to the large literature, started by Diamond and Dybvig (1983), which takes a mechanism design approach to the bank problem and interprets financial fragility as the multiplicity of equilibria in such mechanisms. This literature includes, but is not limited to, Wallace (1988), Peck and Shell (2003), Green and Lin (2003), Andolfatto et al. (2007), Ennis and Keister (2009), Cavalcanti and Monteiro (2011) and Andolfatto et al. (2014). There is also a related literature that investigates the connection between Diamond-Dybvig banks and markets, which includes Allen and Gale (2000) and Allen and Gale (2004). My main contribution to these literatures is to embed the ideas regarding financial fragility to a dynamic model of over-the-counter markets. And, by doing so, I provide a way to relate this market structure to the fragility of the financial system.

More closely related in terms of modeling, there are papers that study financial crisis in over-the-counter markets. For example, Lagos et al. (2011) study financial crises in the context of an over-the-counter market where dealers provide liquidity to the economy and there is an aggregate shock which lowers the investors' valuation of the asset. Feldhütter (2012) proposes and estimates a structural model, which is a variation of Duffie et al. (2005), where there is also an aggregate shock which lowers the investors' valuation of the asset. In these papers the financial crisis is associated with the moment in which the aggregate shock hits the economy. My contribution here is to show how the "lower valuation" can be generated endogeneously, as an equilibrium outcome, and study how the market structure relates with the existence of those equilibria.

Regarding multiple equilibria in over-the-counter markets, Trejos and Wright (2014) generates multiple equilibria in a generalized version of Duffie et al. (2005) where preferences are separable but not quasi-linear. This paper complements this result by showing how the multiplicity can be the outcome of runs against financial institutions.

This paper is also related to the mechanism design literature. In particular, it is related to papers that illustrates how weakly optimal mechanisms can implement perverse outcomes. See Demski and Sappington (1984) and Postlewaite and Schmeidler (1986) for some of the early papers in this issue. And finally, to the best of my knowledge this is the first application of the Athey and Segal (2013) balanced team mechanism to contracts in financial markets.

The rest of the paper is organized as follows. In section 2 I study a simple model of a financial institution and provide intuition for the run equilibria in a context with only one period and without markets. In section 3 I introduce the environment of the general model. In section 4 I introduce the coalition problem, its optimal asset allocation and the balanced team mechanism. In section 5 I define equilibrium and state the main theoretical results. In section 6 I provide the numerical examples. And in section 7 I offer a discussion of my findings in terms of policy implications.

2. A SIMPLE MODEL OF FINANCIAL INSTITUTIONS

In this section I discuss a simple model of financial institutions—a simplified version of the full model designed to provide intuition for the existence of run equilibria. The environment is the following. There is one period and there are $N \in \mathbb{N}$ agents which I call investors. There is a consumption good called *fruit*, which can be consumed in non-negative amounts, and each investor has a positive endowment $\bar{A} > 0$ of assets. The asset should be thought as a Lucas tree, where one unit of asset bears one unit of fruit. Fruits cannot be traded though. In order to consume fruits an investor must hold assets. Investors are *ex-ante* identical. In the beginning of the period $N_h > 0$ investors turn to be of type θ_h and $N_l = N - N_h > 0$ turn to be type θ_l. In this simplified version there is no aggregate uncertainty, so the number of agents of each type is known in advance to all investors. The model follow a transferable utility framework. The utility of an investor of type θ is $u(c;\theta) + m$, where c is the consumption of fruits and m is the transfer of utility to the investor. Note that m can be either positive or negative. For each θ, the utility function $u(\cdot;\theta)$ is twice continuous differentiable, concave and $u'(c;\theta_h) > u'(c;\theta_l) \geq 0$ for all strictly positive c. Investors can commit with contractual arrangements, the only friction is private information.

The Pareto efficient outcome

Since this is a transferable utility framework, an outcome is Pareto efficient if, and only if, it maximizes *ex-ante* utility of investors. Therefore, it solves

$$(1) \quad \max_{a \in \mathbb{R}^2_+} \{N_h u(a_h;\theta_h) + N_l u(a_l;\theta_l); \text{ subject to } N_h a_h + N_l a_l = N\bar{A}\},$$

where a_h denotes the assets allocated to an investor of type θ_h and a_l the assets allocated to an investor of type θ_l. With some abuse of notation, for the rest of the section I call $a = (a_h, a_l)$ the solution to problem (1). The outcome a cannot be directly implemented because types are private information. A mechanism must be set up so investors have incentive to reveal their types.

A money market mutual fund mechanism

In order to implement the Pareto efficient outcome, I will focus on a simple mechanism which resembles how a money market mutual fund (MMMF) operates. Worth mentioning that there are different ways to implement this outcome. The reason I use this particular mechanism is because the intuition built from it can be carried out to the general version of the model I discuss later.

The mechanism works as follows. In the onset, before knowing their types, investors deposit $\bar{A} - a_l$ assets in the MMMF. Note that $\bar{A} - a_l$ is non negative since $u'(c;\theta_h) > u'(c;\theta_l)$ for all strictly positive c. This deposit gives equal shares of the total fund portfolio, $N(\bar{A} - a_l)$, to all investors. The MMMF then commit to buy the investor's shares at the price $p = u'(a_h;\theta_h) \geq u'(a_l;\theta_l)$. That is, once types are realized, investors have two options: they either do not sell their shares to the MMMF and become the residual claimer of its portfolio; or they sell their shares to the MMMF at the price $p = u'(a_h;\theta_h) \geq u'(a_l;\theta_l)$ in terms of utility. If all investors decide to sell their portfolios, then the MMMF cannot meet its obligations and the assets are equally divided among investors. I refer to not sell the portfolio to the MMMF as action s_h and to sell the portfolio as action s_l.

There are alternative interpretations of this mechanism besides the one of a money market mutual fund. For example, one can interpret the financial institution as an asset backed commercial paper (ABCP) program. The program issues ABCP in order to finance the purchase of the $N(\bar{A} - a_l)$ assets. Investors then have the option to either liquidate these papers and get the payment $p(\bar{A} - a_l)$ in utility, or to roll it over to the end of the period and be the residual claimer of the portfolio in the ABCP program. Another interpretation is of life insurers issuing extendable funding agreement backed notes (XFBS). These notes gives investors the option to extend their investment and stay as the residual claimer of the portfolio in the program, or to not extend and get paid a predetermined amount. These financial institutions are considered part of the so called shadow bank system, which suffered with runs during the financial crisis in 2007-08.[4]

Equilibrium

The mechanism above is associated with a Bayesian game of incomplete information, where each investor n strategy consists of an action $s^n(\theta) \in \{s_h, s_l\}$ as a function of his type $\theta \in \{\theta_h, \theta_l\}$.

[4]See Covitz et al. (2013) for the institutional details of ABCP programs and Foley-Fisher et al. (2015) for the institutional details of XFBS programs. Both these papers found evidence of runs in 2007 in the beginning of the financial crisis. Additionally, Schmidt et al. (2014) found evidence of runs against money market mutual funds in 2008.

The payoffs of an investor n implied by an action profile $s = (s^1, \dots, s^N) \in \{s_h, s_l\}^N$ are

$$v_n(s_h; \theta, s^{-n}) = u(\bar{A} + (N - n_h)(\bar{A} - a_l)/n_h; \theta) - p(N - n_h)(\bar{A} - a_l)/n_h$$
$$v_n(s_l; \theta, s^{-n}) = \begin{cases} u(a_l; \theta) + p(\bar{A} - a_l) & ; \text{ if } n_h > 0 \\ u(\bar{A}; \theta) & ; \text{ if } n_h = 0 \end{cases},$$

where n_h is the number of investors taking action s_h in the action profile s, and s^{-n} is the action profile of investors other than n. For the remaining of this section I use $n_h(s)$ and $n_h(s^{-n})$ to denote the number of investors taking action s_h in the profiles s and s^{-n}. An equilibrium of the model is a strategy profile $\{s^n\}_n$ constituting a Bayesian-Nash equilibria of this game.

Truth-telling equilibrium

An strategy s is truth-telling if $s(\theta_h) = s_h$ and $s(\theta_l) = s_l$. It is easy to see that the outcome of the MMMF mechanism is the solution of the planer problem (1) if, and only if, investors are following truth-telling strategies. Additionally, we can show that truth-telling is a Bayesian-Nash equilibrium of the game. To see this note that the asset allocated to an investor who take action s_h under truth-telling strategies is

$$\bar{A} + \frac{(N - n_h)(\bar{A} - a_l)}{n_h} = \frac{N_h\bar{A} + (N - N_h)(\bar{A} - a_l)}{N_h} = \frac{N\bar{A} - N_l a_l}{N_h} = \frac{N\bar{A} - N_l a_l}{N_h} = a_h,$$

where the last equality comes from the resource constraint in problem (1). Combining the above equation with the definition of v_n, we find that $v_n(s_h; \theta, s^{-n}) = u(a_h; \theta_h) - p(a_h - \bar{A})$. Thus,

$$v_n(s_h; \theta, s^{-n}) = u(a_h; \theta_h) - p(a_h - \bar{A}) = \max_{a \in \mathbb{R}_+} \{u(a; \theta_h) - p(a - \bar{A})\},$$

where the last equality is implied by $u'(a_h; \theta_h) = p$ and the first order of the maximization problem. From the above equation we can conclude that, under truth-telling, $v_n(s_h; \theta_h, s^{-n}) \geq v_n(s_l; \theta_h, s^{-n})$ since $v_n(s_h; \theta_h, s^{-n}) \geq u(a_l; \theta_h) - p(a_l - \bar{A}) = v_n(s_l; \theta_h, s^{-n})$. It is analogous to show that s_l is a best response to an investor of type θ_l. That is, $v_n(s_l; \theta_l, s^{-n}) \geq v_n(s_h; \theta_l, s^{-n})$.

Numerical example with runs

The MMMF mechanism implements the Pareto efficient outcome. However, in some cases it also implements an untruthful equilibrium where every investor announce type θ_l, which I call a run. To illustrate this possibility consider the following numerical example. The economy has $N = 10$ investors. The number of type θ_h investors is $N_h = 9$ and the number of type θ_l investors is $N_l = 1$. The utility of an investor of type θ_h is $2\sqrt{c} + m$ and the utility of an investor of type θ_l is just m. That is, investors of type θ_l do not derive utility from the consumption of fruits. For the last, $\bar{A}$ is normalized to be $N_h/N = 0.9$. Under this normalization the Pareto efficient outcome has $a_h = 1$ and $a_l = 0$. Each investor deposits 0.9 assets at the MMMF in the onset of the period and the price they can sell their portfolio is $p = \partial(2\sqrt{a})/\partial a|_{a=1} = 1$.

Figure 1 displays the indifference curves, of an investor of type θ_h, associated with actions s_h and s_l, given different actions profiles of other investors. The blue curves are associated with the investor not selling their portfolio to the MMMF—action s_h. The highest level blue curve gives his payoff if $n_h(s^{-n}) = 8$. The other blue curves gives his payoff if $n_h(s^{-n}) = 2,\ 1$ and 0 investors take action s_h. The red curves give his payoff for action s_l when $n_h(s^{-n}) = 0$, and $n_h(s^{-n}) > 0$.

Regarding a type θ_l investor, selling their portfolio (action s_l) is a strictly dominant action since his payoff is either $v_n(s_l;\theta_l,s^{-n}) = 0.9 > -0.9(10-n_h)/n_h = v_n(s_h;\theta_l,s^{-n})$, if $n_h(s^{-n}) > 0$, or $v_n(s_l;\theta_l,s^{-n}) = 0.0 > -0.9\times 9 = v_n(s_h;\theta_l,s^{-n})$, if $n_h(s^{-n}) = 0$.

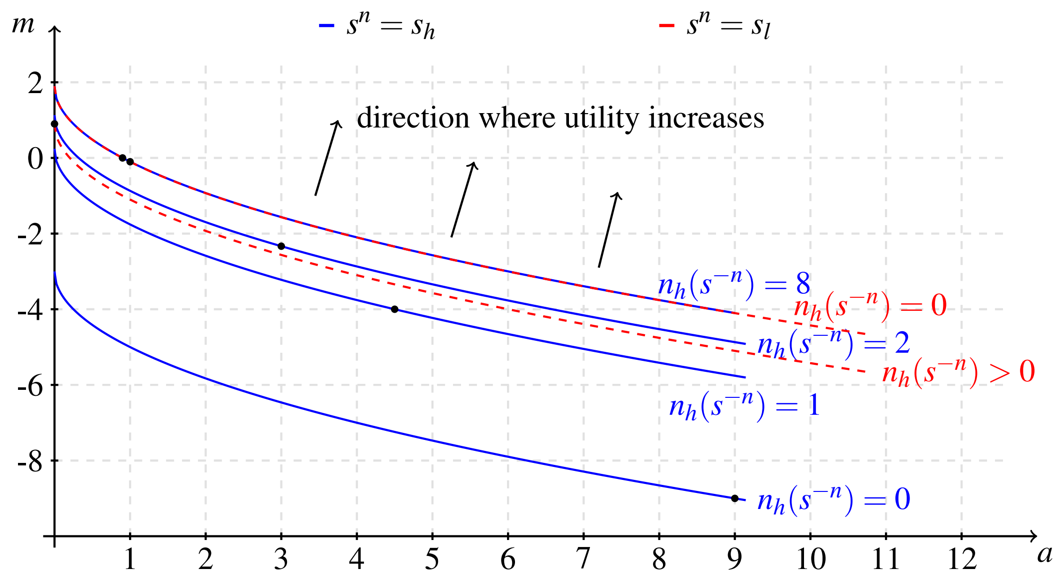


Figure 1: Payoff of type θ_h investor

If an investor n of type θ_h believes that at least other $n_h(s^{-n}) = 2$ investors will not sell their portfolio to the MMMF, his best response is not to sell his portfolio, action $s_h = 0$. However, once his belief of $n_h(s^{-n})$ goes below 2, his best response is to sell his portfolio, action $s_l = 0.9$. This strategic complementary generates an equilibrium where every investor try to sell their portfolio, the MMMF doesn't have funds to cover its obligations and, as a result, assets are equally shared among investors—a form of self-fulfilling run against the money market mutual fund.

Comparison with Diamond and Dybvig (1983)

The model presented in this section has some interesting similarities with Diamond and Dybvig (1983). Both models use mechanisms which resemble contracts observed in the bank/shadow bank industry in order to provide liquidity insurance against preference shocks. But these contracts also have an inefficient equilibrium: a form of run which leads to lower welfare.[5]

The advantage of using this model instead of the classic Diamond and Dybvig (1983) is that, once combined with some recent developments in mechanism design, the model can be easily integrated with the literature of over-the-counter markets. This integration allows us to discuss several features of financial markets such as asset price, trade volume and bid-ask spreads. In the

[5] In the Diamond-Dybvig environment, however, more sophisticate contracts are able to prevent run equilibria under very general assumptions. For example, Andolfatto et al. (2014) propose an indirect mechanism which is able to prevent runs under the same assumptions of the original Diamond-Dybvig model. It is also possible to prevent runs in the version of my model presented in this section. In the general version presented in the next few sections whether a more sophisticated mechanism is able to eliminate multiple equilibria is an open question. Studying this question is an interesting avenue for future research, but it is out of the scope of this paper.

next sections I develop this more general version of the model and use some numerical examples to discuss the mentioned variables and also welfare implications.

3. GENERAL ENVIRONMENT

Time is discrete and infinite. There is a non-atomic measure of investors and a non-atomic measure of dealers. Investors are divided in coalitions of size $N \in \mathbb{N}$ and stay in the same coalition forever. As in the economy with N investors of the previous section, a coalition is interpreted as a financial institution. The measure of coalitions is normalized to one, which implies a measure N of investors, and the measure of dealers is $\alpha \in (0,1)$. There is a consumption good called *fruit*, which can be consumed in non-negative amounts, and a positive aggregate endowment $\bar{A}$ of assets. One unit of asset bears one unit of fruit per period. Fruits cannot be traded, in order to consume it an investor must hold assets. In period zero assets are uniformly distributed among investors.

The model has transferable utility. Investors period utility is $u(c;\theta) + m$, where c is the consumption of fruits, θ is the investor's preference type and m is the transfer of utility to the investor, which can be either positive or negative. For each θ, the utility function $u(\cdot;\theta)$ is twice continuous differentiable, strictly increasing, strictly concave, $\lim_{c\to 0} u'(c;\theta) = \infty$ and $\lim_{c\to\infty} u'(c;\theta) = 0$. Preference types are privately observed, independent across investors, has finite support $\Theta \subset \mathbb{R}$, and follow a Markov chain with transition F. The transition F has a unique ergodic distribution, π^*, and I assume for simplicity that the initial θs are drawn from π^*. This assumption is not used in the theoretical results, but it makes easier to compute equilibria. Dealers do not hold assets and their period utility is just the transfer from investors. Both, investors and dealers, maximize expected utility and have inter-temporal discount $\beta \in [0,1)$. Investors in the same coalition can commit towards future transfers, the only friction is the private information regarding preference shocks.

Agents observe a sunspot variable $x_t \in \mathbb{S} := \{R, NR\}$ in the beginning of period t. The process $\{x_t\}_t$ follows a Markov chain with transition Q. The letters R and NR extend to run and not to run. I consider sunspot equilibria where investors use this variable to coordinate their actions.

In every period there is a random match between coalitions and dealers. Without loss of generality, I assume that every dealer is matched with a coalition. Since there is a measure $\alpha \in (0,1)$ of dealers, this implies that a coalition meets a dealer with probability α. Dealers have access to an inter-dealer competitive market where they can buy/sell assets to trade with the coalitions. I denote the price of assets in terms of utility in the inter-dealer market by $\boldsymbol{p} = \{p_t\}$. It is worthy noticing that the only aggregate uncertainty in the economy comes from the sunspots. As a result, the price in a period t is a function of the history of sunspot realizations $x_0, x_1, \ldots, x_t$.[6]

Figure 2 depicts the sequence of actions. In the beginning of the period investors observe their preference type and sunspot realization. Then they simultaneously announce their type to the coalition. After announcements the coalition either meets a dealer ($\iota_t = 1$) with probability α or it does not meet a dealer ($\iota_t = 0$). The dealer can buy/sell assets for the coalition in the inter-dealer market at the price p_t. But the dealer bargain with the coalition over the gains from trade using the Nash bargaining protocol. The bargaining power of the dealer is $\eta \in [0,1)$. The Nash bargaining between investors and dealers keeps the model tractable; however, since investors have private information, it cannot be applied in general. For this reason I assume in all the propositions in the

[6]The asset distribution is also a state variable in the economy. Thus, price could also be a function of the asset distribution. However, it can be shown that with quasi linear preferences investors decisions are independent of their asset holdings, and, therefore, the asset distribution does not affect prices.

paper that η is in a neighborhood of zero. $\boldsymbol{\phi} = \{\phi_t\}_t$ denotes the transfer from the coalition to the dealer resulting from the bargaining. During the bargaining dealers observe the coalition aggregate asset holdings and the most recent type announcements of investors in the coalition.[7] After the bargaining, transfers of assets and utilities are made.

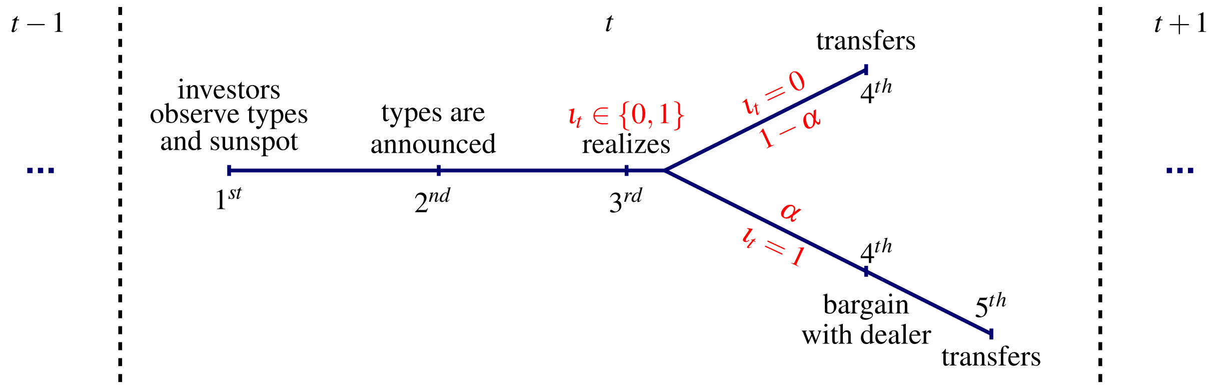


Figure 2: Sequence of actions

The environment is characterized by a family $\mathscr{E} = \{u, N, \alpha, \eta, F, Q\}$. Section 2 environment is a variation of this one where there are no dealers ($\alpha = 0$), investors completely discount the future ($\beta = 0$) and preference shocks follow the distribution described in that section.

4. THE COALITION PROBLEM

The coalition problem is to maximize the ex ante utility of its own investors taking as given what other agents in the economy are doing. Investors face two frictions: they may not have access to a dealer to trade in the inter-dealer market when needed; and investment decisions are distorted because dealers extract part of the gains from trade in the bargaining process. As a result, a contractual arrangement that reallocates assets across investors in the coalition can improve in these two fronts. It increases welfare in the periods where the coalition doesn't meet with a dealer and minimizes the amount of assets traded trough the dealer when the coalition does meet with one, which reduces the rent dealers can extract. The difficulty in implementing such arrangement is that preference types are private information. That is why the balanced team mechanism, proposed by Athey and Segal (2013), is useful in this setting. It implements the efficient allocation of assets among the investors in perfect Bayesian equilibrium (PBE). Pretty much in the same way the money market mutual fund contract did in the simple model of section 2.

The balanced team mechanism has two components. There is the efficient asset policy for the coalition and the transfer scheme that implements this policy outcome in PBE. In the remaining of this section I discuss these two components in detail.

[7]The key assumption is that η is low enough so the coalition don't have incentives to misrepresent the announcement vector during the bargaining. But requiring that the announcement vector is public simplifies the computation of equilibria because you only have to check for individual investors deviations. The assumption that asset holdings are observed is usual and many financial institutions are required by regulators to keep public records of their portfolio.

Asset policy

In this subsection I characterize the asset allocation that maximizes welfare in the coalition for a given price process—all under the assumption that investors' types are truthfully announced. Let me start with notation. Label $\boldsymbol{\theta}_t = (\theta_t^1, \ldots, \theta_t^N) \in \Theta^N := \boldsymbol{\Theta}$ the period t announcement vector. The period t coalition state is a triple $h_t = (x_t, \boldsymbol{\theta}_t, \iota_t) \in H := \mathbb{S} \times \boldsymbol{\Theta} \times \{0,1\}$ of sunspot realization, announcement vector and status of meeting with the dealer ($\iota_t = 1$) or not ($\iota_t = 0$). The history of state realizations up to period t is denoted by $h^t = (h_0, h_1, \ldots, h_t)$. Trough out the paper subscript t denotes a variable realization in period t and superscript t denotes a vector with variable realizations from period zero up to period t. Define the function $U : \mathbb{R}_+ \times \boldsymbol{\Theta} \to \mathbb{R}$ as the maximum aggregate period utility of a coalition with total assets A and vector type $\boldsymbol{\theta}$. That is,

$$(2) \qquad U(A;\boldsymbol{\theta}) = \max_{a \in \mathbb{R}_+^N} \left\{\textstyle\sum_n u(a^n;\theta^n);\ \text{subject to } \sum_n a^n = A\right\}.$$

An asset allocation is a sequence $\boldsymbol{a} = \{a_t\}_t$, where $a_t = (a_t^1, \ldots, a_t^N) : H^t \to \mathbb{R}_+^N$ denotes the amount of assets allocated to each coalition investor in period t contingent on the history h^t. An asset policy is feasible if $(1-\iota_t)[A_t(h^t) - A_{t-1}(h^{t-1})] = 0$ for all histories h^t, where $A_t(h^t) = \sum_n a_t^n(h^t)$ is the aggregate asset holdings in period t and $a_{-1}^n := \bar{A}$ for all n. This condition means that the coalition can only adjust its aggregate asset holdings when it meets a dealer, but it can adjust the asset holdings within its own investors at any period. Label Γ the set of all feasible asset policies. The welfare induce by $\boldsymbol{a} \in \Gamma$ is

$$(3) \qquad W(\boldsymbol{a}) = \mathbb{E}\textstyle\sum_t \beta^t \left[U(A_t;\boldsymbol{\theta}_t) - p_t(A_t - A_{t-1}) - \phi_t(A_{t-1};x^t,\boldsymbol{\theta}_t)\right],$$

where ϕ_t denotes the dealer's fee which is determined by Nash bargaining. Note that ϕ_t depends only on what is observed by the dealer: the aggregate asset holdings of the coalition, the history of sunspots and the current announcement vector. The coalition asset allocation problem is

$$(4) \qquad \max_{\boldsymbol{a} \in \Gamma} W(\boldsymbol{a}).$$

In order to characterize the dealers fee, ϕ_t, it is useful to write problem (4) recursively. The relevant state variables for a coalition in period t are the aggregate asset holding A, the history of sunspots x^t, and the current vector type $\boldsymbol{\theta}_t$. Let $V_t(A;x^t,\boldsymbol{\theta}_t)$ denotes the coalition value function in the end of period t. That is, after any possible trade with a dealer has occurred. By construction, $V_t(A;x^t,\boldsymbol{\theta}_t)$ satisfies the functional equation

$$(5) \qquad \begin{aligned} V_t(A;x^t,\boldsymbol{\theta}_t) = \mathbb{E}_t\textstyle\sum_{T=1}^{\infty}(1-\alpha)^{T-1}\alpha\Big\{\sum_{s=0}^{T-1}\beta^s U(A;\boldsymbol{\theta}_{t+s}) + \\ \beta^T \max_{\hat{A}\in\mathbb{R}_+}\left[V_{t+T}(\hat{A};x^{t+T},\boldsymbol{\theta}_{t+T}) - p_{t+T}(\hat{A}-A) - \phi_{t+T}(A;x^{t+T},\boldsymbol{\theta}_{t+T})\right]\Big\}. \end{aligned}$$

$\phi_t(A;x^t,\boldsymbol{\theta}_t)$ is determined by the Nash bargaining between the coalition and the dealer, where the dealer bargaining power is η and the coalition bargaining power is $1-\eta$. Therefore, it is given by

$$(6) \qquad \phi_t(A;x^t,\boldsymbol{\theta}_t) = \operatorname*{argmax}_{\phi\in\mathbb{R}_+}\Big\{\phi^\eta \max_{\hat{A}\in\mathbb{R}_+}\left[V_t(\hat{A};x^t,\boldsymbol{\theta}_t) - p_t(\hat{A}-A) - V_t(A;x^t,\boldsymbol{\theta}_t) - \phi\right]^{1-\eta}\Big\}.$$

The solution of the above problem implies that

$$\phi_t(A_t, x^t, \boldsymbol{\theta}_t) = \eta \max_{\hat{A}\in\mathbb{R}_+} [V_t(\hat{A}; x^t, \boldsymbol{\theta}_t) - p_t(\hat{A} - A_t) - V_t(A; x^t, \boldsymbol{\theta}_t)]. \tag{7}$$

Note that the dealer in this model is completely passive. He benefits from the rent he can extract from the investors, but there is no decision making besides the one embedded in the Nash bargaining. Replacing (7) in (5) we obtain

$$\begin{aligned} V_t(A; x^t, \boldsymbol{\theta}_t) = \mathbb{E}_t \textstyle\sum_{T=1}^{\infty} (1-\alpha)^{T-1} \alpha \big\{ \textstyle\sum_{s=0}^{T-1} \beta^s U(A; \boldsymbol{\theta}_{t+s}) + \eta \beta^T V_{t+T}(A; x^{t+T}, \boldsymbol{\theta}_{t+T}) + \\ (1-\eta)\beta^T \max_{\hat{A}\in\mathbb{R}_+} \left[V_{t+T}(\hat{A}; x^{t+T}, \boldsymbol{\theta}_{t+T}) - p_{t+T}(\hat{A} - A) \right] \big\}. \end{aligned} \tag{8}$$

Analogous to Lagos and Rocheteau (2009), the functional equation in (8) is equivalent to one in which the coalition has all the bargaining power when trading with the dealer but it only meets a dealer with probability $\hat{\alpha} = \alpha(1-\eta)$. Under this formulation, $V_t(A; x^t, \boldsymbol{\theta}_t)$ can be written as

$$\begin{aligned} V_t(A; x^t, \boldsymbol{\theta}_t) = \mathbb{E}_t \textstyle\sum_{T=1}^{\infty} (1-\hat{\alpha})^{T-1} \hat{\alpha} \big\{ \textstyle\sum_{s=0}^{T-1} \beta^s U(A; \boldsymbol{\theta}_{t+s}) + \\ \beta^T \max_{\hat{A}\in\mathbb{R}_+} \left[V_{t+T}(\hat{A}; x^{t+T}, \boldsymbol{\theta}_{t+T}) - p_{t+T}(\hat{A} - A) \right] \big\}. \end{aligned} \tag{9}$$

And the associated sequential problem is

$$\max_{\boldsymbol{a}\in\Gamma} \hat{W}(\boldsymbol{a}) = \hat{\mathbb{E}} \textstyle\sum_t \beta^t \left[U(A_t; \boldsymbol{\theta}_t) - p_t(A_t - A_{t-1}) \right], \tag{10}$$

where $\hat{\mathbb{E}}$ denotes the expectation of future meetings with dealers using the probability measure induced by $\hat{\alpha} = \alpha(1-\eta)$ instead of α.

A solution to problem (10) doesn't always exist because the problem is unbounded for some prices (for instance, if $\boldsymbol{p}$ is identically zero). However, if it exists it is unique since $u(\cdot, \theta)$ is strictly concave for all θ. For now I assume that $\boldsymbol{p}$ is such that a solution exists and label $\boldsymbol{a}^*_{\boldsymbol{p}}$ this solution. Note that $\boldsymbol{a}^*_{\boldsymbol{p}}$ is a function of the price $\boldsymbol{p}$, but I omit this subscript trough out the text to keep the notation simple. The first order conditions of problem (10) are given by

$$p_t - \hat{\mathbb{E}}\{\beta^{d_k} p_{t+d_k} \mid h^t, \iota_t = 1\} = \hat{\mathbb{E}}\{\textstyle\sum_{d=0}^{d_k-1} \beta^d U'(A_t; \boldsymbol{\theta}_{t+d}) \mid h^t, \iota_t = 1\}, \tag{11}$$

$$[\forall\, n, m]: \; u'(a_t^n, \theta_t^n) = u'(a_t^m, \theta_t^m) \text{ and} \tag{12}$$

$$\lim_{k\to\infty} \hat{\mathbb{E}}\{\beta^{t_k} p_{t_k} A_{t_k}\} = 0; \tag{13}$$

where $A_t = \sum_n a_t^n$ is the aggregate asset holdings of the coalition, t_k is the time period in which the coalition meets with a dealer for the kth time and $d_k = t_k - t_{k-1}$ the time interval between the meetings with dealers. Equation (11) says that the marginal expected cost of buying one additional unit of asset and hold it until the next meeting with a dealer should equal the marginal expected benefit of holding this asset for the same period of time. Equation (12) says that at any period in time the marginal utility of holding an asset should be equalized across investors in the same coalition. And equation (13) gives the usual transversality conditions. It is straightforward to show that equations (11) to (13) provide necessary and sufficient conditions for a feasible asset policy $\boldsymbol{a} \in \Gamma$ to solve problem (10) and, therefore, the coalition problem.

Transfers

The transfers in the balanced team mechanism are designed so investors internalize the impact of their announcements on the other coalition investors. Let

$$(14)\quad v^n(a^{*t},h^t) = u(a_t^{*n}(h^t),\theta_t^i) - p_t(x^t)[a_t^{*n}(h^t) - a_{t-1}^{*n}(h^{t-1})] - \phi_t(A_{t-1}^*;x^t,\boldsymbol{\theta}_t)/N$$

denote the period utility of an investor n given the policy $\boldsymbol{a}^*$ and history h^t. It is implicit in equation (14) that investors pay the cost associated with their changes in asset holdings at the market price $p_t(x^t)$ and share equally the cost associated with the dealer fee. There are different ways to formulate how these costs are shared. For instance, one could have each investor paying an equal share of the aggregate cost, *i.e.*, $\{p_t(x^t)[A_t^*(h^t) - A_{t-1}^*(h^{t-1})] + \phi_t(A_{t-1}^*;x^t,\boldsymbol{\theta}_t)\}/N$. I chose the formulation above because the associated payoffs converge to those in a competitive Walrasian equilibrium when α converges to one and η to zero. This convergence guarantees uniqueness of equilibrium in a neighborhood of these parameters. This result is discussed in section 5.

Given a history h^t, let $\psi_t^{*n}(h^t)$ be the period t utility of all investors in the coalition other than investor n implied by $\boldsymbol{a}^*$. That is,

$$(15)\quad \psi_t^{*n}(h^t) = \textstyle\sum_{i\neq n} v^i(a^{*t},h^t).$$

$\psi_t^{*n}(h^t)$ is the term that investor n must internalize in order to have incentives to truthfully reveal his type. Athey and Segal (2013) refer to the mechanism associated with the transfer $\psi_t^{*n}(h^t)$ as the team mechanism. It is the equivalent to the Vickey-Groves-Clarke (VGC) mechanism for a dynamic setting. A major problem with this transfer is that it is not budget balanced. In order to make it budget balanced, instead of working directly with $\psi_t^{*n}(h^t)$, the balanced team mechanism uses the impact of the announcement in the expected present value of $\psi_t^{*n}(h^t)$, which Athey and Segal (2013) call the incentive term of the agents. Formally, the investor n incentive term is

$$(16)\quad \gamma_t^n(h^{t-1},x_t,\theta_t^n) = \mathbb{E}\left[\textstyle\sum_{s=t}^{\infty}\beta^{s-t}\psi_s^{*n}(h^s) \;\middle|\; h^{t-1},x_t,\theta_t^n\right] - \mathbb{E}\left[\textstyle\sum_{s=t}^{\infty}\beta^{s-t}\psi_s^{*n}(h^s) \;\middle|\; h^{t-1},x_t\right].$$

And the transfer in the balanced team mechanism to investor n in a period t given history h^t is

$$(17)\quad \tau_t^{*n} = \underbrace{p_t\left[a_{t-1}^{*n} - a_t^{*n}\right] - \frac{\phi_t}{N}}_{\text{Market component}} + \underbrace{\frac{(N-1)\gamma_t^n - \sum_{i\neq n}\gamma_t^i}{N-1}}_{\text{Insurance component}}.$$

Let $\boldsymbol{\tau}_{\boldsymbol{p}}^*$ denote the sequence of transfers defined by equation (17). The balance team mechanism is given by the pair $\boldsymbol{\mu}_{\boldsymbol{p}}^* = \{\boldsymbol{a}_{\boldsymbol{p}}^*, \boldsymbol{\tau}_{\boldsymbol{p}}^*\}$ of asset policy and transfer. The mechanism $\boldsymbol{\mu}_{\boldsymbol{p}}^*$ is associated with a price $\boldsymbol{p}$, but I omit the subscript for simplicity when possible. The transfer has a market component and an insurance component, as depict in equation (17). The market component reflects the costs in the inter-dealer market of changes in asset holdings, $p_t\left[a_{t-1}^{*n} - a_t^{*n}\right]$, and the rent imposed by the dealer to the coalition, ϕ_t. The insurance component provides incentives for investors to truthfully reveal types in the insurance arrangement embedded in the optimal asset policy by making the investors internalize the welfare impact of his announcement, γ_t^n.

The coalition game

The price $\boldsymbol{p}$, the dealer fees $\boldsymbol{\phi}$, and the balanced team mechanism $\boldsymbol{\mu}^*$ are associated with a dynamic game of incomplete information. I label it the coalition game and focus on its perfect

Bayesian equilibria (PBE). In this game investors' strategies are sequences of type announcements as functions of sunspots, past histories and type realizations. That is, strategies are sequences $\boldsymbol{\sigma} = \{\sigma_t\}_t \in \boldsymbol{\Sigma}$, where $\sigma_t(x^t, h^{t-1}, \theta_t^n) \in \boldsymbol{\Theta}$ and $\boldsymbol{\Sigma}$ denotes the set of investors' strategies.

The truth-telling strategy is a strategy $\boldsymbol{\sigma} \in \boldsymbol{\Sigma}$ such that $\sigma_t(x^t, h^{t-1}, \theta_t^n) = \theta_t^n$ for all realizations of $(x^t, h^{t-1}, \theta_t^n)$. When η equals zero, Athey and Segal (2013) result applies and the coalition game has a perfect Bayesian equilibrium in truth-telling strategy. However, when η is different from zero, the presence of the dealers fee in equation (14) creates a problem for applying the results in Athey and Segal (2013) and, therefore, for implementing the efficient policy. Types of investors other than n affect the utility of investor n trough the dealers fee ϕ_t unless we are in the limit case where the dealer has no bargaining power, *i.e.*, η equals zero. In other words, this model does not have private values. The reason for this is the following. The announcement of other investors affect investor n trough the optimal policy. The dealer anticipates that this optimal policy will be put in place and charge a fee in accordingly. Investor n is directly affect by the other investors announcement since they all share the dealer's fee.[8] For this reason I consider only economies where the bargaining power of dealers η is close to zero. The idea behind it is that truth-telling is a strict best response of the balanced team mechanism when η equals zero and the payoffs are continuous in η. Therefore, truth-telling is still a best response in a neighborhood of η equal to zero.

5. EQUILIBRIUM

In the previous section I described the coalition problem, the implied coalition game and the outcome of the bargaining between coalitions and dealers, all taking as given the price $\boldsymbol{p}$ in the inter-dealer market. But in equilibrium, the price must be such that excess demand in this market equals zero. In this section I provide a definition of equilibrium for the whole economy taking into account market clearing in the inter-dealer market.

Let $\{\boldsymbol{\sigma}^{c,n}\}_{c,n}$ denote the strategy profile in coalition c. The transition probability of types, F, the probability of meeting with a dealer, α, the distribution of sunspots, Q, the balanced team mechanism, $\boldsymbol{\mu}_{\boldsymbol{p}}^*$, and the strategy profile $\{\boldsymbol{\sigma}^{c,n}\}_n \in \boldsymbol{\Sigma}^n$ of each coalition c, generate a sequence of measures $\boldsymbol{\psi}^c = \{\psi_t^c\}_t$ over the space of histories H^t for each coalition c. These measures are defined in the usual way. The excess demand for assets in the centralized market in period t is

$$(18) \quad ED_t(x^t) := \int \int \textstyle\sum_n a_t^n(h^t) d\psi_t^c(h^t|x^t) dc - \bar{A}.$$

It is worth mentioning that $\boldsymbol{\sigma}^{c,n}$ must be measurable in c, otherwise the integral in (18) is not well defined. Note also that the excess demand for assets is a function of the sunspot history x^t. Excess demand in period t is zero if $ED_t(x^t)$ equals zero for all realized histories of sunspots.

DEFINITION 1: An equilibrium is a triple $\{\{\boldsymbol{\sigma}^{c,n}\}_{c,n}, \boldsymbol{\phi}, \boldsymbol{p}\}\}$ such that: (i) for every coalition c the strategy profile $\{\boldsymbol{\sigma}^{c,n}\}_n$ is a PBE of the coalition game associated with the price $\boldsymbol{p}$ and the balanced team mechanism $\boldsymbol{\mu}^*$; (ii) ϕ_t is the solution to the Nash bargaining between coalition and dealer given in equation (7); and (iii) excess demand is zero in every period.

[8]Another way to interpret this issue is that the payments to the dealer are not designed to generate incentives for truth-telling. It comes from the Nash bargaining protocol. As a result, it can distort the incentives generated by the balanced team mechanism since, without private values, this mechanism cannot be used in general and the efficient allocation may not be implementable. See Jehiel and Moldovanu (2001).

Existence and uniqueness of truth-telling equilibrium

A truth-telling equilibrium is an equilibrium $\{\{\boldsymbol{\sigma}^{c,n}\}_{c,n}, \boldsymbol{\phi}, \boldsymbol{p}\}\}$ such that the perfect Bayesian equilibrium played in every coalition is in truth-telling strategies. In order to understand the existence of a truth-telling equilibrium in this model, it is useful to draw a parallel between the model I develop here and Lagos and Rocheteau (2009). This model extends LR in two ways. The first one is that investors are organized in coalitions of size $N \in \mathbb{N}$. Hence, $N = 1$ is the particular case of LR. But this extension alone does not change LR model in any substantial way if there was complete information regarding the preference shocks. If that was the case, this extension would only relabel the utilities in LR to the sum of the utilities of the members in each coalition. The second extension is the private information of preference shocks. With private information the coalitions become an interesting object because a mechanism is needed in order to induce investors to truthfully announce the preference shocks they receive over time.

I build on these observations to show that a truth-telling equilibrium exists for small enough η. First I construct the equilibrium price of the complete information case, which is just a relabeling of LR. For η equal to zero, all the conditions of Athey and Segal (2013) applies and the balanced team mechanism has a truth-telling equilibrium when taking as given any price and, in particular, given the equilibrium price of the complete information case. The balance team mechanism dos not distort the optimal asset policy, thus it must be the same of the complete information case and the market clearing condition is satisfied. Therefore, the constructed price, the associated balanced team mechanism and truth-telling strategies for all coalitions must constitute an equilibrium of the economy with private information for the case of η equal to zero. Moreover, the associated outcome is constrained Pareto efficient as it was in Lagos and Rocheteau (2009). Since when η is equal to zero truth-telling is a strict best response against truth-telling and payoffs are continuous in η, we can conclude that there is also a truth-telling equilibrium in a neighborhood of η equal to zero. These results are summarized in the following proposition. Proofs are in the appendix.

PROPOSITION 1: *Given u, N, α, F and Q, there exists $\bar{\eta} \in (0,1)$ such that for all $\eta \in [0,\bar{\eta}]$ the economy $\mathscr{E} = \{u, N, \alpha, \eta, F, Q\}$ has a truth-telling equilibrium. Additionally, the associated equilibrium outcome is constrained Pareto efficient if, and only if, η is equal to zero.*

An implication from the proof of proposition 1 is that, in a truth-telling equilibrium, prices and asset allocations are the same of the unique equilibrium of an alternative economy with complete information. Therefore, equilibrium outcomes of the private information model form a super-set of the outcome of the complete information model. But under what conditions is truth-telling also the unique equilibrium of the private information model? In other words, under what conditions do the outcomes of the complete and incomplete information environments coincide?

With complete information the liquidity insurance embed in the asset allocation only relies on observed variables. Therefore, there is no reason for investors to announce their types and the only possible sources of multiplicity in the model are the bargaining with the dealer and the trades in the inter-dealer market. Regarding the bargaining with the dealer, with complete information Nash bargaining has a unique outcome. Regarding the inter-dealer market, this market is competitive so agents take prices as given, they don't act strategically. Hence, there is no strategic complementary and the usual assumptions that spur uniqueness in general equilibrium models apply. As a result, the model with complete information generates a unique equilibrium.

With incomplete information the Nash bargaining protocol does not apply unless η is small, but even under this assumption one cannot rule out multiplicity. The reason is that the balanced team mechanism may generate multiple equilibria in the same way that the model in section 2 does. However, this mechanism only goal is to provide insurance to investors against the risk of needing to trade an asset and not finding a dealer to do so. Hence, one can conjecture that if the probability of finding a dealer is high enough, the need for this insurance is low and the same market forces that spur uniqueness in the complete information case also spur uniqueness in the incomplete information one. I find this conjecture to be true.

PROPOSITION 2: *Given u, N, F and Q, there exists $\bar{\alpha}$, $\bar{\eta} \in (0,1)$ such that for all $\alpha \in [0,1]$ and $\eta \in [0,\bar{\eta}]$ the economy $\mathscr{E} = \{u, N, \alpha, \eta, F, Q\}$ has a truth-telling equilibrium and it is unique.*

6. EXAMPLES OF NON TRUTH-TELLING EQUILIBRIA

In the previous section we saw that if search frictions are small enough, the unique equilibrium will have investors playing truth-telling strategies and those outcomes are the same as the model without private information. But what happen to the economy if that is not the case and other equilibria arise? In this section I study numerical examples and compute different equilibria of the model in order to provide a better understand of what happens with market variables in non truth-telling equilibria. In particular, I compute equilibria that resemble financial crises—where investors run against their financial institutions in the same spirit of the runs in section 2.

Computation

I compute equilibria in the following way. I first guess strategy profiles for the coalitions. Then I compute the prices and asset policy that clear the market given those profiles. Finally, I verify that the guessed strategy profiles constitute a perfect Bayesian equilibrium.

The coalition game is a dynamic game of incomplete information. In this type of game, strategies can be very complicated objects (even to guess) since they are functions of all past histories. For this reason I consider only strategy profiles where announcements are functions of the investors current type and sunspot. Specifically, in a fraction $\upsilon \in [0,1]$ of the coalitions, investors follow a run strategy $\boldsymbol{\sigma}^r$, where

$$(19) \quad \sigma_t^r(x^t, h^{t-1}, \theta_t^n) = \begin{cases} \theta_t^n & ; \quad \text{if } x_t = NR \\ \theta_L & ; \quad \text{if } x_t = R \end{cases}$$

for all t and $(x^t, h^{t-1}, \theta_t^n)$. And in a fraction $1-\upsilon$, investors follow truth-telling strategies.

Given the strategy profile, I numerically check if it constitutes and perfect Bayesian equilibrium. In order to do this, I must find beliefs that are consistent with Bayesian updates given other investors strategies, and check if each strategy is a best response given those beliefs. Since the game is dynamic, it is also necessary to keep track of those beliefs over time.[9] This makes computing equilibria challenging. But there are two particular cases in which keeping track of beliefs is not

[9]For example, if the sunspot realization is R from period t to $t+5$ and all investors in a coalition are following the run strategy $\boldsymbol{\sigma}^r$, then the beliefs over the distribution of types in period $t+6$, consistent with Bayesian updates, will be the composition of the Markov chain of types for five periods $F^5 = F \circ F \circ \cdots \circ F$. This is the case because, under $\boldsymbol{\sigma}^r$ strategy, announcements are only informative when the sunspot is NR so investors truthfully announce their types. Therefore, to check if the strategy profile $\{\boldsymbol{\sigma}^r\}_n$ is a PBE one would have to check if it is a best response given beliefs F^l, where $l = 1, \ldots,$ is the number of consecutive periods with sunspot realization R.

necessary. First, if types are *i.i.d.* over time. In this case the distribution of types is the same in every period so beliefs must be the same. The second case is when the state R is an absorbing state for the sunspot. In this case, after the first realization of R, announcements are independent of types. Therefore, the beliefs over types are irrelevant to identify whether a strategy is a best response or not. In this paper I will focus only the *i.i.d.* example.

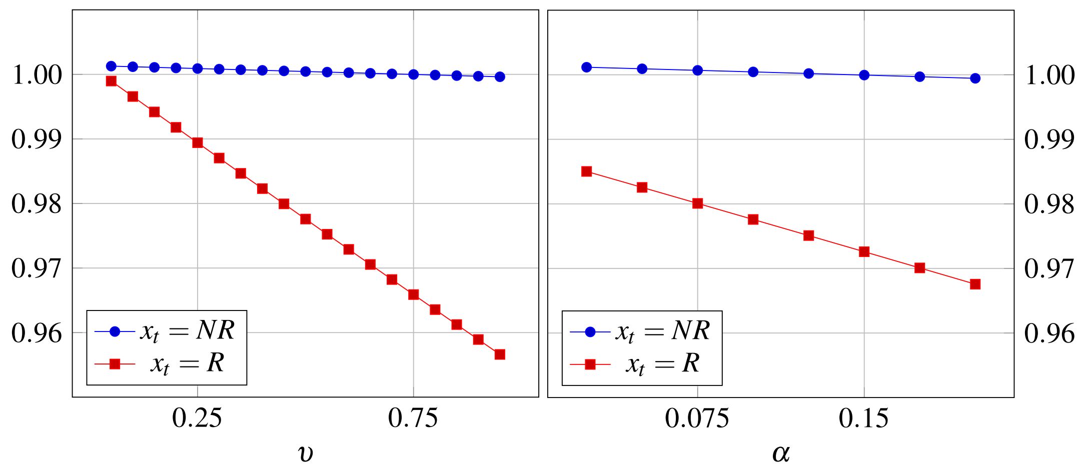
Parametrization

There are $N = 3$ investors per coalition and two types, $\theta_L = 0.7$ and $\theta_H = 1.0$. The utility function is a constant relative risk aversion $u(a;\theta) = \theta\frac{a^{1-\delta}-1}{1-\delta}$ with parameter $\delta = 5.0$. The discount factor β equals 0.9. The endowment of assets is $\bar{A} = 3.0$ (one unit per investor). The bargaining power of the dealers is set to $\eta = 0.1$. In order to understand the impact of search frictions, I compute numerical examples with α going from 0.025 to 0.175 in increments of 0.025. In a similar way, the fraction υ of the coalitions which follow the run strategy is computed from 0.05 to 0.95 in increments of 0.05. Preference shocks are *i.i.d.* with $F(\theta_L,\theta_L) = F(\theta_H,\theta_L) = 0.1$. And the sunspot distribution is $Q(NR,NR) = 0.99$ and $Q(R,R) = 0.97$. I simulate the model for 200 periods. In the simulation, x_t equals R from period 37 to 111 and from period 130 to 136. The initial distribution of asset holdings is set to be what would be the distribution after a long period of time in which x_t equals NR. Prices, trade volume and welfare are relative to the values in the friction-less economy where α equals 1 and η equals 0. In this case the over-the-counter market is equivalent to a competitive Walrasian market, hence there is a unique equilibrium and those variables are well defined. In the figures from 3 to 6 the first graph displays the variables as a function of υ, the fraction of coalitions under a run, for α fixed at 0.1. And the second graph displays the variables as a function of α, the probability of meeting with a dealer, for υ fixed at 0.5. It is worthy mentioning that there is nothing special in these parameters and I had simulated the economy with several others. The qualitative results I obtained seems to be quite general.

Prices

Figure 3 depicts the price for different values of υ and α. Since preferences are quasi linear and strategies are only function of types and sunspots, the equilibrium price process has a simple structure taking only two values. One for periods in which the sunspot realization is $x_t = NR$ and one for periods in which the sunspot realization is $x_t = R$. The price in periods with runs is lower than the price in periods without runs, as it should be expected. Besides, both prices are decreasing in υ. The reason the price in periods with runs is decreasing in υ is straightforward. Due to a higher number of coalitions with investors announcing low valuation for the asset, more coalitions try to sell assets to dealers which creates a selling pressure and reduces price. The reason the price in periods with no runs is also decreasing on υ is that coalitions anticipate that the price will collapse if there is a run. This generate the possibility of capital losses and, as a consequence, the price must go down in advance. And a higher υ, implies a higher capital loss and more the price responds in advance. Interestingly, the increase in the fraction of coalitions in which investors run not only decreases prices. It also increases price volatility since the price in periods with runs decreases more with υ than the price in periods with no runs. The same is true regarding increases in α, as we can see from the second graph in figure 3. The result that higher α, or lower search frictions, could reduce prices were already present in LR. This numerical examples shows that it can also increase the price volatility in the presence of runs.

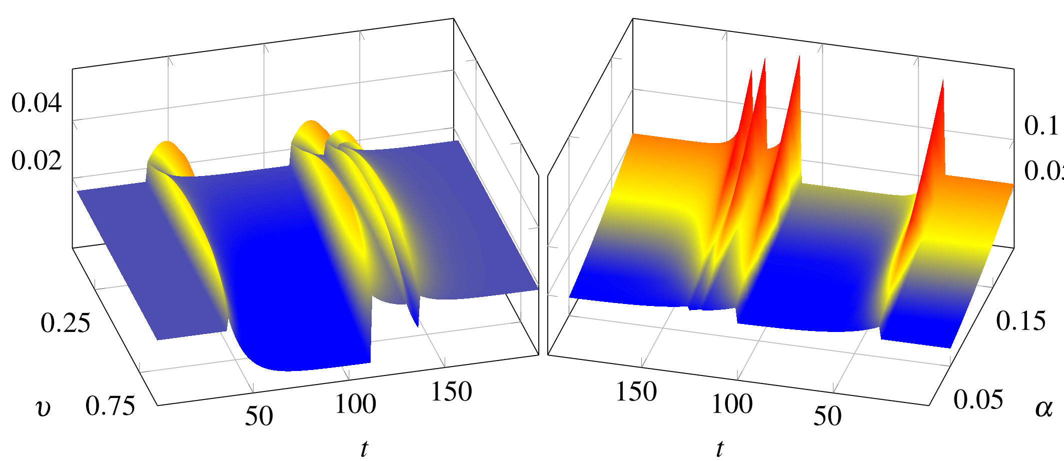
Figure 3: Prices



Trade volume

Figure 4 depicts a simulation of trade volume over the two hundred periods for different values of υ and α. When the sunspot $x_t = R$ hits the economy there is an initial spike in trade volume, which is accompanied by a following contraction. This spike is due to a fire sale effect. Since several coalitions try to sell assets at the same time, price collapses and some coalitions end up buying assets, which increases volume. After this initial reallocation of assets, the coalitions under a run stop trading which in turn decreases trade volume. The spike in trade volume also occurs in recovery periods when the economy switches from $x_t = R$ to $x_t = NR$. In these periods investors stop the run and coalitions try to buy back some of the assets sold in the run periods. As a result, the asset price goes up and there is an initial spike in trade volume which which then bounces back to the average in periods where $x_t = NR$. The first graph in figure 4 shows that higher is the fraction of coalitions under a run, higher is the collapse in trade volume for $x_t = R$. The second graph shows that the spikes in trade volume are more substantial when search frictions are smaller.

Figure 4: Trade volume

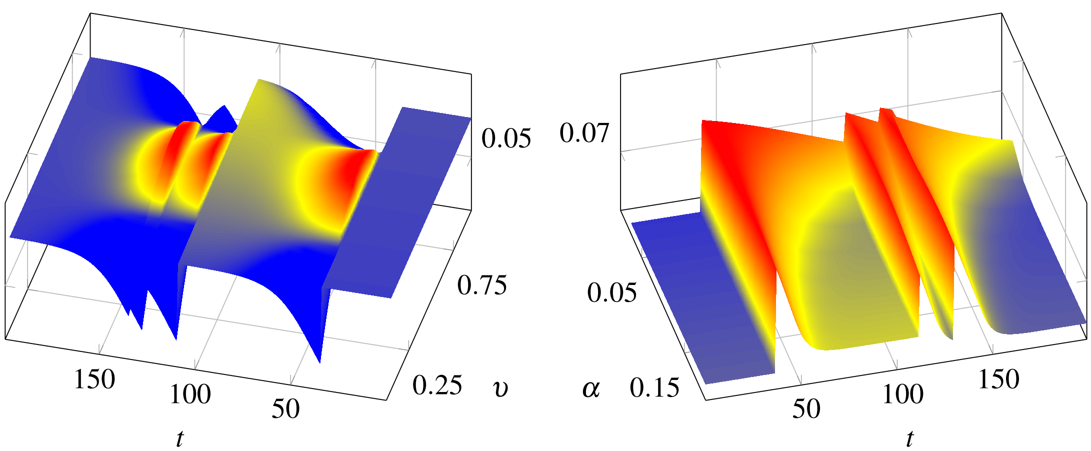


Bid-ask spread

The bid-ask spread is a traditional measure of liquidity used in financial markets. In the context of this model I define it as the average difference between the price faced by the coalitions when buying/selling assets to dealers and the price faced by dealers in the inter-dealer market. The unit price for a coalition buying assets from a dealer is $p_t + \phi_t/dA_t$, where dA_t is the total amount of asset bought. Similarly, the unit price for a coalition selling assets to the dealer is $p_t - \phi_t/dA_t$, where dA_t is the total amount of asset sold. Therefore, the bid-ask spread is defined as $\phi_t/p_t dA_t$.

Figure 5 depicts the average bid-ask spread for each period in the simulation. The direction in which runs impact the bid-ask spread is not uniform, as we can see from the first graph in the figure. If the fraction of coalitions under a run is either small or large, a run decreases the average bid-ask spread. On the other hand, for intermediary values, a run increases the average bid-ask spread. The intuition is that coalitions under a run want to sell their assets and, as a consequence, the price collapses. Then the coalitions that are not under a run have huge gains in buying those assets at this lower price. Dealers take advantage of those coalitions by charging them high bid-ask spread. If there are too many coalitions under a run, then there are not many coalitions to take advantage of the lower prices and dealers are able to charge this high bid-ask spreads. If there are too little coalitions under a run, then the price doesn't collapse so there is no margin for dealers to increase the bid-ask spreads. The intermediary case is where a run causes the bid-ask spreads to go up. The same analysis apply for when the economy switches back from a run state to a no run state. From the second graph in figure 5 we can see that if search frictions are smaller (higher α) these effects fade away faster over time. Which is typical in this class of over-the-counter market models.

Figure 5: Bid-ask spread

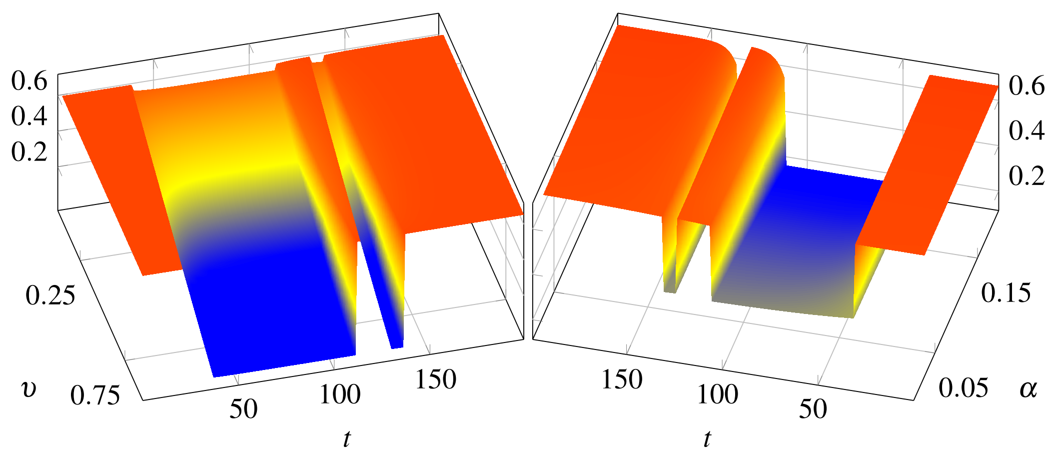


Welfare

From a welfare perspective a run has two effects and they are both due to misallocation of assets in the economy. The first effect is within a coalition under a run. The assets in this coalition are not well allocated because investors with high valuation of the asset end up holding the same amount as investors with low valuation of the asset since they all announce the lower type θ_L. The second effect is across coalitions. When investors in a fraction υ of the coalitions run, these coalitions sell assets in response to the announcements of low valuation. But the coalitions that end up buying those assets not necessarily have investors with higher valuation than the investors in the coalitions facing a run. It just happens that their investors are not misrepresenting their types. So, for example, you could have a coalition with all investors of type θ_H selling assets and ending up with a smaller asset position than a coalition with only one investor of type θ_H.

Figure 6 depicts the period aggregate welfare in the simulation. Both effects of runs discussed above makes the welfare decreasing in the fraction of coalitions under a run in periods where $x_t = R$, as we can see from the first graph. However, the welfare in the periods of run is also decreasing in α. In other words, a reduction in search friction lowers welfare in periods of runs. The reason is if the search friction is smaller (α higher) it is easier for coalitions to sell assets. But it is inefficient for coalitions under a run to do so and, as a result, the welfare can go down during periods of runs.

Figure 6: Period aggregate welfare



Search friction has an ambiguous effect on period welfare. In periods with no run the welfare is increasing in α because it allow investors to sell their assets if they need it. And in periods with runs the welfare can be decreasing in α because coalitions sell their assets inefficiently. But what effect dominates? In other words, is the expected welfare increasing or decreasing in α?

Figure 7: Average aggregate welfare

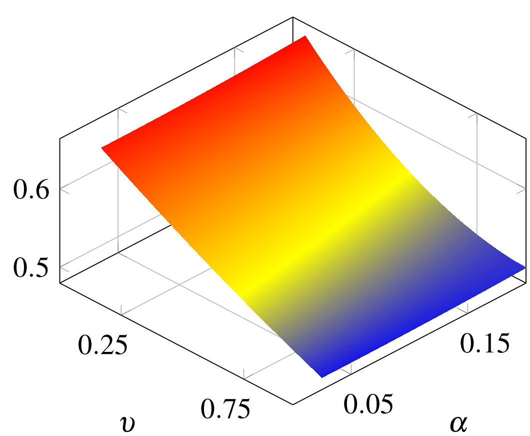


Figure 7 shows the expected aggregate welfare of the economy in the long run. I find that depending on the fraction of coalitions playing the run equilibrium, υ, the expected welfare can be increasing or decreasing in α. For low or high values of υ the welfare is increasing in α. While for intermediary values of υ the welfare is decreasing in α. Proposition 2 states that, as long as the dealer's bargaining power is not too high, if the search frictions is small truth-telling is the unique equilibrium. This proposition could lead someone to think that a good way to improve welfare is by reducing search frictions and, consequently, eliminating all the run equilibria. These numerical simulations show that such policy could have unintended consequences. If the reduction in search friction is not enough to completely eliminate the run equilibria, it may reduce welfare by allowing more coalitions to inefficiently sell there assets during episodes of runs.

7. DISCUSSION

In this paper I identify a connection between runs and over-the-counter markets. I show that, in the model, financial institutions are more vulnerable to runs when search frictions are severe. It is interesting to notice that during the 2007/08 financial crisis the institutions that suffered runs were engaged in trade of asset-backed securities (ABSs), which were mostly traded over the counter. Annual issuance of ABS went from \$10 billions in 1986 to \$893 billions in 2006, as reported by Agarwal et al. (2010). And a growing shadow bank sector has purchased most of these assets. As a result, in 2007 the financial sector featured a large number of financial institutions operating in a market with severe over-the-counter market frictions. My model suggests that these features are important elements to understand the runs during the period.

The most common prescription for enhancing financial stability is to regulate the contracts offered by financial institutions. For example, recently, the Securities and Exchange Commission (SEC) announced a set of proposals to enhance financial stability, which includes a recommendation for the MMF board of directors to impose fees and gate payments in times of heavy redemption activity.[10] And Cochrane (2014) calls for an narrow bank sector funded 100% by equity.

There are two downsides of directly regulating contracts. First, each type of financial institution serves a different type of investor and, therefore, requires a different contract. As a result, the regulation needs to be specific to the type of institution. That is, we need one particular regulation for commercial banks, one for mutual funds, one for structured investment vehicles, etc. Which results in a complex regulatory system doomed to feature loopholes and regulatory arbitrage possibilities. The second downside is that, even if we are willing to write complex regulations to every type of financial institution, it is not clear what regulations we should impose. Even a glimpse through the Diamond-Dybvig literature shows that the optimal contract depends on several details of the environment. When you consider models other than Diamond-Dybvig, the possible regulations grow exponentially. Besides, more often than not, regulations have a welfare cost. In which case, the optimal regulation also depends on how the policy-maker evaluates welfare.

My results suggest a different avenue in which a policy maker can enhance financial stability. Instead of focusing on the particular contract that financial institutions offer, it can intervene on the market for the underlying assets. That is, if we reduce trade frictions enough, we enhance financial stability with no need to regulate individual institutions. And we can interpret some of the policies set in place during the 2007-8 financial crisis as a step in this direction. The Federal Reserve responded to the financial crisis by implementing a number of programs designed to support the liquidity of financial institutions. An example of this kind of program is the Asset-Backed Commercial Paper Money Market Mutual Fund Liquidity Facility (AMLF). In the words of the

[10] See SEC (2013).

Board of Governors of the Federal Reserve System, *"The AMLF was designed to provide a market for ABCP that MMMFs sought to sell."* and *"These institutions used the funding to purchase eligible ABCP from MMMFs. Borrowers under the AMLF, therefore, served as conduits in providing liquidity to MMMFs, and the MMMFs were the primary beneficiaries of the AMLF."*[11] This seems to suggest an attempt to increase the chances of an MMMF to find a buyer for their ABCP. In light of the model, I interpret this as an attempt to increase the probability of a MMMF of meeting a dealer. Of course, much research still need to be done to understand how robust are these results to alternative specifications and how relevant the mechanism proposed here is in practice.

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[11] See `http://www.federalreserve.gov/newsevents/reform_amlf.htm`

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APPENDIX A

I am rewriting the proofs after changes in notation. Please email me if you would like to see the old version, or download it from my website `www.sultanum.com`. I will post the new version online as soon as possible.