

Benchmarks in Search Markets^{*}

Darrell Duffie[†] *Piotr Dworczak*[‡] *Haoxiang Zhu*[§]

July 27, 2015

Abstract

We characterize the price-transparency role of benchmarks in over-the-counter markets. A benchmark can, under conditions, raise total social surplus by increasing the volume of beneficial trade, facilitating more efficient trade matching between dealers and customers, and reducing total search costs. Although the improvement in market transparency caused by benchmarks lowers dealer profit margins on each trade, dealers may introduce a benchmark in order to encourage greater market participation by investors. Low-cost dealers may introduce a benchmark in order to increase their market share through reducing entry by high-cost dealers, a further source of efficiency gain.

Keywords: Benchmark, Search, OTC markets, LIBOR

JEL codes: G12, G14, G18, G21, G23, D43, D47, D83

^{*}We are grateful for useful discussions with and comments from members of the Market Participants Group on Reference Rate Reform (MPG), as well as David Bowman, Willie Fuchs, Will Gornall, Brett Green, Terry Hendershott, Gustavo Manso, Jose Moraga-Gonzalez, Konstantin Milbradt, Paul Milgrom, Andy Skrzypacz, Chester Spatt, Jeremy Stein, Gabor Virag, and Xavier Vives. Helpful comments were offered by seminar and conference participants at Stanford University, Princeton University, U.C. Berkeley, Harvard University, MIT, Finance Theory Group, SFS Cavalcade, European Central Bank, Paul Woolley Centre Annual Conference, Barcelona GSE Summer Forum, Western Finance Association, Bonn Workshop on Information Aggregation, Society for Economic Dynamics meeting, and NBER Summer Institute Asset Pricing meeting.

[†]Dean Witter Distinguished Professor of Finance, Graduate School of Business, Stanford University, and research associate, National Bureau of Economic Research. Duffie was appointed chair of the Market Participants Group on Reference Rate Reform by the Financial Stability Board. For other potential conflicts of interest, see www.darrellduffie.com/outside.cfm.

[‡]Stanford University, Graduate School of Business, dworczak@stanford.edu.

[§]MIT Sloan School of Management and NBER, zhuh@mit.edu.

1 Introduction

An enormous quantity of over-the-counter (OTC) trades are negotiated by counterparties who rely on the observation of benchmark prices. This paper explains how benchmarks affect pricing and trading behavior by reducing market opaqueness, characterizes the welfare impact of benchmarks, and shows how the incentives of regulators and dealers to support benchmarks depend on market structure.

Trillions of dollars in loans are negotiated at a spread to LIBOR or EURIBOR, benchmark interbank borrowing rates.¹ The WM/Reuters daily fixings are the dominant benchmarks in the foreign exchange market, which covers over \$5 trillion per day in transactions.² There are popular benchmarks for a range of commodities including silver, gold, oil, and natural gas, among others.³ Benchmarks are also used to provide price transparency for manufactured products such as pharmaceuticals and automobiles.⁴

Among other roles, benchmarks mitigate search frictions by lowering the informational asymmetry between dealers and their “buy-side” customers. We consider a market for an asset in which dealers offer price quotes to customers who are relatively uninformed about the typical cost to dealers of providing the asset. We provide conditions under which adding a benchmark to an opaque OTC market can improve efficiency by encouraging entry by customers, improving matching efficiency, and reducing total search costs.

Recent major scandals over the manipulation of benchmarks for interest rates, foreign currencies, commodities, and other assets have made the robustness of benchmarks a major concern of international investigators and policymakers. This paper offers a theoretical foundation for public-policy support of transparent financial benchmarks. [Section 6](#) discusses the manipulation of benchmarks in more detail.

¹LIBOR is the London Interbank Offered Rate. EURIBOR is the Euro Interbank Offered Rate. For U.S. dollar Libor alone, the [Market Participants Group on Reference Rate Reform \(2014\)](#) (chaired by one of the authors of this paper) reports that over 3 trillion dollars in syndicated loans and over 1 trillion dollars in variable-rate bonds are negotiated relative to LIBOR. The MPG report lists many other fixed-income products that are negotiated at a spread to the “interbank offered rates” known as LIBOR, EURIBOR, and TIBOR, across five major currencies. As of the end of 2013, [Bank for International Settlements \(2014\)](#) report a total notional outstanding of interest rate derivatives of 583 trillion U.S. dollars, the vast majority of which reference LIBOR or EURIBOR. These swap contracts and many other derivatives reference benchmarks, but are not themselves benchmark products. Other extremely popular benchmarks for overnight interest rates include SONIA, the Sterling OverNight Index Average, and EONIA, the Euro OverNight Index Average.

²See [Foreign Exchange Benchmark Group \(2014\)](#), which reports that 160 currencies are covered by the WM/Reuters benchmarks. These benchmarks are fixed at least daily, and by currency pair within the 21 major “trade” currencies.

³The London Bullion Market Association provides benchmarks for gold and silver. Platts provides benchmarks for oil, refined fuels, and iron ore (IODEX). Another major oil price benchmark is ICE Brent. ICIS Heren provides a widely used price benchmark for natural gas.

⁴For a discussion of the Average Wholesale Price (AWP) drug-price benchmarks, see [Gencarelli \(2005\)](#). The Kelly Blue Book publishes the “Fair Purchase Price” of automobiles, based on the average transaction price by model and location.

Our model works roughly as follows. In an over-the-counter market with a finite number of dealers and a continuum of investors that we call “traders,” the cost to a dealer of providing the asset to a trader is the sum of a dealer-specific (idiosyncratic) component and a component that is common to all dealers. (In practice the clients of financial intermediaries may be buying or selling the asset. We take the case in which traders wish to buy. The opposite case is effectively the same, up to sign changes.) The existence of a benchmark is taken to mean that the common cost component is publicly announced. Each trader observes, privately, whether her search cost is high or low. Traders are searching for a good price, and dealers offer them price quotes that depend endogenously on the presence of a benchmark. Each dealer posts an offer price, available for execution by any trader, anonymously. Traders, who have a commonly known value for acquiring the asset, contact the dealers sequentially, expending a costly search effort, or costly delay, with each successive dealer contacted. At each point in time the trader, given all of the information available to her at that time (including past price offers and, if published, the benchmark) decides whether to buy, keep searching, or exit the market. All market participants maximize their conditional expected net payoffs, at all times, in a perfect Bayesian equilibrium.

Under natural parameter assumptions, which vary with the specific result, we show that publishing the benchmark is socially efficient because of the following effects. First, publication of the benchmark encourages efficient entry by traders, thus increasing the realized gains from trade. The benchmark improves the information available to traders about the likely price terms they will face. This assists traders in deciding whether to participate in the market, based on whether there is a sufficiently large conditional expected gain from trade. The increased transparency of prices created by the benchmark causes dealers to compete more aggressively in their quotes. In this sense, publication of the benchmark mitigates the hold-up problem caused by dealers’ incentives to quote less attractive prices once the search costs of traders have been sunk.

Second, benchmarks can improve matching efficiency, leading to a higher market share for low-cost dealers. When the benchmark is not observed by traders, high-cost dealers exploit the ignorance of traders about the cost of providing the asset and may conduct sales despite the presence of more efficient competitors. The benchmark allows traders to decompose a price offer into a common-cost component and a dealer-specific component for cost and profit margin. As a result, if search costs are sufficiently small, customers trade with the most efficient dealers. Third, benchmarks reduce wasteful search by *(i)* alerting traders that gains from trade are too small to justify entry, and *(ii)* helping traders infer whether they should stop searching because they have likely encountered a low-cost dealer.

Our result that benchmarks promote market efficiency through improved price transparency is consistent with empirical evidence that post-trade transparency introduced in

2003 by TRACE in the U.S. corporate bond market often lowered bid-ask spreads. Key results include those of Bessembinder, Maxwell and Venkataraman (2006), Edwards, Harris and Piowar (2007), Goldstein, Hotchkiss and Sirri (2007), and Asquith, Covert and Pathak (2013).

We also characterize cases in which the introduction of a benchmark *lowers* welfare. This can happen when the market is already relatively efficient without the benchmark. This finding is consistent with the insight of Asriyan, Fuchs, and Green (2015) (in a very different model) that welfare can be non-monotone in the degree of transparency. Asquith, Covert and Pathak (2013) show that the introduction of TRACE lowered transaction volumes in some less liquid segments of the corporate bond market. They speculate that some dealers may have reduced their commitment of capital to the market due to the adverse impact of additional price transparency on their intermediation rents. Additional arguments for and against greater price transparency are discussed by Bessembinder and Maxwell (2008) in the context of corporate bond markets.

Although a published benchmark reduces the informational advantage of dealers over buy-side traders, dealers may sometimes prefer to commit to a benchmark, assuming they are able to coordinate among themselves to do so. Typically, by reducing market opaqueness, a benchmark reduces the local monopoly power of a dealer when facing a customer, and hence decreases each dealer's average profit margin. Thus, dealers prefer to introduce a benchmark only when the resulting reduction in profit margin is more than offset by the increased volume of trade. We provide supporting conditions on model parameters.

In the simplest version of our model, in which dealers have homogeneous costs, we demonstrate that dealers never want to introduce a benchmark when doing so would reduce social surplus. On the other hand, there are cases in which benchmarks would enhance welfare, but dealers lack the incentives to introduce them. Thus, there may be scope for regulators to promote benchmarks in order to improve market efficiency. Recently, Japan and the United Kingdom introduced legislation in support of financial benchmarks, and the European Union has announced plans to do so.

Finally, we analyze how incentives to commit to a benchmark differ across different types of dealers. Given the improvement in matching efficiency caused by benchmarks, we show that the most efficient dealers can use a benchmark as a "price transparency weapon" that drives inefficient competitors out of the market and draw trades to dealers in the "benchmark club." This may help explain why benchmarks such as LIBOR were first introduced into the Eurodollar loan market by large London-based banks, well before the introduction of LIBOR swaps, without support by regulators (see Hou and Skeie 2013).

Benchmarks serve important purposes beyond those modeled in this paper. As discussed by Duffie and Stein (2014), the existence of a benchmark makes it possible to contract in

advance for the exchange of an asset at a formulaic price that depends on the benchmark. For example, with the benefit of a price benchmark, a forward contract for oil can be cash-settled, rather than settled in a more costly way by physical delivery of oil. A benchmark also permits investors to monitor the effectiveness of trade execution by agents acting as their asset managers.

Our analysis draws upon techniques first used in search-based models of labor markets, in a literature surveyed by [Rogerson, Shimer and Wright \(2005\)](#). The framework that we consider features mixed strategies in pricing (as modeled by [Varian \(1980\)](#), [Burdett and Judd \(1983\)](#), and [Stahl \(1989\)](#), among others) and uncertainty about the distribution of prices, as in [Rothschild \(1974\)](#). Our model builds on that of [Janssen, Pichler and Weidenholzer \(2011\)](#), with two important differences that allow us to study welfare implications. First, we introduce endogenous entry to study efficient participation in the market.⁵ With endogenous entry, we show that the result of [Janssen, Pichler and Weidenholzer \(2011\)](#), that sellers never wish to disclose their costs to the market, may fail. Indeed, in our model setting, the fact that dealers often wish to publish a benchmark is consistent with the historical emergence of dealer-supported financial benchmarks. Second, we permit heterogeneity in dealers' costs. We show that benchmarks promote the direction of trade toward more efficient dealers.

[Section 4](#) of our paper on matching efficiency is related to [Benabou and Gertner \(1993\)](#), who analyze the influence of inflationary uncertainty (similar in spirit to the effect of cost uncertainty in our model) on welfare and on the split of surplus between consumers and firms. The relationship between their approach and ours with regard to uncertainty can be described as “local” versus “global.” [Benabou and Gertner \(1993\)](#) analyze the marginal effect on welfare when uncertainty is reduced slightly, while the introduction of a benchmark in our setting reduces this source of uncertainty significantly. A limitation of their model is its restriction to only two sellers.

The remainder of the paper is organized as follows. [Section 2](#) states the model. In [Section 3](#) we analyze the role of benchmarks in markets with relatively high search costs, focusing on how benchmarks encourage market participation by traders. [Section 4](#) focuses on the impact of a benchmark on matching efficiency. In [Section 5](#) we show that dealers may have a total-profit incentive to commit to a benchmark, and we analyze how benchmarks may be endogenously introduced by dealers. [Section 6](#) addresses benchmark manipulation. [Section 7](#) concludes. All proofs are relegated to appendices, which also contain supplementary supporting results and examples.

⁵[Janssen, Moraga-González and Wildenbeest \(2005\)](#) model entry of buyers when sellers' cost is common knowledge but they do not focus on the effect of information disclosure about dealers' costs.

2 Model

This section describes a search-based model of an over-the-counter market, beginning with primitive definitions that cover market participants, the trading protocol, and the definition of market equilibrium.

Our market participants consist of a finite number $N \geq 2$ of dealers and an infinite set of traders, distributed uniformly on $[0, 1]$. For concreteness, we model encounters in which a dealer sells and a trader buys. The model can be equivalently formulated with the buying and selling roles reversed. The important distinction between the two types of agents is that dealers make markets by offering executable price quotes, whereas traders contact dealers sequentially and accept their quotes or not, in a manner to be described.

All trades are for a unit amount of a given asset. Dealer i can supply the asset at a per-unit cost of $c_i = c + \epsilon_i$, where c is common to all dealers and ϵ_i is idiosyncratic. The common cost component c has a cumulative distribution function G with support $[\underline{c}, \bar{c}]$, for some $\underline{c} \geq 0$, with $\underline{c} < \bar{c} < \infty$. High-cost dealers are those whose outcome for ϵ_i is some constant $\Delta > 0$. Low-cost dealers are those with $\epsilon_i = 0$. The common probability of a low-cost outcome is $\gamma > 0$. The cost components $c, \epsilon_1, \dots, \epsilon_N$ are independent. Dealer i observes c and ϵ_i , but does not observe the cost type ϵ_j of any other dealer j .

All traders have a known constant value $v > 0$ for acquiring the asset. Traders have no information concerning which dealers are low-cost. Trader $j \in [0, 1]$ incurs a search cost of s_j for making each contact with a new dealer. For tractability, we suppose that $s_j = 0$ with some probability μ in $(0, 1)$, and that $s_j = s$ with probability $1 - \mu$, for some constant $s > 0$. Search costs are independent across almost every pair of traders. By the exact law of large numbers of Sun (2006), μ is also the fraction of traders with zero search cost, almost surely.⁶ The presence of some traders with zero search cost overcomes the usual Diamond paradox.⁷ Because search costs in practice often arise from delay costs, we refer for simplicity and concreteness to traders with zero search cost as “fast traders,” and to those with non-zero search cost as “slow traders.”

The presence of a benchmark is taken to mean the publication of the common component c of the dealers’ costs. We will compare two market designs: the benchmark case and the no-benchmark case.

The game proceeds as follows. If there is a benchmark, its value c is first revealed. Each dealer i posts a price p_i that constitutes a binding offer to sell one unit of the asset at this price to any trader. This offer price is observed only by those traders who contact the dealer.

⁶We adopt throughout Sun’s construction of the agent space and probability space, and the measurable subsets of the product of these two spaces, so as to allow without further comment various applications of the exact law of large numbers for a continuum of essentially pairwise-independent random variables.

⁷The Diamond paradox (Diamond 1971) refers to cases in which all dealers charge the monopoly price in a unique equilibrium with no search.

In OTC financial markets, this trade protocol is sometimes called “click for trade.”

Traders (without yet having observed the quotes of any dealers) make entry decisions. A failure to enter the market ends the game for the trader. With entry, a trader contacts one of the dealers, with equal likelihood across the N dealers. Upon observing a dealer’s offer, the trader can accept that offer or the offer of any previously contacted dealer, in which case the corresponding transaction is made and the trader leaves the market. A trader may alternatively continue searching by contacting another randomly selected dealer, again with the uniform distribution over the yet-to-be-visited dealers. The order of dealer contacts is independent across traders. A trader may exit the market at any point without trading, even after having contacted all N dealers. Dealers observe neither the price offers posted by other dealers nor the order in which traders contact dealers. Traders observe nothing about the searches or transactions of other traders.

In many over-the-counter financial markets, traders are not anonymous and dealers’ quotes are good only when offered. In [Appendix A](#), we discuss the implications of this alternative protocol.

A (mixed) strategy for dealer i is a measurable function mapping the dealer’s cost type ϵ_i and the common cost component c to a probability distribution over price offers. In the absence of a benchmark, a strategy for trader j maps the trader’s search cost s_j and any prior history of observed offers to a choice from: *(i)* accept one of the observed offers, *(ii)* continue searching, or *(iii)* exit. (If the trader has not yet visited any dealer, the decision to continue searching is equivalent to the decision to enter the market.) In the presence of a benchmark, the strategy of a trader may also depend on the published benchmark c . The payoff of dealer i is $(p_i - c_i)Q_i$, where Q_i is the total quantity of sales⁸ by dealer i . If trader j successfully conducts a purchase, say from dealer i , then her payoff is $v - p_i - s_j K_j$, where K_j is the number of dealers that she contacted. If she does not purchase the asset, then her payoff is $-s_j K_j$.

An equilibrium is a collection of strategies for the respective agents, possibly mixed (allowing randomization), with the property that each agent’s strategy maximizes at each time that agent’s expected payoff conditional on the information available to the agent at that time, and given the strategies of the other agents. We focus on symmetric perfect Bayesian equilibria. We also assume, essentially without loss of generality, that fast traders play their weakly dominant strategy of always entering the market and contacting all dealers.⁹ As is

⁸That is, $Q_i = \int_0^1 1_{(i,j)} dj$, where $1_{(i,j)}$ has outcome 1 if trader j accepts the offer of dealer i , and otherwise has outcome 0. This integral is always well defined and, under our equilibrium strategies, satisfies the exact law of large numbers, using the Fubini property of [Sun \(2006\)](#).

⁹This assumption is without loss of generality in that for every equilibrium in which fast traders do not play this strategy, there exists a payoff-equivalent equilibrium in which they do. The only exception is the degenerate Diamond-paradox equilibrium, in which all dealers quote the price v , fast traders contact no more than one dealer, and slow traders do not enter.

conventional in the literature for search-based markets, we restrict attention to reservation-price equilibria unless otherwise indicated. These are equilibria in which a trader’s decision to continue searching can be based at any time on a cutoff for the best observed offer to that point.

Concerning our definition of a benchmark as the common component c , a more realistic but far less tractable formulation is one in which the benchmark is the dealers’ average cost, $\sum_i c_i/N$, or average quoted price, $\sum_i p_i/N$. These alternatives provide less precise information to the buy-side than c , although both converge to perfect revelation of c as N gets large, in the sense of the law of large numbers. We expect the qualitative thrust of our results would carry over to these alternative definitions of a benchmark.

3 With High Search Costs, Benchmarks Improve Entry Efficiency

This section considers how benchmarks affect the efficiency of entry by traders. We thus focus on cases in which search costs are relatively high compared to gains from trade.¹⁰ In particular, we maintain throughout this section that gains from trade may fail to exist for sufficiently high cost realizations, in that $\bar{c} \geq v$.

The results include conditions under which having the benchmark dominates the no-benchmark case in terms of expected total social surplus, defined as the expected sum of the payoffs of all agents, both dealers and traders, net of costs.

In order to simplify and isolate the effect of a benchmark on entry decisions, we also assume throughout this section that $\gamma = 1$, that is, all dealers have the same supply cost c . Online Appendix H proves a version of the main result of this section without that assumption. The general case adds technical complications but does not offer any additional insights when it comes to entry. Later, in Section 4, we consider the general case in which the dealers’ costs are heterogeneous and explain how the introduction of a benchmark can improve the efficiency with which traders are matched to low-cost dealers.

3.1 The benchmark case

We first characterize equilibrium in the benchmark case. A considerable part of the analysis here draws upon the work of Janssen, Moraga-González and Wildenbeest (2005) and Janssen, Pichler and Weidenholzer (2011).

In the event that $c > v$, there are no gains from trade, and in light of the benchmark

¹⁰A note on terminology: When we say “gains from trade” we refer to the random variable $(v - c)^+ \equiv \max(v - c, 0)$, representing the “potential” or “maximal” gains from trade. Unless the meaning is clear from the context, we use the term “realized gains from trade” to refer to gains that are generated in the actual equilibrium of the game.

information, slow traders do not enter. Obviously, there can be no trade in equilibrium. If $v - s \leq c \leq v$, because dealers never quote prices below their costs, slow traders still do not enter. Fast traders enter and buy from the dealer that offers the lowest price. It is easy to show that the only equilibrium is one in which all dealers quote a price of c , amounting to Bertrand competition among dealers. From this point we therefore concentrate on the interesting case, the event in which $c < v - s$.

We fix some candidate probability λ_c of entry by slow traders. This entry probability will be determined in equilibrium. Conditional on entry, the optimal policy of a slow trader is characterized by [Weitzman \(1979\)](#): Search until she contacts a dealer whose offer is no higher than a certain cutoff r_c , which depends neither on the history of received offers nor on the number of dealers that have not yet been visited.

A standard search-theory argument—found, for example, in [Varian \(1980\)](#) and elaborated in [Appendix B](#)—implies that the only possible equilibrium response of dealers is a mixed strategy in which offers are drawn from a continuous distribution whose support has r_c as its maximum. Because, in equilibrium, a dealer's price is never worse than a slow trader's reservation price, a slow trader buys from the first dealer that she contacts.

Let $F_c(\cdot)$ be the equilibrium cumulative distribution function of a dealer's price offer. Given the traders' strategies, a contacted dealer assigns the posterior probability

$$q(\lambda_c) = \frac{\mu}{\mu + \frac{1}{N}\lambda_c(1 - \mu)} \quad (3.1)$$

that the visiting trader is fast. Here, we used the property that a slow trader enters with probability λ_c and visits this particular dealer with probability $1/N$. Because, in equilibrium, dealers must be indifferent between all price offers in the support $[\underline{p}_c, r_c]$ of the distribution, we have

$$\left[\underbrace{(1 - q(\lambda_c))}_{P(\text{Sell to slow trader})} + \underbrace{q(\lambda_c)(1 - F_c(p))^{N-1}}_{P(\text{Sell to fast trader})} \right] (p - c) = \underbrace{(1 - q(\lambda_c))}_{P(\text{Sell to slow trader})} (r_c - c). \quad (3.2)$$

We used the fact that a slow trader accepts a price $p \leq r_c$ for sure, but a fast trader accepts p if and only if all other dealers offer worse prices. Thus, the equilibrium cumulative distribution function F_c of price offers is given by

$$F_c(p) = 1 - \left[\frac{\lambda_c(1 - \mu)}{N\mu} \frac{r_c - p}{p - c} \right]^{\frac{1}{N-1}}. \quad (3.3)$$

The lowest price \underline{p}_c in the support is determined by the boundary condition $F_c(\underline{p}_c) = 0$.

We can now calculate the optimal reservation price r_c^* of slow traders. Because traders

value the asset at v , we must have $r_c^* \leq v$. The optimality condition of [Weitzman \(1979\)](#) implies that after observing a quote of $p = r_c^*$, a trader must be indifferent between immediately accepting the offer and continuing to search. We thus have the condition

$$v - r_c^* = -s + v - \int_{\underline{p}_c}^{r_c^*} p dF_c(p). \quad (3.4)$$

Substituting the solution for $F_c(p)$ and conducting a change of variables yields

$$r_c^* = c + \frac{1}{1 - \alpha(\lambda_c)} s, \quad (3.5)$$

where

$$\alpha(\lambda_c) = \int_0^1 \left(1 + \frac{N\mu}{\lambda_c(1-\mu)} z^{N-1} \right)^{-1} dz < 1. \quad (3.6)$$

By direct calculation, the expected offer conditional on c is

$$\int_{\underline{p}_c}^{r_c^*} p dF_c(p) = (1 - \alpha(\lambda_c))c + \alpha(\lambda_c)r_c^*.$$

Equation (3.5) states that the maximum price that a slow trader is willing to accept is the cost of the asset plus a dealer profit margin equal to the trader's search cost s multiplied by a proportionality factor that reflects an entry externality, represented through the function α . This "entry externality" arises as follows. If the slow-trader entry probability λ_c is low, the market consists mainly of fast traders, and competition among dealers pushes the expected profit margins of dealers to zero, in that $\lim_{\lambda \rightarrow 0} \alpha(\lambda) = 0$. (That is, the trading protocol converges to an auction run by fast traders.) On the other hand, if λ_c is close to 1, then slow traders constitute a considerable part of the market, and the existence of search frictions allows dealers to exert their local monopoly power and sell at prices bounded away from their costs.

To complete the description of equilibrium, we must specify the optimal entry decisions of slow traders. Holding the entry probability λ_c fixed, the expected payoff of a slow trader conditional on c and on entry is

$$\pi(\lambda_c) = v - s - \int_{\underline{p}_c}^{r_c^*} p dF_c(p) = v - \frac{1}{1 - \alpha(\lambda_c)} s - c.$$

It can be verified that $\pi(\lambda_c)$ is strictly decreasing in λ_c through the role of $\alpha(\lambda_c)$.

If $\pi(\lambda_c)$ is strictly positive at $\lambda_c = 1$, then the equilibrium slow-trader entry probability

λ_c^* must be 1. Because α is maximized at $\lambda_c = 1$, this happens if and only if

$$c \leq v - \frac{1}{1 - \bar{\alpha}}s,$$

where

$$\bar{\alpha} = \alpha(1) = \int_0^1 \left(1 + \frac{N\mu}{1 - \mu} z^{N-1}\right)^{-1} dz. \quad (3.7)$$

If the profit $\pi(\lambda_c)$ is negative at $\lambda_c = 0$, then there is no entry by slow traders, that is, $\lambda_c^* = 0$. Since $\alpha(0) = 0$, this happens whenever $c > v - s$.

Finally, if $c \in (v - s, v - s/(1 - \bar{\alpha}))$, then we have “interior entry,” in that $\lambda_c^* \in (0, 1)$ is uniquely determined by the equation

$$s = (1 - \alpha(\lambda_c^*))(v - c). \quad (3.8)$$

We summarize these results in the following proposition.

Proposition 1. *In the benchmark case, the equilibrium payoffs are unique, and there exists a reservation-price equilibrium in which the following properties hold.*

1. *Entry.* In the event that $c \geq v - s$, no slow traders enter. If

$$v - \frac{s}{1 - \bar{\alpha}} < c < v - s,$$

then slow traders enter with the conditional probability $\lambda_c^* \in (0, 1)$ determined by equation (3.8). If $c \leq v - s/(1 - \bar{\alpha})$, then slow traders enter with conditional probability 1.

2. *Prices.* In the event that $c > v$, dealers quote arbitrary offers no lower than c . If $c \in [v - s, v]$, then dealers quote offers equal to c . If $c < v - s$, then every dealer quotes offers drawn with the conditional probability distribution function F_c given by (3.3).

3. *Traders' reservation prices.* In the event that $c < v - s$, conditional on entry, a slow trader's reservation price r_c^* is given by (3.5).

4. *Social surplus.* The conditional expected total social surplus given c is

$$\lambda_c^*(1 - \mu)(v - c - s) + \mu(v - c)^+,$$

where $(v - c)^+ \equiv \max(v - c, 0)$. The conditional expected profit of each dealer is

$$\frac{\lambda_c^*(1 - \mu)}{N} \frac{s}{1 - \alpha(\lambda_c^*)}.$$

An immediate implication of Proposition 1 is that entry is inefficient. In equilibrium, if $c \in (v - s/(1 - \bar{\alpha}), v - s)$, the gain from trade for any slow traders is larger than the search cost, but we do not observe full entry. This inefficiency can be understood as a hold-up problem. Once traders enter, search costs are sunk and dealers make higher-than-efficient price offers. Taking into account this hold-up problem, slow traders enter only if gains from trade $v - c$ are significantly higher.

3.2 The no-benchmark case

When the absence of a benchmark prevents traders from observing the common component c , traders potentially make complicated Bayesian inferences based on the observed price offers in order to assess the attractiveness of these offers. To keep the model tractable we restrict attention to equilibria in which traders, when on the equilibrium path, follow a reservation-price strategy.¹¹ That is, in the k -th round of search a slow trader has a reservation price of the form $r_{k-1}(p_1, p_2, \dots, p_{k-1})$, where $(p_1, p_2, \dots, p_{k-1})$ is the history of prior price offers. According to this reservation-price strategy, any offer $p_k > r_{k-1}(p_1, p_2, \dots, p_{k-1})$ is not immediately accepted and any offer $p_k < r_{k-1}(p_1, p_2, \dots, p_{k-1})$ is immediately accepted. An offer $p_k = r_{k-1}(p_1, p_2, \dots, p_{k-1})$ is accepted with some (mixing) probability that is determined in equilibrium. For simplicity, from this point we describe an offer that is not immediately accepted as “rejected,” bearing in mind that the trader retains the option to later accept the offer.

We first characterize reservation-price equilibria, assuming one exists. Then we provide conditions under which a reservation-price equilibrium does exist. The following lemma is an important step in characterizing a reservation-price equilibrium.

Lemma 1. *In every reservation-price equilibrium in which slow traders enter with strictly positive probability, (i) the first-round reservation price r_0^* is equal to v and (ii) for each outcome of c strictly below v , the upper limit of the support of the conditional distribution of price offers is v .*

Without the benchmark, a trader’s ignorance of the common component c of dealers’ costs makes it more difficult for her to evaluate the attractiveness of price offers. Lemma 1 states that this information asymmetry causes a slow trader to accept any price offer below

¹¹Although this restriction is standard in the literature, [Janssen, Parakhonyak and Parakhonyak \(2014\)](#) analyzed non-reservation-price equilibria in a consumer-search model with two firms. They assume that the customer’s value is sufficiently high relative to the firms’ cost, so there is no issue of entry efficiency, a key focus of our model. They also assume that the two firms have identical costs, drawn with the same outcome from a binomial distribution. This shuts down the matching efficiency on which we focus in the next section. Because of these assumptions and the technical difficulties in solving non-reservation-price equilibria in our setting, we follow the more usual convention in the literature of focusing on reservation-price equilibrium.

her value v for the asset, in a reservation-price equilibrium. Thus, only two things can happen if a positive mass of slow traders enter. If $c \leq v$, a slow trader buys from the first dealer that she contacts. If $c > v$, then a slow trader will observe a price offer above her value for the asset, conclude that there is no gain from trade, and exit the market. This outcome—slow traders entering only to discover that there is no gain from trade—is a waste of costly search that would be avoided if there were a benchmark. With a benchmark, as seen in Proposition 1, slow traders do not enter unless the conditional expected gain from trade exceeds the cost s of entering the market and making contact with a dealer.

Using Lemma 1, we can describe the reservation-price equilibrium without the benchmark, analogously with Proposition 1. We define the expected gain from trade

$$X = G(v) [v - \mathbb{E}(c | c \leq v)], \quad (3.9)$$

that is, the probability of a positive gain from trade multiplied by the expected gain given that it is positive. Let λ^* denote the equilibrium probability of entry by slow traders.

Proposition 2. *In the no-benchmark case, if a reservation-price equilibrium exists, it must satisfy the following properties:*

1. *Entry. If $s \geq X$, no slow traders enter, that is, $\lambda^* = 0$. If $s \in ((1 - \bar{\alpha})X, X)$, the fraction λ^* of entering slow traders solves*

$$s = (1 - \alpha(\lambda^*))X. \quad (3.10)$$

If $s \leq (1 - \bar{\alpha})X$, all slow traders enter with probability $\lambda^ = 1$.*

2. *Prices. In the event that $c > v$, dealers quote an arbitrary price offer no lower than c . If $c \leq v$, dealers quote prices drawn with the cumulative distribution:*

$$F_c(p) = 1 - \left[\frac{\lambda^*(1 - \mu) v - p}{N\mu (p - c)} \right]^{\frac{1}{N-1}}. \quad (3.11)$$

3. *Traders' reservation prices. Conditional on entry, a slow trader has a reservation price of v at her first dealer contact. If this first dealer's price offer is no more than v , the slow trader accepts it. Otherwise the slow trader rejects it and exits the market.*
4. *Surplus. The expected total social surplus is $\lambda^*(1 - \mu)(X - s) + \mu X$, and the expected profit of each dealer is $\lambda^*(1 - \mu)X/N$.*

The markets with and without benchmarks, characterized by Propositions 1 and 2, respectively, share some common features. In both, dealers' strategies depend on the realization

of the benchmark c , and slow traders never contact more than one dealer on the equilibrium path. The distribution of quoted prices and the entry probability of slow traders are characterized by functions whose forms, with and without a benchmark, are similar.

That said, there are two crucial differences. First, slow traders' entry decisions in the presence of the benchmark depend on the realization (through publication of the benchmark) of the gains from trade. By contrast, without a benchmark, entry depends only on the (unconditional) expected gain from trade. Second, with the benchmark, the reservation price of slow traders generally depends on the realization of the benchmark c . Absent the benchmark, however, a slow trader's reservation price is always v , so that an offer of v is in the support of price offers regardless of the outcome of c .

Existence of reservation-price equilibria in the no-benchmark case

Before comparing welfare with and without the benchmark, it remains to characterize conditions under which a reservation-price equilibrium exists without the benchmark. Providing general conditions for existence in this setting is challenging. While significant progress on existence has been made by [Janssen, Pichler and Weidenholzer \(2011\)](#), their results do not apply in our setting because they assume that the trader value v is so large that varying its level has no effect on the equilibrium. We cannot make this assumption because the size of gains from trade plays a key role in our analysis of entry. [Benabou and Gertner \(1993\)](#) also provide partial existence results for the case of two dealers, but in a different setting.

[Appendix B](#) provides a necessary and sufficient condition for the existence of reservation-price equilibrium in the case of two dealers, and an explicit sufficient condition for existence with $N > 2$ dealers. The main conclusion is summarized as follows.

Proposition 3. *There exists some $\underline{s} < X$ such that for any search cost s greater than \underline{s} , a reservation-price equilibrium in the no-benchmark case exists and is payoff-unique.*

Proposition 3 states that the equilibrium described in Proposition 2 exists if the search cost is sufficiently large. The condition $\underline{s} < X$ ensures that there exists an equilibrium with strictly positive probability of entry by slow traders. If $s \geq X$ there exists a trivial reservation-price equilibrium in which slow traders do not enter.

3.3 Welfare comparison

We now show that if search costs are high relative to the expected gain from trade, then introducing the benchmark raises the social surplus by encouraging the entry of slow traders.

As noted above, entry may be inefficiently low under search frictions due to the hold-up problem and the negative externality in the entry decisions of slow traders. Because a search

cost is sunk once a slow trader has visited a dealer, a dealer can more heavily exploit its local-monopoly pricing power. Expecting this outcome, slow traders may refrain from entry despite the positive expected gain from trade. The hold-up problem is more severe when more slow traders enter (because this raises the posterior belief of a dealer that he faces a slow trader). These effects apply both with and without the benchmark. The question is whether benchmarks alleviate or exacerbate this situation.

We now state the main result of this section, giving conditions under which adding the benchmark improves welfare by encouraging entry.

Theorem 1. *Suppose that (i) $s \geq (1 - \bar{\alpha})(v - \underline{c})$ or (ii) $s \geq (1 - \psi)X$ holds, where $\psi \in (0, \bar{\alpha})$ is a constant that depends¹² only on μ and N . Then a reservation-price equilibrium in the no-benchmark case (if it exists) yields a lower social surplus than that of the equilibrium in the benchmark case. Condition (i) holds if there are sufficiently many dealers or if the fraction μ of fast traders is small enough.*

There are two key sources of intuition behind Theorem 1. First, the presence of a benchmark allows slow traders to make their entry decisions contingent on additional information about magnitude of gains from trade. In equilibrium with the benchmark, entry is higher precisely when gains from trade are larger. In other words, if the unconditional probability of entry were the same across the two settings, then social surplus would be higher in the benchmark case because, in the equilibrium with the benchmark, volume is positively correlated with gains from trade. Second, adding the benchmark reduces the information asymmetry between dealers and traders. Without the benchmark, a slow trader is not sure whether an unexpectedly high price offer is due to a high outcome for the common cost c of dealers, or is due to an unlucky draw from the dealer's offer distribution. Dealers exploit this informational advantage, which exacerbates the hold-up problem. By providing additional information about dealers costs, benchmarks give more bargaining power to slow traders.

The proof of the theorem is illustrated in Figure 3.1, which depicts the dependence of the benchmark-market social welfare function $W_b(x)$ on the realized gain from trade $x = \max\{v - c, 0\}$. The proof first shows that the expected social surplus in the no-benchmark case is actually equal to $W_b[\mathbb{E}(x)]$. We thus want to show that $\mathbb{E}[W_b(x)] \geq W_b(\mathbb{E}(x))$. Because slow traders increase their entry probability when the benchmark-implied gain from trade is large, we can prove that $W_b(\cdot)$ is convex over the set of x for which the entry probability is interior. Condition (i) ensures the convexity of $W_b(\cdot)$ on its entire domain, allowing an application of Jensen's Inequality. The alternative condition (ii) ensures that $W_b(\cdot)$ is subdifferentiable at $X = \mathbb{E}(x)$, yielding the same comparison. Both conditions require that

¹²We have $\psi = \frac{1}{2} \left[\sqrt{(1 - \bar{\alpha} + \bar{\alpha}\beta)^2 + 4\bar{\alpha}(1 - \bar{\alpha})} - (1 - \bar{\alpha} + \bar{\alpha}\beta) \right]$, where $\beta = N\mu/(1 - \mu)$, and $\bar{\alpha}$ is defined by equation (3.7).

place a hyperplane above the graph of $W_b(\cdot)$, tangent to it at X . That is, we get super-differentiability rather than sub-differentiability, reversing the welfare inequality. Condition (i) guarantees that the inequality is strict.

The reverse welfare ranking of Proposition 4 relies on the fact that there is a bounded mass of slow traders. In an alternative model in which the potential mass of slow traders is unbounded, “full entry” is impossible, and the function $W_b(\cdot)$ in Figure 3.1 is globally convex. In this unbounded-entry model, a reservation-price equilibrium in the no-benchmark case (if one exists) yields a lower social surplus than the equilibrium in the benchmark case. A formal proof of this claim is omitted as it follows directly from the proof of Theorem 1.

3.4 Separating the entry-promoting roles of a benchmark

As argued in our discussion of Theorem 1, introducing a benchmark encourages entry through two channels: (i) signaling when gains from trade are high and (ii) increasing the slow traders’ share of gains from trade by reducing the informational advantage of dealers concerning the cost of the asset. In order to distinguish between these two effects, we study in this subsection (only) an intermediate “costly-benchmark-observation” setting in which traders observe the benchmark only upon making their first contact with a dealer (after making the entry decision but before accepting or rejecting an offer). Essentially, this means that slow traders must pay the search cost s to learn the outcome of the benchmark. This artificial costly-benchmark-observation setting allows us to characterize in the next proposition the specific entry screening effect (ii) of benchmarks, while keeping the other entry effect (i) “switched off.”

Proposition 5. *A reservation-price equilibrium always exists (and is payoff-unique) in the costly-benchmark-observation setting. Moreover, under the condition that $(1-\bar{\alpha})X < s < X$, the equilibrium in the costly-benchmark-observation setting has a strictly higher expected social surplus than that of the reservation-price equilibrium without the benchmark.*

The proposition states that channel (ii), reducing information asymmetry between dealers and traders, always works in favor of introducing a benchmark. By providing slow traders with information about the market-wide cost of the asset to dealers, the presence of a benchmark increases traders’ expected payoffs off the equilibrium path, thus encouraging their entry and raising total social surplus on the equilibrium path.

The next result states that role (i) of a benchmark, signaling when there are high gains from trade, is also relevant.

Proposition 6. *There exists $\underline{s} < X$ such that for any search cost $s \in (\underline{s}, v - \underline{c})$ the expected social surplus is strictly higher in the benchmark case than in the costly-benchmark-observation case.*

4 When Do Benchmarks Improve Matching?

This section characterizes the matching-efficiency role of benchmarks in search-based markets. For this purpose, we must analyze the full-fledged model in which dealers' costs are heterogeneous. So, from this point, we assume that the probability γ that a dealer has a low cost for providing the asset is in $(0, 1)$. Throughout this section we maintain the following two assumptions.¹³

Assumption A.1. *Search is socially optimal, in that $s < \gamma\Delta$.*

Assumption A.2. *Gains from trade exist with probability 1. That is, $\bar{c} < v - \Delta$.*

Together, these conditions imply full entry by slow traders in equilibrium, with the benchmark. This allows us to separately identify the welfare effect of matching efficiency. Assumption A.2 is adopted for expositional purposes only. We give generalized statements (weakening Assumption A.2) of the results of this section in Appendix C. We will show that if search costs are relatively low then adding a benchmark raises social surplus by making it easier for traders to find efficient (that is, low-cost) dealers. Having a low search cost is important because contacting a low-cost dealer is socially optimal only if the search cost is lower than the potential improvement in matching efficiency, that is, under Assumption A.1.

The benchmark case. In the presence of a benchmark, the key intuition for the equilibrium construction from Section 3 carries over to this setting, but the supporting arguments are more complicated, and several cases need to be considered. For that reason, we focus here on parameter regions that are relevant for social-surplus comparisons, and relegate a full characterization to Appendix C. Figure 4.1 summarizes pricing schemes that arise in equilibrium as a function of search cost s . We begin with the following result.

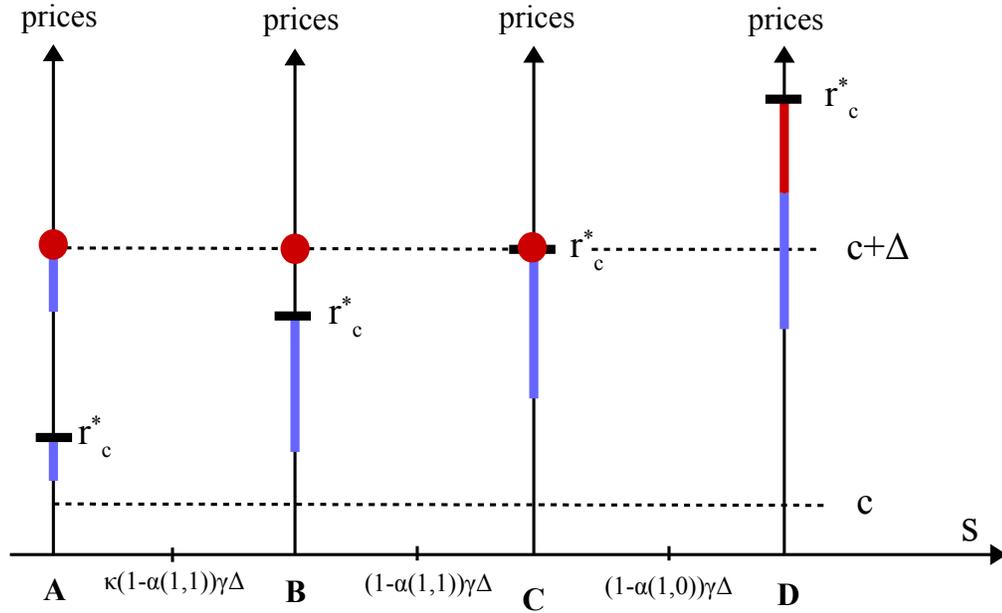
Proposition 7. *In the presence of a benchmark, the equilibrium is payoff-unique and slow traders use a reservation-price strategy.*

Proposition 7 is not surprising given the analysis of Section 3.1. There is, however, a subtle but important difference. Under a reservation-price strategy, a trader is typically indifferent between accepting an offer and continuing to search when the offer is equal to her reservation price. In the setting of Section 3 it does not matter whether traders accept such an offer or not because this event has zero probability. With idiosyncratic costs, however, there are parameter regions in which the only equilibrium requires traders who face an offer

¹³Appendix C provides the supporting analysis when Assumption A.1 fails. In that case, there will be no search in the equilibrium with the benchmark. While the absence of search is socially optimal in this case, this is not the case in which we are most interested.

at their reservation price to mix between accepting and continuing to search. The mixing probabilities are important when there is an atom in the probability distribution of offers located at a trader's reservation price. In equilibrium, these atoms may arise if the reservation price is equal to the high outcome of dealer costs (see panel C in Figure 4.1). This affects the inference made by dealers when they calculate the probability of facing a fast trader.

Fig. 4.1: Price supports in different equilibrium regimes. Lines represent the non-atomic (“continuous”) portions of distributions. Dots represents atoms. Low-cost dealers are shown in blue. High-cost dealers are shown in red.



To account for heterogeneity in dealers' costs, we need to adjust the probability that a dealer's counterparty is fast (as opposed to slow), from that given by equation (3.1). This probability now depends on both the entry probability λ_c and the c -conditional probability, denoted θ_c , that a slow trader rejects an offer from a high-cost dealer. As θ_c gets larger, slow traders search more, and the posterior probability that a dealer is facing a fast trader falls. We will denote by $q(\lambda_c, \theta_c)$ the probability that a contacting trader is fast. Accordingly, the definition of the function $\alpha(\lambda_c)$ from equation (3.6) is generalized to a two-argument function $\alpha(\lambda_c, \theta_c)$ with values in $(0, 1)$. Explicit formulas are provided by equations (C.5) and (C.6) in Appendix C. The role of $\alpha(\lambda_c, \theta_c)$ is analogous to that of $\alpha(\lambda_c)$ in Section 3. Here, $\alpha(\lambda_c, \theta_c)$ is strictly increasing in both arguments. As λ_c and θ_c increase, the probability that a counterparty is slow rises, leading dealers to quote higher prices in equilibrium. The constant $\alpha(1, 1)$ is an analogue of $\bar{\alpha}$ in Section 3, and bounds $\alpha(\lambda_c, \theta_c)$ from above. For the

sake of simplifying upcoming expressions, we denote

$$\hat{\alpha} = \alpha(1, 1).$$

We can now state one of the main results of this section.

Proposition 8. *If $s \leq (1 - \hat{\alpha})\gamma\Delta$, then the equilibrium in the benchmark case leads to efficient matching: slow traders always enter, and all traders buy from a low-cost dealer in the event that there is at least one such dealer present in the market. Additionally, if $s \geq \kappa(1 - \hat{\alpha})\gamma\Delta$, where $\kappa < 1$ is a constant¹⁴ depending only on γ , μ , and N , the equilibrium with the benchmark achieves the second best, in the sense that each slow trader buys from the first low-cost dealer that she contacts, minimizing search costs subject to matching efficiency.*

In order to understand how benchmarks lead to efficient matching and second-best performance in the above sense, consider first the case in which the search cost s is in the interval

$$(\kappa(1 - \hat{\alpha})\gamma\Delta, (1 - \hat{\alpha})\gamma\Delta).$$

This case is illustrated in panel B of Figure 4.1. In equilibrium, slow traders follow a reservation-price strategy with a reservation price r_c^* that is below $c + \Delta$. Low-cost dealers quote prices according to a continuous probability distribution whose support is below this reservation price. Thus, if there are any low-cost dealers in the market, slow traders buy from the first low-cost dealer that they contact. In the unlikely event that there are only high-cost dealers in the market, which happens with probability $(1 - \gamma)^N$, slow traders search the entire market and then trade with one of the high-cost dealers at the price $c + \Delta$. This second-best equilibrium outcome is therefore fully efficient at matching.

The key role of the benchmark in this case is to introduce enough transparency to permit traders to distinguish between efficient and inefficient dealers. The benchmark not only ensures that traders ultimately transact with the “right” sort of counterparty, but also ensures that no search cost is wasted while looking for this transaction. This last conclusion is true under the weaker condition that $s \geq \kappa(1 - \hat{\alpha})\gamma\Delta$.

If $s < \kappa(1 - \hat{\alpha})\gamma\Delta$, however, slow traders may search excessively. As the search cost s get smaller, the equilibrium reservation-price r_c^* also gets smaller (closer to c), and low-cost dealers are forced to quote very low prices if they want to sell at the first contact of any slow trader. Because of their cost advantage, low-cost dealers always have the “outside option” of trying head-on competition by quoting a price above the reservation price (and just below $c + \Delta$), hoping that all other dealers have high costs (in which case low-cost dealers win the resulting effective auction, making positive profits). It turns out that low-cost dealers wish

¹⁴We have $\kappa = (1 - \gamma)^{N-1} / [\mu(1 - \gamma)^{N-1} + (1 - \mu)[1 - (1 - \gamma)^N] / (N\gamma)]$.

to deviate to this strategy when $s < \kappa(1 - \hat{\alpha})\gamma\Delta$. In the resulting equilibrium, which we illustrate in panel A of Figure 4.1 and describe formally in part C.1 of Appendix C, matching remains efficient but we do not achieve the second best, because of the higher-than-efficient amount of search.

The intuition described above indicates that a low-cost dealer's incentive to quote a high price should disappear as the number N of dealers gets large. Indeed, as N becomes large the probability that all other dealers have high costs goes to zero quickly. We confirm in Appendix C.3 that an upper bound on the potential surplus loss (compared to first best) goes to zero exponentially fast with N when $s < \kappa(1 - \hat{\alpha})\gamma\Delta$. In sharp contrast, surplus losses are potentially unbounded in N when s is close to $(1 - \hat{\alpha})\gamma\Delta$. Hence, for practical purposes, it is natural to focus on the case $s \geq \kappa(1 - \hat{\alpha})\gamma\Delta$.

The no-benchmark case. We now show that without the benchmark, it is impossible to achieve the second best.

Proposition 9. *In the absence of a benchmark, if $\bar{c} > \underline{c} + \Delta$ there does not exist an equilibrium that achieves the second best.*

The proof of the proposition explores the simple idea that when there is no benchmark for traders to observe, they cannot recognize a low-cost dealer when they contact one. In the absence of a benchmark, traders can rely only on Bayesian inference based on the observed price quotes. This Bayesian inference, however, can be relatively ineffective. With low realizations of the common cost component c , high-cost dealers may make offers that “imitate” the offers that low-cost dealers would make at higher realizations of c . As a result, slow traders buy from inefficient dealers or engage in socially wasteful search. The benchmark adds enough transparency to allow traders to distinguish between high offers from low-cost dealers and low offers from high-cost dealers.

Welfare comparison. As a corollary of Propositions 8 and 9, we obtain the following result, providing conditions under which adding a benchmark improves welfare.

Theorem 2. *If (i) $\kappa(1 - \hat{\alpha})\gamma\Delta \leq s \leq (1 - \hat{\alpha})\gamma\Delta$ and (ii) $\bar{c} > \underline{c} + \Delta$ both hold, then the equilibrium in the benchmark case yields a strictly higher expected social surplus than that of any equilibrium in the no-benchmark case.*

The theorem does not cover the entire search-cost space. We discuss the remaining cases in Online Appendix F.1, where we show in particular that the second best is not achieved if $s > (1 - \hat{\alpha})\gamma\Delta$, even if the benchmark is present. Nonetheless, with a benchmark, if search costs are not too large, partial efficiency applies to the matching of traders to low-cost dealers. The (unique) equilibrium supporting this outcome has an interesting structure.

High-cost dealers post a price $c + \Delta$ equal to the reservation price r_c^* of slow traders (see panel C of Figure 4.1). Slow traders accept that price with some nontrivial (mixing) probability that is determined in equilibrium.

When search costs are sufficiently high, as illustrated in panel D of Figure 4.1, both types of dealers sell at a strictly positive profit margin, and slow traders buy from the first encountered dealer. Thus, in this case, matching is inefficient. To make welfare comparisons for this parameter region, it is necessary to explicitly characterize the no-benchmark equilibrium. This is a difficult task because traders can potentially search multiple times and their posterior beliefs about c become intractable.

That said, for the case of two dealers, we can provide a full characterization of reservation-price equilibria in the no-benchmark case. Under the condition $s \geq \kappa(1 - \hat{\alpha})\gamma\Delta$, we show that matching is more efficient with a benchmark than without, provided that traders use a reservation-price strategy in equilibrium. Because the details are complicated, we relegate them to Online Appendix F.2.

5 Incentives of Dealers to Introduce a Benchmark

To this point we have taken the presence or absence of a benchmark as given. In practice, benchmarks are often introduced by market participants, such as dealers in OTC financial markets. In this section we explore the incentives of dealers to introduce a benchmark.

5.1 Introducing benchmarks to encourage entry

As we have seen in previous sections, the introduction of a benchmark reduces the informational advantage of dealers relative to traders, and increases the expected payoffs of slow traders. It might superficially seem that dealers have no incentive to introduce the benchmark. In this subsection we show that the contrary can be true. Under certain conditions dealers want to introduce a benchmark in order to increase their volume of trade. We assume that dealers are able to commit to a mechanism leading to truthful revelation of c , so the question of whether they prefer to have the benchmark boils down to comparing dealers' profits with and without the benchmark. We address the implementability of adding a benchmark in Section 6.

For simplicity of exposition we concentrate on the effects of entry on dealers' profits in the setting in which dealers have homogeneous costs for supplying the asset, that is, with $\gamma = 1$. We discuss in Online Appendix G and show formally in Online Appendix H that the same conclusions hold if dealers' costs are heterogeneous. (Just as in Section 3, the heterogeneity of dealers' costs does not "interact" with the effects of entry.)

Theorem 3. *Suppose that (i) $s \geq (1 - \bar{\alpha})(v - c)$ or (ii) $s \geq (1 - \eta)X$, where $\eta \in (0, \bar{\alpha})$ is a constant that depends only on N and μ . If all dealers have the same cost (that is, $\gamma = 1$), then a reservation-price equilibrium in the no-benchmark case (whenever it exists) yields a lower expected profit for dealers than in the setting with the benchmark. Condition (i) holds if there are sufficiently many dealers or if the fraction μ of fast traders is small enough.*

The benchmark raises the profits of dealers by encouraging the entry of slow traders. If search costs are large relative to gains from trade (assumption (i) or (ii) of Theorem 3), dealers benefit from the increased volume of trade arising from the introduction of the benchmark. In order for dealers' total profits to rise with the introduction of a benchmark, entry by slow traders must be sufficiently low without the benchmark, for otherwise the benchmark-induced gain in trade volume does not compensate for the dealers' drop in profit margin on each trade.

A benchmark can be viewed as a commitment device, by which dealers promise higher expected payoffs to traders in order to encourage entry. In particular, a benchmark partially solves the hold-up problem by reducing market opaqueness and hence by giving more bargaining power to traders.

It can be shown that the conclusion of Theorem 3 implies the conclusion of Theorem 1. That is, whenever dealers would opt for the benchmark, it must be the case that the introduction of the benchmark raises social surplus. The opposite is not true. There generally exists a range of search costs in which the benchmark raises social surplus but dealers would have no incentive to commit to it. This is intuitive. Whenever the gain from trade $v - c$ exceeds the search cost s , any increase in entry probability is welfare-enhancing. If, however, this increase is too small to compensate for the reduction in dealers' profit margins, dealers would not opt to introduce the benchmark. The above discussion is illustrated with a numerical example found in Online Appendix I.2.

5.2 Low-cost dealers may compete by introducing a benchmark

This subsection analyzes the incentives of low-cost dealers to introduce a benchmark on their own—despite opposition from high-cost dealers—as a powerful device to compete for business. We show that under certain conditions the collective decision of low-cost dealers to add a benchmark drives high-cost dealers' profits to zero and forces them out of the market. As a result, low-cost dealers make more profits, and the market becomes more efficient overall. This may explain why emergent “benchmark clubs” are often able to quickly attract the bulk of trades in some OTC markets, as was the case with LIBOR.

In order to explain how “benchmark clubs” may emerge, we augment our search-market game of the previous sections with an earlier stage in which dealers decide whether to intro-

duce a benchmark and, after calculating their expected profits, whether to enter the market themselves. To simplify the modeling, we suppose that there are two types of environments, with respect to the cross-sectional distribution of dealer cost efficiency. With some probability $\Gamma \in (0, 1)$, there is a relatively low-cost environment in which the number L of low-cost dealers is at least 2. Otherwise, there are no low-cost dealers ($L = 0$). We rule out the case in which there is exactly one low-cost dealer in the market because, for a high enough cost difference Δ , the low-cost dealer would in that case be an effective monopolist, complicating the analysis. A formal description of the game follows:

1. Pre-trade stage: the introduction of a benchmark and entry by dealers.
 - (a) Nature chooses the dealer-cost environment, whose outcome is not observed. With probability $1 - \Gamma$, all dealers have high costs. With probability Γ , the number L of low-cost dealers is drawn from a truncated binomial distribution with parameters (N, γ) , where the truncation restricts the support to the set $\{2, 3, \dots, N\}$. Conditional on L , the identities of dealers with low costs are drawn independently of L and symmetrically.¹⁵ The idiosyncratic component ϵ_i of dealer i is the private information of dealer i .
 - (b) Dealers simultaneously vote, anonymously, whether to have a benchmark or not. If there are at least two votes in favor, the benchmark is introduced. (We explain in [Section 6](#) how dealers could implement a benchmark, provided that there are at least two of them.) In this case, c immediately becomes common knowledge. If the number of votes in favor is zero or one, the benchmark is not introduced.
 - (c) Dealers make entry decisions. For simplicity, we adopt a tie-breaking rule that dealers enter if and only if their expected trading profits are strictly positive.
 - (d) After dealers' entry decisions, the number of dealers that enter, denoted M , becomes common knowledge among dealers and traders.
2. Trading stage. The game proceeds according to the baseline model described in [Section 2](#), but with N replaced by M .

We denote by

$$X_\Delta = G(v - \Delta)\mathbb{E}(v - c - \Delta \mid c \leq v - \Delta)$$

¹⁵This implies that $\epsilon_1, \dots, \epsilon_N$ are no longer i.i.d. Our results would hold under more general distributions of dealer types. The only properties required of the unconditional distribution of L are (i) symmetry with respect to dealer identities, (ii) that the events $L = 0$ and $L \geq 2$ both have positive probability, and (iii) that the event $L = 1$ has zero probability.

the expected gain from trade with high-cost dealers. The following theorem establishes conditions that are sufficient to induce low-cost dealers to collectively introduce the benchmark and drive their high-cost competitors out of the market.

Theorem 4. *Suppose that $s < (1 - \bar{\alpha})(v - \bar{c})$. Then there is a constant Δ^* such that, for any dealer cost difference $\Delta \geq \Delta^*$, the following are true.*

- *There exists an equilibrium of the extended game in which all low-cost dealers vote in favor of the benchmark and all high-cost dealers vote against it. There are no profitable group deviations in the voting stage.*
- *If the environment is competitive (that is, $L \geq 2$), the benchmark is introduced, all high-cost dealers stay out of the market, all low-cost dealers enter the market, and all traders enter the market.*
- *If the environment is uncompetitive ($L = 0$), the benchmark is not introduced, and high-cost dealers enter the market if and only if $X_\Delta > s$.*

A proof is provided in [Appendix D](#). Here, we explain the intuition of the result.

To start, we note that the theorem makes economically significant predictions about the role of the benchmark only in the case $X_\Delta > s$. This case arises if s is sufficiently small. (The proof provides details.) In the opposite case of $X_\Delta < s$, high-cost dealers earn zero profits regardless of whether the benchmark is introduced, so they are indifferent between voting in favor of, or against, the benchmark, and they never enter. In the discussion below, we focus on the interesting case of $X_\Delta > s$, in which high-cost dealers can make positive profits and strictly prefer not to introduce the benchmark.

The benchmark serves as a signaling device for low-cost dealers to announce to traders that the environment is competitive. The signal is credible because traders, expecting low prices conditional on introducing the benchmark, set a low reservation price in equilibrium. Therefore, high-cost dealers cannot imitate low-cost dealers by deviating and announcing the benchmark. Instead, they prefer to trade in opaque markets without the benchmark and with low participation by slow traders, which allows them to make positive profits.

Low-cost dealers have two distinct incentives to add the benchmark. First, adding the benchmark encourages the entry of slow traders. In addition to the intuition conveyed in [Section 3](#), in the setting of this section the benchmark plays the additional role of signaling the types of active dealers, because the benchmark is added endogenously. On the equilibrium path, once a benchmark is introduced, slow traders believe with probability one that all active dealers have low costs. If a benchmark is not introduced, slow traders believe that all dealers have high costs. As a consequence, the (correctly) perceived gain from trade by slow

traders goes up considerably if a benchmark is added. This channel encourages entry. The condition $s < (1 - \bar{\alpha})(v - \bar{c})$ ensures full entry by traders if the benchmark is introduced.

Second, low-cost dealers capture additional market share by adding the benchmark. With a large enough dealer cost difference Δ , the expected gains from trade are small if the benchmark is not introduced. As a result, we show that slow traders who enter will set a reservation price r^* equal to v in the trading-stage subgame, and high-cost dealers inevitably capture a large proportion of trades with slow traders. If, however, the benchmark is introduced, a sufficiently large Δ makes high-cost dealers' quotes highly uncompetitive, which drives trades to low-cost dealers. Thus, although low-cost dealers' profit per trade may be lower with the benchmark, they capture an additional amount of trade. In fact, in equilibrium, if the environment is competitive, high-cost dealers drop out completely because they cannot make any profit, and low-cost dealers handle all of the trades.

The first part of Theorem 4 asserts that in the equilibrium that we construct there are no profitable *group deviations* in the voting stage. In the usual Nash equilibrium of the voting game, if everyone is voting against or in favor, no dealer is pivotal, and each outcome may be supported in equilibrium. This arbitrariness is eliminated by allowing group deviations.

6 Benchmark Manipulation and Implementation

Recent scandals involving the manipulation of interest-rate benchmarks such as LIBOR and EURIBOR, as well as currency price fixings provided by WM/Reuters, have shaken investor confidence in financial benchmarks. Serious manipulation problems or allegations have also been reported for other major benchmarks, including those for term swap rates, gold, silver, oil, and pharmaceuticals.¹⁶ Major banks are now more reluctant to support these benchmarks in the face of potential regulatory penalties and private litigation. For example, of the 44 banks contributing to EURIBOR before the initial reports of manipulation, 18 have already dropped out of the participating panel.¹⁷ Regulators have responded not only with sanctions,¹⁸ but also by taking action to support more robust benchmarks. The Financial Stability Board has set up several international working groups charged with recommending reforms to interest-rate and foreign-exchange benchmarks that would reduce their susceptibility to manipulation while maintaining their usefulness in promoting market efficiency.¹⁹

¹⁶See, respectively, [Patterson and Burne \(2013\)](#), [Vaughn \(2014\)](#), [Hurtado \(2014\)](#), [Scheck and Gross \(2013\)](#), and [Gencarelli \(2002\)](#).

¹⁷See [Brundsen \(2014\)](#).

¹⁸See [Finch and Larkin \(2014\)](#).

¹⁹See [Official Sector Steering Group \(2014\)](#), [Market Participants Group on Reference Rate Reform \(2014\)](#), and [Foreign Exchange Benchmark Group \(2014\)](#).

The United Kingdom is preparing a comprehensive regulatory framework for benchmarks.²⁰

So far, we have assumed that dealers can credibly commit to the truthful revelation of c . We show in [Appendix E](#) that in our simple setting there exists a mechanism that truthfully implements a benchmark, provided that there are at least two dealers, and a benchmark administrator who can impose transfers. In the mechanism, dealers report the common component c to the administrator individually, and are punished if the reports disagree. The challenge of that mechanism is that it does not preclude collusion by dealers, who may all report a distorted benchmark in a coordinated fashion. That said, a benchmark administrator may use the tool of post-trade reporting to detect such manipulation with certain confidence. For example, if the reported cost c implies a distribution of transaction prices that differs substantially from the empirically observed distribution of transaction prices, there could be scope for further investigation by the benchmark administrator. Explicit models of benchmark manipulation in different settings are offered by [Coulter and Shapiro \(2014\)](#) and [Duffie and Dworczak \(2014\)](#).

7 Concluding Remarks

Benchmarks underlie a significant fraction of transactions in financial and non-financial markets, particularly those with an over-the-counter structure that rules out a common trading venue and a publicly announced market-clearing price. This paper provides a theory of the effectiveness and endogenous introduction of benchmarks in search-based markets that are opaque in the absence of a benchmark. Our focus is the role of benchmarks in improving market transparency, lowering the informational asymmetry between dealers and their customers regarding the true cost to dealers of providing the underlying asset.

In the absence of a benchmark, traders have no information other than their own search costs and what they learn individually by “shopping around” for an acceptable quote. Dealers exploit this market opaqueness in their price quotes. Adding a benchmark alleviates information asymmetry between dealers and their customers. We provide naturally motivated conditions under which the publication of a benchmark raises expected total social surplus by encouraging greater market participation by buy-side market participants, improving the efficiency of matching, and reducing wasteful search costs.

²⁰See [Bank of England \(2014\)](#). The report provides a list of over-the-counter-market benchmarks “that should be brought into the regulatory framework originally implemented in the wake of the LIBOR misconduct scandal.” (See page 3 of the report.) A table listing the benchmarks that are recommended for regulatory treatment is found on page 15. In addition to LIBOR, which is already regulated in the United Kingdom, these are the overnight interest rate benchmarks known as SONIA and RONIA, the ISDAFix interest-rate-swap index, the WM/Reuters 4pm closing foreign exchange price indices (which cover many currency pairs), the London Gold Fixing, the LBMA Silver Price, and ICE Brent (a major oil price benchmark).

In some cases, dealers have an incentive to introduce benchmarks despite the associated loss of local monopoly advantage, because of a more-than-offsetting increase in the trade volume achieved through greater customer participation. When dealers have heterogeneous costs for providing the asset, those who are more cost-effective may introduce benchmarks themselves, in order to improve their market share by driving out higher-cost competitors.

Which markets have a benchmark is not an accident of chance, but rather is likely to be an outcome of conscious decisions by dealers, case by case, when trading off the costs and benefits of the additional market transparency afforded by a benchmark. Our analysis also suggests that there may be a public-welfare role for regulators regarding which markets should have a benchmark, and also in support of the robustness of benchmarks to manipulation.

References

- Asquith, Paul, Thomas R. Covert, and Parag Pathak**, “The Effects of Mandatory Transparency in Financial Market Design: Evidence from the Corporate Bond Market,” Working paper, MIT 2013.
- Asriyan, Vladimir, William Fuchs, , and Brett Green**, “Information Spillovers in Asset Markets with Correlated Values,” Working paper 2015.
- Bank for International Settlements**, “[OTC Derivatives Statistics at End-December 2013](#),” Monetary and Economic Department, Statistical Release May 2014.
- Bank of England**, “[Recommendations on Additional Financial Benchmarks](#) to be Brought into UK Regulatory Scope,” Fair and Effective Markets Review, Report to Her Majesty’s Treasury August 2014.
- Benabou, Roland and Robert Gertner**, “Search with Learning form Prices: Does Increased Inflationary Uncertainty Lead to Higher Markups?,” *The Review of Economic Studies*, January 1993, *60* (1), 69–93.
- Bessembinder, Hendrik and William Maxwell**, “Transparency and the Corporate Bond Market,” *Journal of Economic Perspectives*, 2008, *22* (2), 217–234.
- , – , and **Kumar Venkataraman**, “Market transparency, liquidity externalities, and institutional trading costs in corporate bonds,” *Journal of Financial Economics*, 2006, *82*, 251–288.

- Brundsen, Jun**, “[ECB Seeks Rules to Stem Bank Exodus from Benchmark Panels](#),” *Bloomberg*, June 19 2014.
- Burdett, Kenneth and Kenneth L. Judd**, “Equilibrium Price Dispersion,” *Econometrica*, 1983, 51 (4), 955–969.
- Coulter, Brian and Joel Shapiro**, “[A Mechanism for LIBOR](#),” 2014. Working Paper, University of Oxford.
- Diamond, Peter**, “A Model of Price Adjustment,” *Journal of Economic Theory*, 1971, 3, 156–168.
- Duffie, Darrell and Jeremy Stein**, “[Reforming LIBOR and Other Financial-Market Benchmarks](#),” Working Paper September 2014.
- and **Piotr Dworczak**, “[Robust Benchmark Design](#),” 2014. Working Paper, Stanford University.
- Edwards, Amy K., Lawrence E. Harris, and Michael S. Piwowar**, “Corporate Bond Market Transaction Costs and Transparency,” *Journal of Finance*, 2007, 62 (3), 1421–1451.
- Finch, Gavin and Nicholas Larkin**, “[U.K. Seeks to Criminalize Manipulation of 7 Benchmarks](#),” *Bloomberg*, September 25 2014.
- Foreign Exchange Benchmark Group**, “[Foreign Exchange Benchmarks – Final Report](#),” Financial Stability Board September 2014.
- Fudenberg, Drew and Jean Tirole**, *Game Theory*, MIT Press, 1991.
- Gencarelli, Dawn**, “Average Wholesale Price for Prescription Drugs: Is There a More Appropriate Pricing Mechanism,” *NHPF Issue Brief*, June 7 2002, 775, 1–19.
- , “One Pill, Many Prices: Variation in Prescription Drug Prices in Selected Government Programs,” *National Health Policy Forum, Issue Brief*, August 2005, 807, 1–20.
- Goldstein, Michael A., Edith S. Hotchkiss, and Erik R. Sirri**, “Transparency and Liquidity: A Controlled Experiment on Corporate Bonds,” *Review of Financial Studies*, 2007, 20 (2), 235–273.
- Hou, David and David Skeie**, “[LIBOR: Origins, Economics, Crisis, Scandal and Reform](#),” *The New Palgrave Dictionary of Economics, Online Edition*, 2013. edited by Steven N. Durlauf and Lawrence E. Blume.

- Hurtado, Patricia**, “[Deutsche Bank, HSBC Accused of Silver Fix Manipulation](#),” *Bloomberg*, July 25 2014.
- Janssen, Maarten, Alexei Parakhonyak, and Anastasia Parakhonyak**, “Non-Reservation Price Equilibria and Consumer Search,” Working paper, Basic Research Program 2014.
- **and** – , “Consumer search markets with costly revisits,” *Economic Theory*, June 2013, 55 (2), 481–514.
- **and José Luis Moraga-González**, “Strategic Pricing, Consumer Search and the Number of Firms,” *Review of Economic Studies*, 2004, 71, 1089–1118.
- , – , **and Matthijs R. Wildenbeest**, “Truly costly sequential search and oligopolistic pricing,” *International Journal of Industrial Organization*, 2005, 23, 451–466.
- , **Paul Pichler, and Simon Weidenholzer**, “Oligopolistic Markets with Sequential Search and Production Cost Uncertainty,” *RAND Journal of Economics*, 2011, 42 (3), 444–470.
- Market Participants Group on Reference Rate Reform**, “[Final Report](#),” Financial Stability Board March 2014.
- Maskin, Eric**, “Nash Equilibrium and Welfare Optimality,” *Review of Economic Studies*, 1999, 66, 23–38.
- Official Sector Steering Group**, “[Reforming Major Interest Rate Benchmarks](#),” Financial Stability Board July 2014.
- Patterson, Scott and Katy Burne**, “[CFTC Probes Potential Manipulation](#),” *Wall Street Journal*, April 8 2013.
- Rogerson, Richard, Robert Shimer, and Randall Wright**, “Search-Theoretic Models of the Labor Market: A Survey,” *Journal of Economic Literature*, December 2005, 43, 959–988.
- Rothschild, Michael**, “Searching for the Lowest Price When the Distribution of Prices is Unknown,” *Journal of Political Economy*, July 1974, 82 (4), 689–711.
- Scheck, Justin and Jenny Gross**, “[Traders Try to Game Platts Oil-Price](#),” *Wall Street Journal*, June 19 2013.
- Stahl, II Dale O.**, “Oligopolistic Pricing with Sequential Consumer Search,” *The American Economic Review*, September 1989, 79 (4), 700–712.

- Sun, Yeneng**, “The Exact Law of Large Numbers via Fubini Extension and Characterization of Insurable Risks,” *Journal of Economic Theory*, 2006, 126, 31–69.
- Varian, Hal R.**, “A Model of Sales,” *American Economic Review*, 1980, 70 (4), 651–659.
- Vaughn, Liam**, “[Gold Fix Study Shows Signs of Decade of Bank Manipulation](#),” *Bloomberg*, February 8 2014.
- Weitzman, Martin L.**, “Optimal Search for the Best Alternative,” *Econometrica*, 1979, 47 (3), 641–654.
- Zhu, Haoxiang**, “Finding a Good Price in Opaque Over-the-Counter Markets,” *Review of Financial Studies*, 2012, 25 (4), 1255–1285.

Appendix

A Further Discussion of the Trading Protocol

Our model assumes that dealers’ price offers remain valid at any time, the “recall” assumption often used in models of search markets. A more realistic alternative is the good-only-when-offered protocol, in which dealers are allowed to quote a new price upon the second (and any subsequent) visit by the same trader. As shown by [Zhu \(2012\)](#), revisiting a dealer constitutes a negative signal about the trader’s outside option and tends to worsen the dealer’s quote.

Suppose that slow traders cannot recall earlier offers, but fast traders still can. This change is inconsequential if slow traders search only once on the equilibrium path when recall is available. When dealers’ costs are homogeneous, slow traders indeed search only once, and thus our conclusions from [Section 3](#) and [Section 5.1](#) continue to hold without adjustment. [Janssen and Parakhonyak \(2013\)](#) formally show that the recall assumption has no effect on the equilibrium outcome. However, when dealers’ costs are heterogeneous (and search costs are low enough), slow traders may search multiple times on the equilibrium path when recall is available. For instance, if an unlucky slow trader keeps meeting high-cost dealers, his optimal strategy is to keep searching for a low-cost dealer. In this case relaxing the recall assumption can change the equilibrium outcome by changing the bargaining power of dealers. In particular, because a dealer knows that with a positive probability he is the last to be visited, he can quote a higher price, profiting from the slow trader’s inability to recall an earlier offer. This dealer incentive does not apply to the homogenous-cost setting, in which a slow trader only meets one dealer on equilibrium path.

Suppose, instead, that no trader, fast or slow, can recall earlier offers. In this case, fast traders can no longer use the strategy of visiting all dealers and accepting the lowest offer.

This alternative model would run into the Diamond paradox and is no longer suitable for the analysis of benchmarks.

Overall, although relaxing the recall assumption may change search and pricing behaviors in some cases, we have no reasons to expect this to change our main conclusions concerning benchmarks, as the role of benchmarks and recall are related to distinct types of frictions. Building a general and tractable framework that incorporates the good-only-when-offered protocol is a desirable research direction that is outside the scope of this paper.

B Proofs for Section 3

B.1 Proof of Proposition 1

We fill in the gaps in the derivation of the equilibrium in the benchmark case. We focus on the non-trivial case $\underline{c} \leq c < v - s$.

As argued in Section 3, regardless of the price distribution that dealers use in a symmetric equilibrium, slow traders play a reservation-price strategy with some reservation price r_c . Fast traders play their weakly dominant strategy of searching the entire market. (Thus, if the trader is a fast trader, the dealers are essentially participating in a first-price auction.)

Given this strategy of traders, the following Lemma establishes the properties of the equilibrium response of dealers.

Lemma 2. *If slow traders enter with a strictly positive probability, the equilibrium price distribution cannot have atoms or gaps, and the upper limit of the distribution is equal to r_c .*

Proof. Suppose there is an atom at some price p in the distribution of prices $F_c(\cdot)$ for some cost level $c \in (\underline{c}, v - s)$. Suppose further that $p > c$. In this case a dealer quoting p can profitably deviate to a price $p - \epsilon$, for some small $\epsilon > 0$ (because slow traders play a reservation-price strategy, the probability of trade jumps up discontinuously). Because dealers never post prices below their costs, we must have $p = c$. But that is also impossible, because a dealer could then profitably deviate to r_c (clearly, $r_c \geq c + s$ in equilibrium). Thus, there are no atoms in the distribution.

Second, suppose that $\bar{p}_c > r_c$. In this case the dealer posting \bar{p}_c makes no profits, so she could profitably deviate to r_c . On the other hand, if $\bar{p}_c < r_c$, a dealer can increase profits by quoting r_c instead of \bar{p}_c as this does not effect the probability of selling. Thus $\bar{p}_c = r_c$.

Third, suppose that there is an open gap in the support of the distribution of prices conditional on some cost level c , that is, an interval $(p_1, p_2) \subset [\underline{p}_c, \bar{p}_c] \setminus \text{supp}(F_c(\cdot))$. Take this interval to be maximal, that is, such that p_1 is infimum and p_2 is a supremum, both subject to being in the support of $F_c(\cdot)$. Then we get a contradiction because the probability of selling is the same whether the dealer posts p_1 or p_2 . \square

The rest of the equilibrium characterization follows from the derivation in Section 3.1.

B.2 Proof of Lemma 1

Let r_0^* be the equilibrium first-round reservation price for slow traders. Note that, unlike in the benchmark case, r_0^* is a number, not a function of c .

We take $c < r_0^*$. Such a c exists because $r_0^* \geq \underline{c} + s$. Suppose that the upper limit of the support of the distribution F_c of offer prices, \bar{p}_c , is (strictly) larger than r_0^* . Since traders follow a reservation-price strategy, and because fast traders visit all dealers, there can be no atoms in the distribution of prices (otherwise a dealer could profitably deviate by quoting a price just below the atom). Thus, a dealer setting the price \bar{p}_c never sells in equilibrium, and hence makes zero profit. However, she could make positive profit by setting a price equal to r_0^* . Thus, $\bar{p}_c \leq r_0^*$. Because we took an arbitrary $c < r_0^*$, it follows that whenever $c < r_0^*$, traders do not observe prices above r_0^* on the equilibrium path.

Suppose that $r_0^* < v$. Whenever the realization of c lies above r_0^* , the offer in the first round must be rejected by a slow trader (dealers cannot offer prices below their costs). In particular, a slow trader must reject the price $p^* \in \text{supp}(F_c(\cdot))$ with $r_0^* < p^* \leq \inf\{p \in \text{supp}(F_c(\cdot)) : c > r_0^*\} + \delta < v$, for a sufficiently small $\delta > 0$.²¹ This is a contradiction. Indeed, by the previous paragraph, conditional on observing a price $p > r_0^*$ in the first round, the trader believes that c must lie above r_0^* with probability 1. But in this case, the price p^* is within δ of the best possible price that the trader can ever be offered, so this offer should be accepted by a slow trader (if $\delta < s$), contrary to $p^* > r_0^*$. This shows that $r_0^* = v$.

Finally, suppose that $\bar{p}_c < v$ for some $c < v$. Then a dealer quoting the price \bar{p}_c could profitably deviate by posting a price v (the probability of trade is unaffected). This justifies the second claim.

B.3 Proof of Proposition 2

Fix a fraction λ of slow traders that enter. By Lemma 1 and the arguments used in the derivation of equilibrium prices in the benchmark case, the cdf of offered prices must be

$$F_c(p) = 1 - \left[\frac{\lambda(1-\mu)v-p}{N\mu(p-c)} \right]^{\frac{1}{N-1}} \quad (\text{B.1})$$

²¹Such p^* exists. As long as $c < v$, in equilibrium dealers must be posting prices below v with positive probability.

with support $[\underline{p}_c, v]$, where $\underline{p}_c = \varphi(\lambda)v + (1 - \varphi(\lambda))c$ and

$$\varphi(\lambda) = \frac{\lambda(1 - \mu)}{N\mu + \lambda(1 - \mu)}.$$

We note that the only difference with the equilibrium pricing under the benchmark is that the reservation price and probability of entry are constants, not functions of c .

We can now calculate the expected profits of slow traders if they choose to enter:

$$\pi(\lambda) = -s + \int_{\underline{c}}^v \left[\int_{\underline{p}_c}^v (v - p) dF_c(p) \right] dG(c) = -s + (1 - \alpha(\lambda))X,$$

where

$$X = G(v) [v - \mathbb{E}[c | c \leq v]]$$

is the expected gains from trade. By reasoning analogous to that in the benchmark case, we determine that:

- If $s \leq (1 - \bar{\alpha})X$, there must be full entry by slow traders ($\lambda^* = 1$).
- If $s \geq X$, there cannot be entry by slow traders ($\lambda^* = 0$).
- If $s \in ((1 - \bar{\alpha})X, X)$, then the entry of slow traders is interior, with probability λ^* determined uniquely by the equation (3.10).

B.4 Proof of Proposition 3

Given Proposition 2, in order to prove existence in our setting we need only show that a slow trader does not want to search after observing a price $p \leq v$ in the first round. After observing a price p , the slow trader forms a posterior probability distribution of c , given by the cdf

$$G(c|p) = \frac{\int_{\underline{c}}^c f_y(p) dG(y)}{\int_{\underline{c}}^{\bar{c}_p} f_y(p) dG(y)},$$

where $f_c(p)$ denotes the density of the distribution defined by the cdf (3.11), and

$$\bar{c}_p = \frac{1}{1 - \varphi(\lambda^*)}p - \frac{\varphi(\lambda^*)}{1 - \varphi(\lambda^*)}v$$

is the upper limit of the support of the posterior distribution.

With two dealers, it is easy to provide a sufficient and necessary condition for existence.

A price p is accepted in the first round if and only if

$$v - p \geq -s + \int_{\underline{c}}^{\bar{c}_p} \left[\int_{\underline{p}_c}^p (v - \rho) f_c(\rho) d\rho + (v - p)(1 - F_c(p)) \right] dG(c|p),$$

or

$$s \geq \frac{\int_{\underline{c}}^{\bar{c}_p} \int_{\underline{p}_c}^p F_c(\rho) d\rho (v - c)(p - c)^{-2} dG(c)}{\int_{\underline{c}}^{\bar{c}_p} (v - c)(p - c)^{-2} dG(c)}. \quad (\text{B.2})$$

Thus, a reservation-price equilibrium with two dealers exists if and only if inequality (B.2) holds for all $p \in (\underline{p}_c, v)$. The condition can be easily verified, as the expression on the right hand side of (B.2) is directly computable.

With more than two dealers, an additional difficulty arises because it is not easy to calculate the continuation value when an offer p is rejected in the first round. We can nevertheless provide a sufficient condition based on the following argument. Suppose that after observing p and forming the posterior belief about c , the slow trader is promised to find, in the next search, an offer equal to the lower limit of the price distribution. This provides an upper bound on the continuation value; thus, if the trader decides not to search in this case, she would also not search under the actual continuation value. Thus, a sufficient condition for existence is that

$$s \geq (p - v) + (1 - \varphi(\lambda^*)) \frac{\int_{\underline{c}}^{\bar{c}_p} (v - c)^2 (p - c)^{-\frac{N}{N-1}} dG(c)}{\int_{\underline{c}}^{\bar{c}_p} (v - c)(p - c)^{-\frac{N}{N-1}} dG(c)}, \quad (\text{B.3})$$

for all $p \in (\underline{p}_c, v)$. Again, inequality (B.3) can be directly computed and verified.

The last step in the proof is to show that inequality (B.3) holds for s in some range below X . To this end, we analyze the behavior of the posterior distribution of costs $G(c|p)$ after a price p is observed by a slow trader in the first round when probability of entry λ^* is small. As $\lambda^* \searrow 0$, conditional on p , the upper limit of the support of the posterior cost distribution, \bar{c}_p , converges to p . Thus $G(c|p)$ converges pointwise to 0 for $c < p$ and to 1 for $c > p$. By one of the (equivalent) definitions of weak* convergence of probability measures, the posterior distribution converges in distribution to an atom at p . Thus, in the limit, inequality (B.3) becomes

$$s \geq (p - v) + (1 - \varphi(0))(v - p) = 0,$$

and is thus vacuously satisfied. By continuity of the right-hand side of inequality (B.3), the inequality holds if λ^* is smaller than some $\underline{\lambda} > 0$. Recall that λ^* is determined uniquely by equation (3.10). Moreover, it is continuous and strictly decreasing in s for $s \in ((1 - \bar{\alpha})X, X)$, and equal to zero at $s = X$. Thus, there exists $\underline{s} < X$ such that for all $s > \underline{s}$, λ^* is smaller than $\underline{\lambda}$.

B.5 Proof of Theorem 1

We first outline the main steps of the argument, and leave the technical details for the two lemmas that follow. To make the proof concise, we make a change of variables by defining $x = (v - c)^+ \equiv \max(v - c, 0)$ as the realized gain from a trade given the common cost c .

Note first that conditions (i) and (ii) both imply that $s > (1 - \bar{\alpha})X$. The case $s \geq X$ is trivial to analyze as there is no entry of slow traders without the benchmark (see Proposition 2). Thus, we focus on the range $(1 - \bar{\alpha})X < s < X$, within which Proposition 2 implies interior entry in the absence of the benchmark.

The total expected surplus in the no-benchmark case is

$$W_{nb} \equiv [\lambda^*(1 - \mu) + \mu] X - \lambda^*(1 - \mu)s.$$

With the benchmark, we let $\lambda(x)$ denote the probability of entry by slow traders conditional on a realized gain from trade of x . By Proposition 1,

$$\lambda(x) \begin{cases} = 0, & \text{if } x \leq s, \\ \text{solves } s = (1 - \alpha(\lambda(x)))x, & \text{if } s < x < \frac{s}{1 - \bar{\alpha}}, \\ = 1, & \text{if } x \geq \frac{s}{1 - \bar{\alpha}}. \end{cases}$$

The conditional expected social surplus in the benchmark case conditional on x is

$$W_b(x) \equiv [\lambda(x)(1 - \mu) + \mu] x - \lambda(x)(1 - \mu)s.$$

The crucial observation, demonstrated in Lemma 3 below, is that W_b is a convex function on $[0, s/(1 - \bar{\alpha})]$. Figure 3.1 depicts a typical shape of that function.

Under condition (i), W_b is convex on its entire domain. (This corresponds to cutting off the part of the domain that upsets convexity, as shown in Figure 3.1.) We can thus apply Jensen's Inequality to obtain

$$\mathbb{E}[W_b(x)] \geq W_b[\mathbb{E}(x)] = W_b\left(\int_{\underline{c}}^{\bar{c}} (v - c)^+ dG(c)\right) = W_b(X) = W_{nb}.$$

To justify the last equality, one notes that λ^* is precisely $\lambda(X)$, by equations (3.8) and (3.10). (This inequality is actually strict because G is a non-degenerate distribution and because $\lambda(x) > 0$ with positive probability under G .)

Under condition (ii), W_b may fail to be convex on its entire domain. However, an inspection of the proof of Jensen's Inequality shows that all that is required to achieve the

inequality is that the function W_b is subdifferentiable²² at $\mathbb{E}(x)$. The slope of W_b is increasing on $[0, s/(1 - \bar{\alpha})]$ and equal to 1 on $(s/(1 - \bar{\alpha}), v - \underline{c}]$. Thus, a sufficient condition for existence of a supporting hyperplane of W_b at X is that $W'_b(X) \leq 1$. We thus want to solve the equation $W'_b(x_0) = 1$ for $x_0 \in (s, s/(1 - \bar{\alpha}))$ and impose $X \leq x_0$. (See Figure 3.1.) An explicit solution is not available, so instead we show in Lemma 4 below (by approximating the functions α and λ) that this condition is implied by $s \geq (1 - \psi)X$.

Finally, a simple application of the Lebesgue Dominated Convergence Theorem shows that $\bar{\alpha}$ converges (monotonically) to 1 when either $N \rightarrow \infty$ or $\mu \rightarrow 0$. Thus, condition (i) holds if N is large enough or if μ is small enough.

Lemma 3. $W_b(x)$ and $\lambda(x)$ are convex functions on $[0, s/(1 - \bar{\alpha})]$.

Proof. First we prove that $\lambda(x)$ and $W_b(x)$ are convex on $(s, s/(1 - \bar{\alpha})]$. By the Implicit Function Theorem λ is twice differentiable on this interval and we have

$$\frac{\partial \lambda}{\partial x} = \frac{(1 - \alpha(\lambda))}{\alpha'(\lambda)x} > 0,$$

and

$$\frac{\partial^2 \lambda}{\partial x^2} = \frac{-\alpha'(\lambda)(1 - \alpha(\lambda)) - (1 - \alpha(\lambda)) \left[\alpha'(\lambda) + \alpha''(\lambda) \frac{(1 - \alpha(\lambda))}{\alpha'(\lambda)} \right]}{[\alpha'(\lambda)x]^2}.$$

Hence, $\frac{\partial^2 \lambda}{\partial x^2} \geq 0$ for all $x \in (s, s/(1 - \bar{\alpha}))$ if and only if, for all $\lambda \in (0, 1)$,

$$2[\alpha'(\lambda)]^2 + \alpha''(\lambda)(1 - \alpha(\lambda)) \leq 0. \tag{B.4}$$

Letting $\beta = N\mu/(1 - \mu)$, and computing the derivatives of $\alpha(\lambda)$, we rewrite (B.4) as

$$\left(\int_0^1 \frac{\beta z^{N-1}}{(\lambda + \beta z^{N-1})^2} dz \right)^2 \leq \left(\int_0^1 \frac{\beta z^{N-1}}{(\lambda + \beta z^{N-1})^3} dz \right) \left(\int_0^1 \frac{\beta z^{N-1}}{\lambda + \beta z^{N-1}} dz \right).$$

Hölder's Inequality states that for all measurable and square-integrable functions f and g , $\|fg\|_1 \leq \|f\|_2 \|g\|_2$. By letting

$$f(z) = \sqrt{\frac{\beta z^{N-1}}{(\lambda + \beta z^{N-1})^3}} \quad \text{and} \quad g(z) = \sqrt{\frac{\beta z^{N-1}}{\lambda + \beta z^{N-1}}},$$

we have proven inequality (B.4) and thus the convexity of $\lambda(x)$.

²²A function $f : [a, b] \rightarrow \mathbb{R}$ is said to be subdifferentiable at x_0 if there exists a real number ξ such that, for all x in $[a, b]$, we have $f(x) - f(x_0) \geq \xi(x - x_0)$. If W_b is convex, then it is subdifferentiable on the interior of its domain, by the Separating Hyperplane Theorem.

Now it becomes straightforward to check that $W_b(x)$ is convex on $[s, s/(1 - \bar{\alpha})]$. Notice that $W_b(x)$ and $\lambda(x)$ are trivially convex on $[0, s]$ (because, on this interval, $\lambda(x)$ is identically zero and $W_b(x)$ is affine). Therefore, to finish the proof it is enough to make sure that $\lambda(x)$ and $W_b(x)$ are differentiable at s . We can verify this by computing the left and right derivatives: $\partial_- W_b([s]) = \mu = \partial_+ W_b([s])$, and $\partial_- \lambda([s]) = 0 = \partial_+ \lambda([s])$.

□

Lemma 4. *If $x \leq \frac{s}{1-\psi}$, where $\psi = \frac{1}{2} \left[\sqrt{(1 - \bar{\alpha} + \bar{\alpha}\beta)^2 + 4\bar{\alpha}(1 - \bar{\alpha})} - (1 - \bar{\alpha} + \bar{\alpha}\beta) \right]$ and $\beta = \frac{N\mu}{1-\mu}$, then $W'_b(x) \leq 1$.*

Proof. The claim is true for $x \leq s$, and since $\psi \leq \bar{\alpha}$, we can focus on the region where $\lambda(x)$ is defined as the solution to the equation (3.8) which can be written as

$$\alpha(\lambda(x)) = 1 - \frac{s}{x}.$$

Since $\alpha(\cdot)$ is a strictly increasing function, if we replace $\alpha(\cdot)$ in the above equation by a lower bound, any solution of the new equation will be an upper bound on $\lambda(x)$. Because $W_b(x)$ is convex in the relevant part of the domain (by Lemma 3), to make sure that $W'_b(x) \leq 1$, it's enough to require that $x \leq x_0$, where x_0 solves $W'_b(x_0) = 1$ (such x_0 exists and is unique). We have

$$W'_b(x_0) = \mu + \lambda'(x_0)(1 - \mu)(x_0 - s) + \lambda(x_0)(1 - \mu) = 1. \quad (\text{B.5})$$

We cannot solve this equation explicitly, so we will provide a lower bound on the solution. Because $W'_b(x)$ is increasing, we need to bound $W'_b(x)$ from above. Since $\alpha(\lambda) \geq \lambda\bar{\alpha}$, by the above remark, the solution of the equation

$$\bar{\alpha}\bar{\lambda}(x) = 1 - \frac{s}{x}$$

provides an upper bound on $\lambda(x)$. That is,

$$\lambda(x) \leq \bar{\lambda}(x) = \frac{1}{\bar{\alpha}} - \frac{s}{\bar{\alpha}x}.$$

Moreover,

$$\lambda'(x) = \frac{1}{\alpha'(\lambda(x))} \frac{s}{x^2},$$

and we have, for all $\lambda \in [0, 1]$,

$$\alpha'(\lambda) = \int_0^1 \frac{\beta z^{N-1}}{(\lambda + \beta z^{N-1})^2} dz \geq \frac{1}{\lambda + \beta} \int_0^1 \left(\frac{\lambda + \beta z^{N-1}}{\lambda + \beta z^{N-1}} - \frac{\lambda}{\lambda + \beta z^{N-1}} \right) dz$$

$$= \frac{1}{\lambda + \beta}(1 - \alpha(\lambda)) \geq \frac{1 - \bar{\alpha}}{\lambda + \beta}.$$

Plugging these bounds into equation (B.5) and rearranging, we obtain

$$\frac{\beta + \frac{1}{\bar{\alpha}} - \frac{s}{\bar{\alpha} x_0}}{1 - \bar{\alpha}} \frac{s}{x_0} \left(1 - \frac{s}{x_0}\right) + \frac{1}{\bar{\alpha}} \left[1 - \frac{s}{x_0}\right] = 1.$$

Denoting $y = 1 - s/x_0$, bounding the left hand side from above one more time, and rearranging, we get

$$y^2 + (1 - \bar{\alpha} + \bar{\alpha}\beta)y - \bar{\alpha}(1 - \bar{\alpha}) = 0.$$

The relevant solution is ψ . □

B.6 Proof of Proposition 4

This result follows directly from Propositions 1 and 2.

B.7 Proof of Proposition 5

Using the same arguments used in the derivation of equilibrium from Proposition 1 we can show that in the costly-benchmark case there exists a reservation-price equilibrium, and that equilibrium payoffs are unique. Fixing the probability of entry at λ (and noting that it is independent of c), we compute the reservation price

$$r_c^{cb} = \min \left\{ v, c + \frac{1}{1 - \alpha(\lambda)} s \right\}.$$

A slow trader buys from the first contacted dealer if $c \leq v$. The profit of a slow trader who enters, conditional on c , can be shown to be

$$\pi_c^{cb}(\lambda) = \max \left\{ v - \frac{1}{1 - \alpha(\lambda)} s - c, -s + (1 - \alpha(\lambda))(v - c) \right\}$$

if $c \leq v$, and $-s$ if $c > v$. When $s \geq X$, there can be no entry in equilibrium. If the equilibrium probability of entry λ^{cb} is interior, then it must be determined by the indifference condition, analogous to (3.8) and (3.10), given by

$$\mathbb{E}\pi_c^{cb}(\lambda^{cb}) = 0. \tag{B.6}$$

The solution to that equation exists and is unique if $X \geq s \geq (1 - \bar{\alpha})X + \phi$, where

$$\phi = \frac{\bar{\alpha}}{1 - \bar{\alpha}} \int_{\underline{c}}^{v - \frac{s}{1 - \bar{\alpha}}} [(1 - \bar{\alpha})(v - c) - s] dG(c) \geq 0.$$

When $s < (1 - \bar{\alpha})X + \phi$, we must have entry with probability one.

To show that surplus is higher in the costly-benchmark case than in the no-benchmark case, it is enough to show that entry is higher. Because the function \max is convex, we can apply Jensen's Inequality to conclude that, for all λ ,

$$\mathbb{E}\pi_c^{cb}(\lambda) \geq -s + (1 - \alpha(\lambda))X = \pi^{nb}(\lambda),$$

that is, the expected profit is always higher in the costly-benchmark setting (and is strictly higher provided that $(1 - \bar{\alpha})X < s < X$). It follows that equilibrium entry of slow traders must also be higher (from equations (3.10) and (B.6)).

B.8 Proof of Proposition 6

By Theorem 1 we know that when s is higher than $(1 - \psi)X$, surplus under the benchmark is higher than in the reservation-price equilibrium of the no-benchmark case.²³ It is easy to observe that the difference in surpluses is bounded away from zero as a function of s (under the assumption that $v - \underline{c} > s > (1 - \psi)X$). Given Proposition 5, it suffices to show that the surplus of the costly-benchmark case converges to the surplus of the no-benchmark case as s goes to X (when $s \geq X$, they coincide). It is enough to prove that λ^{cb} , the solution of equation (B.6), converges to λ^* , the solution of equation (3.10), as $s \rightarrow X$. Because the solution of equation (B.6) is continuous in s and equal to 0 at $s = X$, λ^{cb} converges to 0, and so does λ^* .

C Proofs and Supporting Content for Section 4

C.1 Proof of Proposition 7 and equilibrium characterization in the benchmark case

Because the distribution of costs is i.i.d. across dealers conditional on observing the benchmark, slow traders must follow a reservation-price strategy with some reservation price r_c . A stationary²⁴ reservation-price strategy of slow traders will now be characterized by three

²³Even if the latter equilibrium does not exist, the comparison between surpluses is valid, and that is all we need for the proof.

²⁴Requiring stationarity, that is, the same mixing probability at every search round, simplifies the exposition and is without loss of generality. Without stationarity, there is an indeterminacy in specifying the

numbers: λ_c , the probability of entry; r_c , the reservation price; and $\hat{\theta}_c$, the probability of rejecting an offer equal to the reservation price r_c . Fixing the strategy of the dealers and the reservation price r_c , the rejection probability $\hat{\theta}_c$ determines the probability θ_c that a slow trader rejects an offer from a high-cost dealer, and vice versa. Given the one-to-one correspondence between θ_c and $\hat{\theta}_c$, for convenience we will abuse the notation for the strategy of a slow trader, denoting it by the triple $(r_c, \lambda_c, \theta_c)$. Again without loss of generality, we can assume that fast traders play their weakly dominant strategy of always entering and visiting all dealers. We ignore the issue of off-equilibrium beliefs, as it is fairly trivial to deal with.

Fixing c and a strategy $(r_c, \lambda_c, \theta_c)$ we will characterize the equilibrium best-response of dealers. We start with two technical lemmas.

Lemma 5. *In equilibrium, conditional on c (for $c < v$), if dealers of a certain type (high-cost or low-cost) make positive expected profits, then the probability distribution of price offers for that type is atomless. If high-cost dealers make zero expected profits, then in equilibrium they must quote a price equal to their cost, provided that $c + \Delta < v$.*

Proof. The first part of the Lemma can be proven using the argument from the proof of Lemma 2. To prove the second part, suppose that, for some $c < v - \Delta$, a price above $c + \Delta$ is in the support of the equilibrium strategy of high-cost dealers. The probability of selling at that price (or some lower price above $c + \Delta$) must be positive since with probability $(1 - \gamma)^N$ only high-cost dealers are present in the market. Thus, we get a contradiction with the assumption that high-cost dealers make zero expected profits. \square

Lemma 6. *In equilibrium, conditional on c , if $c < v$, for any equilibrium price p_l of a low-cost dealer, and any equilibrium price p_h of a high-cost dealer, we have $p_l \leq p_h$.*

Proof. The claim is true by a standard “revealed-preference” argument. Suppose that $p_l > p_h$. Fix an equilibrium, and let $\varrho(p)$ (for some fixed $c \leq v$) be the probability that a dealer sells the asset when posting the price p . Since dealers are optimizing in equilibrium, we must have

$$\varrho(p_l)(p_l - c) \geq \varrho(p_h)(p_h - c), \quad (\text{C.1})$$

$$\varrho(p_h)(p_h - c - \Delta) \geq \varrho(p_l)(p_l - c - \Delta). \quad (\text{C.2})$$

We have, if $\varrho(p_h) \neq 0$,

$$\varrho(p_h)(p_h - c - \Delta) < \varrho(p_h)(p_l - c - \Delta).$$

probability of rejecting the reservation price in equilibrium. Traders can use different mixing probabilities in every search round, as long as they lead to the same posterior beliefs of dealers. This indeterminacy does not change expected equilibrium payoffs, so without loss of generality we get rid of it by requiring stationarity.

If $p_l > c + \Delta$, then $\varrho(p_h) > \varrho(p_l)$. From inequality (C.1),

$$\varrho(p_l)(p_l - c) + \Delta(\varrho(p_h) - \varrho(p_l)) > \varrho(p_h)(p_h - c)$$

which contradicts inequality (C.2).

We are left with two cases. First, suppose that $p_l \leq c + \Delta$. Then $p_h < c + \Delta$ which is impossible in equilibrium. Second, suppose that $\varrho(p_h) = 0$. Then it must be the case that $\varrho(p_l) = 0$ as well, which is a contradiction if $c < v$. \square

Finally, we prove a lemma about the possibility of gaps in the distribution of prices. Let \underline{p}_c^i and \bar{p}_c^i denote the lower and upper limit of the support of the distribution of prices for dealer of type $i \in \{l, h\}$.

Lemma 7. *In equilibrium, conditional on c (for $c < v$), there can be no gaps in the distribution of prices except for the case in which the support of the distribution of prices of low-cost dealers consists of two intervals, $[\underline{p}_c^l, r_c]$ and $[\bar{p}_c^l, \min\{c + \Delta, v\}]$, and in which either (i) high-cost dealers post $c + \Delta$, or (ii) $c > v - \Delta$.*

Proof. Suppose that there is a gap in the distribution of prices conditional on some cost level c for some type of dealers, that is, an interval $(p_1, p_2) \subset [\underline{p}_c^i, \bar{p}_c^i] \setminus \text{supp}(F_c^i(\cdot))$, $i \in \{l, h\}$. We take this interval to be maximal, that is, such that p_1 and p_2 are in the support of $F_c^i(\cdot)$. It must be the case that probability of selling is strictly larger at p_1 than at p_2 , and thus, in a reservation-price equilibrium, $p_1 \leq r_c \leq p_2$ (we made use here of Lemma 6). It cannot be that $p_1 < r_c$ because then the dealer posting p_1 could profitably deviate to r_c . Thus $p_1 = r_c$.

By Lemma 6, \bar{p}_c^h is the highest price that can be observed on equilibrium path, and it lies above r_c . It follows, using Lemma 5, that high-cost dealers make zero expected profits (if the price distribution for high-cost dealers were atomless, the probability of selling at the price $\bar{p}_c^h > r_c$ would be zero). Moreover, either (i) high-cost dealers post $c + \Delta$, or (ii) $c > v - \Delta$. In either case we can conclude that $i = l$, i.e. the gap occurs in the price distribution of low-cost dealers.

By the above, if there is a gap, then the support of the distribution for low-cost dealers consists of two intervals, the first of which must be $[\underline{p}_c^l, r_c]$. To prove that $\bar{p}_c^l = \min\{c + \Delta, v\}$, we use the fact that $\bar{p}_c^l > r_c$, and thus if $\bar{p}_c^l < \min\{c + \Delta, v\}$, the dealer quoting \bar{p}_c^l would want to deviate to $\min\{c + \Delta, v\}$. \square

Using the above observations, we can now show, case by case, that the equilibrium pricing strategies are uniquely pinned down when there are gains from trade. (We assume throughout that $c < v$; the opposite case is trivial.) We let $F_c^l(p)$ denote the cdf of prices for low-cost dealers, and $F_c^h(p)$ the cdf of prices for high-cost dealers. In most cases it is a routine exercise to rule out the possibility of a gap in the distribution, using Lemma 7.

We will therefore only comment on this possibility explicitly in the two cases when a gap actually occurs in equilibrium.

Case 1: $\lambda_c = 0$. When $\lambda_c = 0$, only fast traders enter. In this case, we have a standard first-price auction between dealers. There are two subcases.

When $c > v - \Delta$, high-cost dealers cannot sell in equilibrium, and the specification of their strategy is irrelevant (they can choose any price above $c + \Delta$). In this case low-cost dealers randomize according to a distribution $F_c^l(p)$ that solves the equation

$$\left[\sum_{k=0}^{N-1} \binom{N-1}{k} \gamma^k (1-\gamma)^{N-1-k} (1 - F_c^l(p))^k \right] (p - c) = (1-\gamma)^{N-1} (v - c).$$

Let us define the function

$$\Phi(z) = \frac{1}{1 - (1-\gamma)^{N-1}} \sum_{k=1}^{N-1} \binom{N-1}{k} z^k \gamma^k (1-\gamma)^{N-1-k}, \quad (\text{C.3})$$

which can be viewed as a generalization of the function z^{N-1} that appears in the definition (3.6). It is easy to see that $\Phi(z)$ is a (strictly) increasing polynomial with $\Phi(0) = 0$, $\Phi(1) = 1$, and $\Phi(z) = z^{N-1}$ when $\gamma = 1$. Moreover, using the binomial identity, we can write $\Phi(z)$ alternatively as

$$\Phi(z) = \frac{(z\gamma + 1 - \gamma)^{N-1} - (1-\gamma)^{N-1}}{1 - (1-\gamma)^{N-1}}. \quad (\text{C.4})$$

Using definition (C.3), we can write

$$F_c^l(p) = 1 - \Phi^{-1} \left(\frac{(1-\gamma)^{N-1} (v-p)}{1 - (1-\gamma)^{N-1} (p-c)} \right)$$

with upper limit $\bar{p}_c^l = v$, and lower limit $\underline{p}_c^l = (1-\gamma)^{N-1}v + (1 - (1-\gamma)^{N-1})c$.

When $c \leq v - \Delta$, high-cost dealers can sell in equilibrium, but a standard result from auction theory (see for example [Fudenberg and Tirole, 1991](#)) says that in the unique equilibrium they will make zero profit by bidding $c + \Delta$. In this case, the distribution $F_c^l(p)$ solves

$$\left[\sum_{k=0}^{N-1} \binom{N-1}{k} (1 - F_l(p|c))^k \gamma^k (1-\gamma)^{N-1-k} \right] (p - c) = (1-\gamma)^{N-1} \Delta,$$

and thus we get

$$F_c^l(p) = 1 - \Phi^{-1} \left(\frac{(1-\gamma)^{N-1} (c + \Delta) - p}{1 - (1-\gamma)^{N-1} (p - c)} \right)$$

with upper limit $\bar{p}_c^l = c + \Delta$, and lower limit $\underline{p}_c^l = c + (1-\gamma)^{N-1} \Delta$.

Case 2: $\lambda_c > 0$. From now on, we assume $\lambda_c > 0$, that is, slow traders enter with positive probability. There are again two subcases.

When $c > v - \Delta$ (case 2.1), high-cost dealers cannot sell in equilibrium, and the specification of their strategy is irrelevant. Low-cost dealers mix according to a continuous distribution $F_c^l(p)$ on an interval with upper limit $\bar{p}_c^l = r_c$, or on a union of two intervals as in Lemma 7.

When $c \leq v - \Delta$ (case 2.2), using Lemmas 5, 6, 7, and the argument from the proof of Lemma 2, we can show that only two subcases are possible:

- If $r_c \leq c + \Delta$, (case 2.2.1), high-cost dealers make zero profit; they post a price $c + \Delta$ with probability 1, while low-cost dealers mix according to a continuous distribution on an interval with upper limit $\bar{p}_c^l = r_c$, or on a union of two intervals as in Lemma 7.
- If $r_c > c + \Delta$ (case 2.2.2), high-cost dealers make positive profits, and in equilibrium both low-cost and high-cost dealers mix according to continuous distributions with adjacent supports ($\bar{p}_c^l = \underline{p}_c^h$), and with r_c being the upper limit of the distribution of prices of high-cost dealers ($\bar{p}_c^h = r_c$).

Below we analyze these cases in detail and characterize the optimal search behavior of slow traders. We first define some key functions that generalize their equivalents from [Section 3](#) to the case of idiosyncratic component in the costs. Let $q(\lambda_c, \theta_c)$ be the posterior probability that a customer is a fast trader, conditional on a visit, given the strategy $(r_c, \lambda_c, \theta_c)$. That is,

$$q(\lambda_c, \theta_c) = \frac{N\mu}{N\mu + \frac{1-\theta_c^N(1-\gamma)^N}{1-\theta_c(1-\gamma)}\lambda_c(1-\mu)}. \quad (\text{C.5})$$

This definition generalizes formula (3.1). We also generalize the definition of the function α from equation (3.6), which now becomes a function of two arguments:

$$\alpha(\lambda_c, \theta_c) = \int_0^1 \left(1 + \frac{q(\lambda_c, \theta_c)(1 - (1-\gamma)^{N-1})}{1 - q(\lambda_c, \theta_c)(1 - (1-\gamma)^{N-1})} \Phi(z) \right)^{-1} dz, \quad (\text{C.6})$$

where $\Phi(z)$ is defined in formula (C.3). Finally, we let $\hat{\alpha} = \alpha(1, 1)$, which corresponds to formula (3.7).

To emphasize the point that we will now deal with equilibrium rather than just best response of dealers to some generic strategy of traders, we add star superscripts to symbols denoting the strategy of traders.

Case 2.1: $\lambda_c^* > 0$, $c > v - \Delta$. In this case, we clearly have $\theta_c^* = 1$. We first suppose that the support of the distribution for low-cost dealers is an interval. Then $F_c^l(p)$ must satisfy

$$\begin{aligned} & \left[1 - q(\lambda_c^*, 1) + q(\lambda_c^*, 1) \sum_{k=0}^{N-1} \binom{N-1}{k} (1 - F_c^l(p))^k \gamma^k (1 - \gamma)^{N-1-k} \right] (p - c) \\ & = [1 - q(\lambda_c^*, 1) + q(\lambda_c^*, 1)(1 - \gamma)^{N-1}] (r_c^* - c). \end{aligned}$$

Solving for $F_c^l(p)$, we obtain

$$F_c^l(p) = 1 - \Phi^{-1} \left(\frac{1 - q(\lambda_c^*, 1) (1 - (1 - \gamma)^{N-1}) r_c^* - p}{q(\lambda_c^*, 1) (1 - (1 - \gamma)^{N-1}) p - c} \right)$$

with $\bar{p}_c^l = r_c^*$, and lower limit

$$\underline{p}_c^l = [1 - q(\lambda_c^*, 1) (1 - (1 - \gamma)^{N-1})] r_c^* + [q(\lambda_c^*, 1) (1 - (1 - \gamma)^{N-1})] c.$$

We can determine r_c^* in this case from the fact that it must solve the following equation (specifying that the trader must be indifferent at r_c^* between buying and searching), analogous to equation (3.4),

$$v - r_c^* = -s + \gamma \left[v - \int_{\underline{p}_c^l}^{r_c^*} p dF_c^l(p) \right] + (1 - \gamma)(v - r_c^*).$$

Using a change of variables, we can transform this equation into the form

$$s = \gamma \left[r_c^* - \int_{\underline{p}_c^l}^{r_c^*} p dF_c^l(p) \right] = (1 - \alpha(\lambda_c^*, 1))\gamma(r_c^* - c).$$

Thus we have

$$r_c^* = c + \frac{s}{(1 - \alpha(\lambda_c^*, 1))\gamma}.$$

The last thing to determine is the probability λ_c^* of entry by slow traders. The profit of a slow trader conditional on entry is equal to

$$\begin{aligned} \pi_c & = (1 - (1 - \gamma)^N) (v - \alpha(\lambda_c^*, 1)r_c^* - (1 - \alpha(\lambda_c^*, 1))c) - \left(\sum_{k=1}^N (1 - \gamma)^{k-1} \gamma k + (1 - \gamma)^N N \right) s \\ & = (1 - (1 - \gamma)^N) \left[v - c - \frac{s}{(1 - \alpha(\lambda_c^*, 1))\gamma} \right]. \end{aligned}$$

When profit is strictly positive, we must have entry with probability one. That is, we have $\lambda_c^* = 1$ if

$$c \leq v - \frac{s}{(1 - \alpha(1, 1))\gamma}.$$

When profit is strictly negative, we must have entry with probability zero, meaning that $\lambda_c^* = 0$ if

$$c \geq v - \frac{s}{(1 - \alpha(0, 1))\gamma}.$$

This takes us back to case 1 analyzed before. Finally, if

$$v - \frac{s}{(1 - \alpha(1, 1))\gamma} < c < v - \frac{s}{(1 - \alpha(0, 1))\gamma},$$

then we must have interior entry $\lambda_c^* \in (0, 1)$, where λ_c^* is the unique solution of the equation

$$s = (1 - \alpha(\lambda_c^*, 1))\gamma(v - c).$$

In this case, slow traders have zero profits and we have $r_c^* = v$.

To check whether the above strategies constitute an equilibrium, we need to verify that the support of price offers by low-cost dealers is indeed an interval, that is, these dealers cannot profitably deviate from posting prices in the range $[p_c^l, r_c^*]$. The only deviation that we need to check is bidding v in the case $r_c^* < v$.²⁵ This leads to the condition

$$\left[\mu(1 - \gamma)^{N-1} + (1 - \mu) \frac{1 - (1 - \gamma)^N}{N\gamma} \right] \frac{s}{(1 - \alpha(1, 1))\gamma} \geq (1 - \gamma)^{N-1}(v - c),$$

where the left hand side is the expected profit from bidding r_c^* , and the right hand side is the expected profit from bidding v (a dealer quoting v can only sell if all other dealers have high costs). We define

$$\kappa = \frac{(1 - \gamma)^{N-1}}{\mu(1 - \gamma)^{N-1} + (1 - \mu) \frac{1 - (1 - \gamma)^N}{N\gamma}}. \quad (\text{C.7})$$

Thus, we have an equilibrium when

$$c \geq v - \frac{s}{\kappa(1 - \alpha(1, 1))\gamma}.$$

Note that $\kappa < 1$, and thus

$$v - \frac{s}{\kappa(1 - \alpha(1, 1))\gamma} < v - \frac{s}{(1 - \alpha(1, 1))\gamma}.$$

²⁵If there is a profitable deviation, this one is the most profitable.

When $c < v - s/(\kappa(1 - \alpha(1, 1))\gamma)$, by Lemma 7, we must have an equilibrium in which the support for low-cost dealers consists of two intervals: $[\underline{p}_c^l, r_c^*]$ and $[\hat{p}_c^l, v]$. Let ζ_c be the conditional probability that a low-cost dealer posts a price in the lower interval. Then, in particular, the dealer must be indifferent between r_c^* and v which pins down ζ_c , in that

$$\left[\mu(1 - \gamma\zeta_c)^{N-1} + (1 - \mu) \frac{1 - (1 - \gamma\zeta_c)^N}{N\gamma\zeta_c} \right] (r_c^* - c) = (1 - \gamma)^{N-1}(v - c). \quad (\text{C.8})$$

We define

$$\vartheta(\zeta_c) = \frac{(1 - \gamma)^{N-1}}{\mu(1 - \gamma\zeta_c)^{N-1} + (1 - \mu) \frac{1 - (1 - \gamma\zeta_c)^N}{N\gamma\zeta_c}}. \quad (\text{C.9})$$

Note that $\vartheta(1) = \kappa$. Then, equation (C.8) becomes

$$r_c^* = (1 - \vartheta(\zeta_c))c + \vartheta(\zeta_c)v. \quad (\text{C.10})$$

We can now determine the exact distribution of prices. In the upper interval we must have

$$\left[\sum_{k=0}^{N-1} \binom{N-1}{k} \gamma^k (1 - \gamma)^{N-1-k} (1 - F_c^l(p))^k \right] (p - c) = (1 - \gamma)^{N-1}(v - c),$$

so we get

$$F_c^l(p) = 1 - \Phi^{-1} \left(\frac{(1 - \gamma)^{N-1} (v - p)}{1 - (1 - \gamma)^{N-1} (p - c)} \right).$$

In the lower interval, the distribution must satisfy

$$\begin{aligned} & \left[\mu \sum_{k=0}^{N-1} \binom{N-1}{k} (\gamma\zeta_c)^k (1 - \gamma\zeta_c)^{N-1-k} \left(1 - \frac{F_c^l(p)}{\zeta_c} \right)^k + \frac{1 - \mu}{N} \frac{1 - (1 - \gamma\zeta_c)^N}{\gamma\zeta_c} \right] (p - c) \\ & = \left[\mu(1 - \gamma\zeta_c)^{N-1} + \frac{1 - \mu}{N} \frac{1 - (1 - \gamma\zeta_c)^N}{\gamma\zeta_c} \right] (r_c^* - c), \end{aligned}$$

which gives

$$F_c^l(p) = \zeta_c - \zeta_c \Phi^{-1} \left(\frac{(1 - \gamma)^{N-1} (r_c^* - p)}{1 - (1 - \gamma\zeta_c)^{N-1} \mu \vartheta(\zeta_c) (p - c)}; \zeta_c \right),$$

where

$$\Phi(z; \zeta_c) = \frac{1}{1 - (1 - \gamma\zeta_c)^{N-1}} \sum_{k=1}^{N-1} \binom{N-1}{k} z^k (\gamma\zeta_c)^k (1 - \gamma\zeta_c)^{N-1-k}.$$

That is, $\Phi(z; \zeta_c)$ is the analogue to $\Phi(z)$ when replacing γ with $\gamma\zeta_c$.

Finally, the reservation price is determined by

$$v - r_c^* = -s + \gamma \zeta_c \left[v - \int_{\underline{p}_c^l}^{r_c^*} p d \left(\frac{F_c^l(p)}{\zeta_c} \right) \right] + (1 - \gamma \zeta_c)(v - r_c^*). \quad (\text{C.11})$$

Using a change of variable $z = (\zeta_c - F_c^l(p))/\zeta_c$, we obtain

$$\int_{\underline{p}_c^l}^{r_c^*} p d \left(\frac{F_c^l(p)}{\zeta_c} \right) = c + (r_c^* - c) \tilde{\alpha}(\zeta_c),$$

where

$$\tilde{\alpha}(\zeta_c) = \int_0^1 \left(1 + \frac{1 - (1 - \gamma \zeta_c)^{N-1}}{(1 - \gamma)^{N-1}} \mu \vartheta(\zeta_c) \Phi(z; \zeta_c) \right)^{-1} dz.$$

Note that $\tilde{\alpha}(1) = \alpha(1, 1)$. From this we can calculate the optimal reservation price, determined by equation (C.11), as

$$r_c^* = c + \frac{s}{(1 - \tilde{\alpha}(\zeta_c)) \gamma \zeta_c}. \quad (\text{C.12})$$

Equations (C.10) and (C.12) together pin down r_c^* and ζ_c . Combining them, we get a single equation that pins down ζ_c , in the form

$$s = \vartheta(\zeta_c)(1 - \tilde{\alpha}(\zeta_c)) \gamma \zeta_c (v - c).$$

A unique solution $\zeta_c^* \in (0, 1)$ exists if and only if $0 < s < \kappa(1 - \alpha(1, 1)) \gamma (v - c)$ which is precisely our assumption for that case.

Note that in this range the equilibrium level ζ_c^* will be close to 1 when s is close to $\kappa(1 - \alpha(1, 1)) \gamma (v - c)$ and will converge to 0 as s goes to 0.

Case 2.2.1: $c \leq v - \Delta$, $r_c^* \leq c + \Delta$. In this case, high-cost dealers offer the price $c + \Delta$. We have two cases to consider, and call them (a) and (b).

Case (a). When $r_c^* < c + \Delta$, we must have $\theta_c^* = 1$. Suppose that low-cost dealers mix on an interval. Then the distribution of prices is

$$F_c^l(p) = 1 - \Phi^{-1} \left(\frac{1 - q(\lambda_c^*, 1) (1 - (1 - \gamma)^{N-1})}{q(\lambda_c^*, 1) (1 - (1 - \gamma)^{N-1})} \frac{r_c^* - p}{p - c} \right),$$

just as in the previous case. What differs from the previous case is the profit of a slow trader conditional on entry. In the event that there are no low-cost dealers in the market, a trader

buys from a high-cost dealer instead of exiting. Accordingly, the profit now becomes

$$\pi_c = v - c - (1 - \gamma)^N \Delta - (1 - (1 - \gamma)^N) \frac{s}{(1 - \alpha(\lambda_c^*, 1))\gamma}.$$

We can have strictly positive entry by slow traders only if

$$v \geq c + (1 - \gamma)^N \left[\Delta - \frac{s}{(1 - \alpha(\lambda_c^*, 1))\gamma} \right] + \frac{s}{(1 - \alpha(\lambda_c^*, 1))\gamma}. \quad (\text{C.13})$$

Recall that we have

$$r_c^* = c + \frac{s}{(1 - \alpha(\lambda_c^*, 1))\gamma}.$$

Thus, given that we assumed $r_c^* < c + \Delta$, we have an equilibrium with positive entry if inequality (C.13) holds and

$$\Delta > \frac{s}{(1 - \alpha(\lambda_c^*, 1))\gamma}.$$

Notice that we have

$$\begin{aligned} & v - c - (1 - \gamma)^N \Delta - (1 - (1 - \gamma)^N) \frac{s}{(1 - \alpha(\lambda_c^*, 1))\gamma} \\ & > v - c - (1 - \gamma)^N \Delta - (1 - (1 - \gamma)^N) \Delta = v - c - \Delta \geq 0, \end{aligned}$$

which means that profits are always strictly positive in this case. Thus we must have full entry, meaning $\lambda_c^* = 1$, and this can be an equilibrium only if $s < (1 - \alpha(1, 1))\gamma\Delta$.

Finally, we verify the supposition that low-cost dealers mix on an interval. We need to check the deviation to (just below) $c + \Delta$, analogous to deviation to v in the previous case. We require

$$\left[\mu(1 - \gamma)^{N-1} + (1 - \mu) \frac{1 - (1 - \gamma)^N}{N\gamma} \right] \frac{s}{(1 - \alpha(1, 1))\gamma} \geq (1 - \gamma)^{N-1} \Delta.$$

Thus, the above strategies are an equilibrium if $s \geq \kappa(1 - \alpha(1, 1))\gamma\Delta$.

In the case $s < \kappa(1 - \alpha(1, 1))\gamma\Delta$, we will have an equilibrium with low-cost dealers mixing on two intervals $[\underline{p}_c^l, r_c^*]$ and $[\hat{p}_c^l, c + \Delta]$. The analysis is analogous to the one in the previous case 2.1 so we skip some details. First, the indifference condition between r_c^* and $c + \Delta$ ²⁶ is

$$(r_c^* - c) = \vartheta(\zeta_c)\Delta. \quad (\text{C.14})$$

²⁶Note that $c + \Delta$ is the upper limit of the support but prices posted by a low-cost dealer are below $c + \Delta$ with probability one. Thus, when we say that the dealer must be indifferent between posting r_c^* and $c + \Delta$, we really mean $c + \Delta - \epsilon$ for arbitrarily small $\epsilon \rightarrow 0$ which leads to the formula below.

The upper part of the distribution is given by

$$F_c^l(p) = 1 - \Phi^{-1} \left(\frac{(1-\gamma)^{N-1}}{1 - (1-\gamma)^{N-1}} \frac{c + \Delta - p}{p - c} \right),$$

while the lower part is

$$F_c^l(p) = \zeta_c - \zeta_c \Phi^{-1} \left(\frac{(1-\gamma)^{N-1}}{1 - (1-\gamma\zeta_c)^{N-1}} \frac{1}{\mu\vartheta(\zeta_c)} \frac{r_c^* - p}{p - c}; \zeta_c \right).$$

The reservation price is determined by equation (C.11). Simplifying as before, we obtain

$$r_c^* = c + \frac{s}{(1 - \tilde{\alpha}(\zeta_c))\gamma\zeta_c}.$$

Combining with equation (C.14) ζ_c is pinned down by the equation

$$s = \vartheta(\zeta_c)(1 - \tilde{\alpha}(\zeta_c))\gamma\zeta_c\Delta.$$

The equation does not depend on c , so neither does the solution. That is, ζ_c^* is independent of c and solves

$$s = \vartheta(\zeta)(1 - \tilde{\alpha}(\zeta))\gamma\zeta\Delta.$$

This equation has a unique solution in $(0, 1)$ precisely when $0 < s < \kappa(1 - \alpha(1, 1))\gamma\Delta$, which was our assumption for this case.

Case (b). We now look at the second possibility: $r_c^* = c + \Delta$. We can now have $\theta_c^* \in (0, 1)$, and this will matter for equilibrium pricing through the impact on the posterior beliefs of dealers. The probability $F_c^l(p)$ of an offer of p or less by a low-cost dealer solves

$$\begin{aligned} & \left[1 - q(\lambda_c^*, \theta_c^*) + q(\lambda_c^*, \theta_c^*) \sum_{k=0}^{N-1} \binom{N-1}{k} (1 - F_c^l(p))^k \gamma^k (1-\gamma)^{N-1-k} \right] (p - c) \\ & = [1 - q(\lambda_c^*, \theta_c^*) + q(\lambda_c^*, \theta_c^*)(1-\gamma)^{N-1}] (r_c^* - c). \end{aligned}$$

The profit of a slow trader is the same as in the previous case. The condition $r_c^* = c + \Delta$ means that we must have

$$\frac{s}{(1 - \alpha(\lambda_c^*, \theta_c^*))\gamma} = \Delta.$$

This implies that we must again have entry with probability one. Thus, we have an equilibrium with full entry and the probability of rejecting an offer of r_c^* given by θ_c^* that solves

$$s = (1 - \alpha(1, \theta_c^*))\gamma\Delta.$$

Note that $\theta_c^* = \theta^*$ (the equation, and hence the solution, is independent of c). An interior solution exists if and only if $(1 - \alpha(1, 1))\gamma\Delta < s < (1 - \alpha(1, 0))\gamma\Delta$. Notice that θ^* is close to 1 when s is close to $(1 - \alpha(1, 1))\gamma\Delta$, and close to 0 when s is close to $(1 - \alpha(1, 0))\gamma\Delta$.

Case 2.2.2: $c \leq v - \Delta$, $r_c^* > c + \Delta$. This is the case when high-cost dealers make positive profits and mix according to a continuous distribution $F_c^h(p)$ with upper limit r_c^* . We must have $\theta_c^* = 0$. The cdf $F_c^h(p)$ solves

$$[1 - q(\lambda_c^*, 0) + q(\lambda_c^*, 0)(1 - \gamma)^{N-1}(1 - F_c^h(p))^{N-1}] (p - c - \Delta) = [1 - q(\lambda_c^*, 0)] (r_c^* - c - \Delta).$$

Simplifying, we obtain

$$F_c^h(p) = 1 - \left(\frac{1 - q(\lambda_c^*, 0)}{q(\lambda_c^*, 0)(1 - \gamma)^{N-1}} \frac{r_c^* - p}{p - c - \Delta} \right)^{\frac{1}{N-1}}$$

with upper limit $\bar{p}_c^h = r_c^*$, and lower limit

$$\underline{p}_c^h = \frac{1 - q(\lambda_c^*, 0)}{1 - q(\lambda_c^*, 0)(1 - (1 - \gamma)^{N-1})} r_c^* + \frac{q(\lambda_c^*, 0)(1 - \gamma)^{N-1}}{1 - q(\lambda_c^*, 0)(1 - (1 - \gamma)^{N-1})} (c + \Delta).$$

To simplify notation, let us denote

$$\phi(\lambda_c^*) = \frac{1 - q(\lambda_c^*, 0)}{1 - (1 - (1 - \gamma)^{N-1}) q(\lambda_c^*, 0)}. \quad (\text{C.15})$$

Next, $F_c^l(p)$ must solve

$$\begin{aligned} & \left[1 - q(\lambda_c^*, 0) + q(\lambda_c^*, 0) \sum_{k=0}^{N-1} \binom{N-1}{k} (1 - F_c^l(p))^k \gamma^k (1 - \gamma)^{N-1-k} \right] (p - c) \\ & = [1 - q(\lambda_c^*, 0) + q(\lambda_c^*, 0)(1 - \gamma)^{N-1}] (\underline{p}_c^h - c). \end{aligned}$$

Solving for $F_c^l(p)$ we get

$$F_c^l(p) = 1 - \Phi^{-1} \left(\frac{1 - q(\lambda_c^*, 0)(1 - (1 - \gamma)^{N-1}) \underline{p}_c^h - p}{q(\lambda_c^*, 0)(1 - (1 - \gamma)^{N-1}) (p - c)} \right),$$

with $\bar{p}_c^l = \underline{p}_c^h$ and lower limit

$$\underline{p}_c^l = [1 - q(\lambda_c^*, 0)(1 - (1 - \gamma)^{N-1})] \underline{p}_c^h + [q(\lambda_c^*, 0)(1 - (1 - \gamma)^{N-1})] c.$$

We need to define one more function, analogous to $\alpha(\lambda, \theta)$, and corresponding to the distribution of prices used by high-cost dealers. Let

$$\alpha_h(\lambda) = \int_0^1 \left(1 + \frac{q(\lambda, 0)(1 - \gamma)^{N-1}}{1 - q(\lambda, 0)} z^{N-1} \right)^{-1} dz.$$

Then, using a change of variables, we get

$$\int p dF_c^h(p) = (1 - \alpha_h(\lambda_c^*)) (c + \Delta) + \alpha_h(\lambda_c^*) r_c^*,$$

and

$$\int p dF_c^l(p) = (1 - \alpha(\lambda_c^*, 0))c + \alpha(\lambda_c^*, 0) \underline{p}_c^h.$$

As always, r_c^* is determined by the indifference condition

$$v - r_c^* = -s + \gamma \left[v - \int_{\underline{p}_c^l}^{\bar{p}_c^l} p dF_c^l(p) \right] + (1 - \gamma) \left[v - \int_{\underline{p}_c^h}^{\bar{p}_c^h} p dF_c^h(p) \right].$$

From this we can obtain

$$r_c^* = c + \Delta + \frac{s - (1 - \alpha(\lambda_c^*, 0))\gamma\Delta}{\gamma(1 - \phi(\lambda_c^*)\alpha(\lambda_c^*, 0)) + (1 - \gamma)(1 - \alpha_h(\lambda_c^*))}.$$

Next, we consider entry decision of slow traders. The profit conditional on entry is simply $v - r_c^*$. Thus, we have entry with probability one if and only if

$$c < v - \Delta - \frac{s - (1 - \alpha(1, 0))\gamma\Delta}{\gamma(1 - \phi(1)\alpha(1, 0)) + (1 - \gamma)(1 - \alpha_h(1))}.$$

Since we have assumed that $r_c^* > c + \Delta$, we additionally require $s > (1 - \alpha(1, 0))\gamma\Delta$.

Interior entry requires λ_c^* to solve

$$v = c + \Delta + \frac{s - (1 - \alpha(\lambda_c^*, 0))\gamma\Delta}{\gamma(1 - \phi(\lambda_c^*)\alpha(\lambda_c^*, 0)) + (1 - \gamma)(1 - \alpha_h(\lambda_c^*))}. \quad (\text{C.16})$$

An interior solution exists if and only if

$$\frac{s - (1 - \alpha(0, 0))\gamma\Delta}{\gamma(1 - \phi(0)\alpha(0, 0)) + (1 - \gamma)(1 - \alpha_h(0))} < v - c - \Delta < \frac{s - (1 - \alpha(1, 0))\gamma\Delta}{\gamma(1 - \phi(1)\alpha(1, 0)) + (1 - \gamma)(1 - \alpha_h(1))}. \quad (\text{C.17})$$

Noticing that $\alpha_h(0) = 0$ and that $\phi(0) = 0$, we can simplify the inequality on the left to $s - (1 - \alpha(0, 0))\gamma\Delta < v - c - \Delta$.

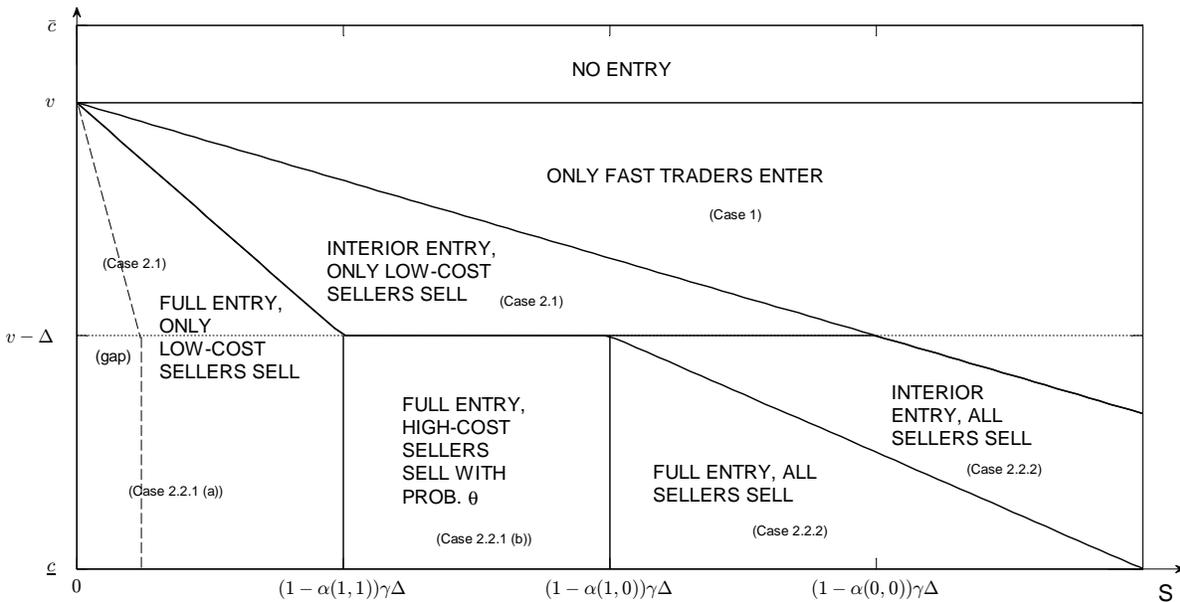
Finally, since we have assumed that $r_c^* > c + \Delta$, we require $s > (1 - \alpha(\lambda_c^*, 0))\gamma\Delta$. This

condition is satisfied vacuously when equation (C.16) holds.

When $s - (1 - \alpha(0, 0))\gamma\Delta \geq v - c - \Delta$, we must have entry with probability zero which brings us back to case 1.

This concludes the analysis of all cases. By direct inspection, we check that for any given pair (s, c) , there is exactly one equilibrium (up to payoff-irrelevant changes in equilibrium strategies). Figure C.1 summarizes our conclusions by depicting the equilibrium correspondence in the (s, c) space. “Full entry” means that $\lambda_c^* = 1$ in the relevant range. “Interior entry” means that $\lambda_c^* \in (0, 1)$. When we say that “only low-cost dealers sell,” we mean that if there is at least one low-cost dealer in the market, then all customers trade with low-cost dealers. When we say that “all dealers sell” or that “high-cost dealers sell with probability θ ,” we refer to the probability of selling to a slow trader upon a visit. Finally, the trapezoidal area denoted by “(gap)” corresponds to the case in which low-cost dealers have a gap in the support of their offer distribution.

Fig. C.1: The benchmark case: Equilibrium correspondence



C.2 Proof of Proposition 8

Generalized statement (without assuming A.2): *If $s \leq (1 - \hat{\alpha})\gamma \min\{\Delta, v - c\}$, then equilibrium in the benchmark case leads to efficient matching. That is, slow traders always enter, and all traders buy from a low-cost dealer, in the event that there is at least one such*

dealer present in the market. Additionally, if $s \geq \kappa(1 - \hat{\alpha})\gamma \min\{\Delta, v - c\}$, where $\kappa < 1$,²⁷ the equilibrium with the benchmark achieves the second best, in the sense that each slow trader buys from the first low-cost dealer that she contacts, thus minimizing search costs subject to matching efficiency.

Proof. The theorem follows directly from the derivation above (cases 2.1 and 2.2.1 (a)). When

$$\kappa(1 - \hat{\alpha})\gamma \min\{\Delta, v - c\} \leq s \leq (1 - \hat{\alpha})\gamma \min\{\Delta, v - c\},$$

we are in the region in which the equilibrium achieves the second best. Slow traders always enter, and search until they find the first low-cost dealer (low-cost dealers always post prices below the reservation price, and high-cost dealers always post prices above the reservation price). If there are no low-cost dealers in the market and $c > v - \Delta$, then traders exit without trading. When $c < v - \Delta$, they buy from a high-cost dealer. When $s < \kappa(1 - \hat{\alpha})\gamma\Delta$, low-cost dealers post prices below the reservation price with probability $\zeta_c^* \in (0, 1)$. Because high-cost dealers still post prices above the reservation-price (and above the prices posted by low-cost dealers), the matching of traders to low-cost dealers is efficient. \square

C.3 Supporting analysis of the case $s < \kappa(1 - \hat{\alpha})\gamma\Delta$

Here, we provide the supporting analysis of the case $s < \kappa(1 - \hat{\alpha})\gamma\Delta$ in the context of [Section 4](#). We show that a low-cost dealer's incentive to quote a high price (leading to higher-than-efficient search by slow traders in equilibrium) disappears as the number N of dealers gets large, in the sense formalized in [Lemma 8](#).

Lemma 8. *Letting $\bar{s}(N) = (1 - \hat{\alpha})\gamma\Delta$ and $\underline{s}(N) = \kappa(1 - \hat{\alpha})\gamma\Delta$, we have*

$$\lim_{N \rightarrow \infty} N\bar{s}(N) = \infty \text{ and } \lim_{N \rightarrow \infty} N\underline{s}(N) = 0,$$

where the convergence to 0 in the second equation is exponentially fast.

The quantity Ns is the upper bound on the search costs incurred by a slow trader. If slow traders adopted the sub-optimal strategy of searching the entire market, we would get the fully efficient outcome of a centralized exchange, before considering the search costs. Thus, $(1 - \mu)Ns$ is an upper bound on the potential welfare loss in our setting. [Lemma 8](#) says that the case $s < \kappa(1 - \hat{\alpha})\gamma\Delta$ in which case the benchmark fails to achieve the second best can be safely ignored for practical purposes, given that the (rough) upper bound of possible inefficiency goes to 0 exponentially fast²⁸ with N . On the other hand, the search-cost range

²⁷ $\kappa = (1 - \gamma)^{N-1} / [\mu(1 - \gamma)^{N-1} + (1 - \mu)[1 - (1 - \gamma)^N] / (N\gamma)]$.

²⁸For example, if $\mu = \gamma = \frac{1}{2}$, $\kappa \approx 0.019$ for just $N = 10$, and $\kappa \approx 1.5 * 10^{-6}$ for $N = 25$.

$(\kappa(1 - \hat{\alpha})\gamma\Delta, (1 - \hat{\alpha})\gamma\Delta)$ is much more important, as the potential welfare gains or losses are unbounded in this region (if we allow v to get large).

To prove the first claim, we show that $(1 - \hat{\alpha})$ converges to zero (as $N \rightarrow \infty$) more slowly than $\log(N)/N$. (That $(1 - \hat{\alpha})$ converges to 0 follows from Lebesgue Dominated Convergence Theorem.) We have

$$1 - \hat{\alpha} = \int_0^1 \frac{a_N \Phi(z)}{1 + a_N \Phi(z)} dz,$$

where

$$a_N = \frac{N\mu(1 - (1 - \gamma)^{N-1})}{\frac{1 - (1 - \gamma)^N}{\gamma}(1 - \mu) + N\mu(1 - \gamma)^{N-1}}.$$

Clearly,

$$1 - \hat{\alpha} \geq \int_{\Phi^{-1}(\frac{1}{N})}^1 \frac{a_N \Phi(z)}{1 + a_N \Phi(z)} dz \geq \left(1 - \Phi^{-1}\left(\frac{1}{N}\right)\right) \frac{a_N}{N + a_N}.$$

The term $a_N/(N + a_N)$ has a finite and strictly positive limit. It is therefore enough to show that

$$\lim_{N \rightarrow \infty} \frac{N}{\log N} \left(1 - \Phi^{-1}\left(\frac{1}{N}\right)\right) > 0.$$

Using equation (C.4) to invert Φ , and applying d'Hospital rule a few times to simplify the expression, we obtain

$$\lim_{N \rightarrow \infty} \frac{N}{\log N} \left(1 - \Phi^{-1}\left(\frac{1}{N}\right)\right) = \lim_{N \rightarrow \infty} \frac{N}{\log N} \left(\left(\frac{1}{N}\right)^{\frac{1}{N}} - 1\right) = \lim_{K \rightarrow \infty} K \left(e^{\frac{1}{K}} - 1\right) = 1.$$

To prove the second claim, recall that

$$N\kappa = \frac{N(1 - \gamma)^{N-1}}{\mu(1 - \gamma)^{N-1} + (1 - \mu)\frac{1 - (1 - \gamma)^N}{N\gamma}} = \frac{1}{\frac{\mu}{N} + (1 - \mu)\frac{1 - (1 - \gamma)^N}{N^2\gamma(1 - \gamma)^{N-1}}}.$$

The above expression goes to 0 as quickly as $N^2(1 - \gamma)^{N-1}$, that is, exponentially.

Clearly, the result is true in the generalized setting without assuming A.2.

C.4 Proof of Proposition 9

Generalized statement (not assuming A.2): *In the absence of a benchmark, if $\min\{v, \bar{c}\} > \underline{c} + \Delta$, there does not exist an equilibrium achieving the second best.*

Proof. In an equilibrium in which the second-best is achieved under the condition $s < \gamma\Delta$, high-cost dealers can only sell when there are no low-cost dealers in the market, and the slow trader searched the entire market. Thus, if an equilibrium like this exists, high-cost

dealers quote prices as if they participated in an auction with all other high-cost dealers. A standard result in auction theory says that in this case they must bid their costs, that is, they must offer to sell for $c + \Delta$.

Consider a situation when a slow trader enters and the first dealer has low costs, for some $c < v$. If the second-best is achieved, that offer needs to be accepted by a slow trader. Under the assumption of the Proposition, we can find a c^* that satisfies $v > c^* > \underline{c} + \Delta$. By the above observation, (almost) all prices in the support of the distribution of the low-cost dealer at $c = c^*$ must be accepted by a slow trader in the first search round. This leads to a contradiction. Since high-cost dealers post a price of $c + \Delta$ conditional on c , they make zero profits. They can profitably deviate at $c = \underline{c}$ by quoting a price in the support of the distribution of a low-cost dealer at $c = c^*$. \square

C.5 Generalized statement of Theorem 2 (not assuming A.2)

Theorem: If (i) $\kappa(1 - \hat{\alpha})\gamma \min\{\Delta, v - \underline{c}\} \leq s \leq (1 - \hat{\alpha})\gamma \min\{\Delta, v - \bar{c}\}$ and (ii) $\bar{c} > \underline{c} + \Delta$ both hold, then the equilibrium in the benchmark case yields a strictly higher social surplus than any equilibrium in the no-benchmark case.

D Supporting Content for Section 5

D.1 Proof of Theorem 3

The proof of Theorem 3 is very similar to the proof of Theorem 1, so we skip some of the details. Denote the expected profits of a dealer in the benchmark case conditional on x (where $x = (v - c)_+$) by $\chi_b(x)$ and in the case with no benchmark by χ_{nb} . Recall from Propositions 1 and 2 that

$$\chi_b(x) = \frac{\lambda(x)(1 - \mu)}{N} \frac{s}{1 - \alpha(\lambda(x))}$$

and $\chi_{nb} = X\lambda^*(1 - \mu)/N$.

Assume that condition (i) holds. Then, using the fact that $\lambda(x)$ is given by $s = (1 - \alpha(\lambda(x)))x$ in the relevant range, we can write $\chi_b(x) = (1 - \mu)\lambda(x)x/N$. By Lemma 3, $\lambda(x)$ is increasing and convex, so $\chi_b(x)$ is also convex. Therefore, applying Jensen's Inequality we get

$$\mathbb{E}[\chi_b(x)] \geq \chi_b(\mathbb{E}[x]) = \chi_b(X) = \chi_{nb}.$$

Now assume that condition (ii) holds. As in the proof of Theorem 1 we want to find a condition on X that would guarantee that the profit function χ_b is subdifferentiable at X .

Using the reasoning from the proof of Theorem 1, we can establish existence of a constant $\eta \in (0, \hat{\alpha})$, that depends only on μ and N , and such that $X \leq s/(1 - \eta)$ guarantees existence of a supporting hyperplane at X (thus allowing us to apply Jensen's Inequality).

D.2 Proof of Theorem 4

To prove the Theorem, we first describe the equilibrium path, and then show the optimality of dealers' strategies under a sufficiently high Δ .

If the environment is efficient, the benchmark is introduced, only low-cost dealers enter and we have a reservation-price equilibrium in the trading-stage subgame described in Section 3.1 (with the exception that N is now replaced by M , which is equal to L in equilibrium). Because $s < (1 - \bar{\alpha})(v - \bar{c})$, we have full entry in this case, and the reservation price of slow-traders is

$$r_c^* = c + \frac{s}{1 - \bar{\alpha}_L},$$

where the subscript L in $\bar{\alpha}_L$ indicates that N is replaced by L in the definition of $\bar{\alpha}$ given by equation (3.7).

If the environment is inefficient (all dealers have high costs), the benchmark is not introduced, and high-cost dealers enter if and only if $X_\Delta > s$. To see this, note that in this case, we can apply the analysis of Section 3.2 with the exception that c is replaced by $c + \Delta$ (correspondingly, X is replaced by X_Δ). In particular, high-cost dealers make strictly positive expected profits if and only if $X_\Delta > s$ because this condition guarantees that there is positive probability of entry by slow traders, according to Proposition 2. Existence follows from Proposition 3 for all $\Delta \geq \Delta_1^*$ for some Δ_1^* with $X_{\Delta_1^*} > s$. Indeed, the inspection of the proof shows that a sufficient condition is that $X_\Delta - s$ is sufficiently small which we can achieve by taking high enough Δ .

On the equilibrium path in the pre-play stage, low-cost dealers vote in favor of the benchmark, and enter if the benchmark is introduced or if the benchmark is not introduced and $X_\Delta > s$. High-cost dealers vote against the benchmark and enter if and only if the benchmark is not introduced and $X_\Delta > s$.

We now verify the optimality of these dealer strategies.

Set $\Delta_0^* = s/(1 - \bar{\alpha})$, and suppose that $\Delta \geq \Delta_0^*$ so that $s < (1 - \bar{\alpha})\Delta$.

First, we show that a high-cost dealer does not want to deviate and enter when the benchmark is introduced. Indeed, when the benchmark is observed, slow traders follow a reservation-price strategy with

$$r_c^* = c + \frac{s}{1 - \bar{\alpha}_M} \leq c + \frac{s}{1 - \bar{\alpha}},$$

using the fact that $\bar{\alpha}_M$ is increasing in M .²⁹ Since $s \leq (1 - \bar{\alpha})\Delta$ for $\Delta \geq \Delta_0^*$, we conclude that $c + \Delta \geq r_c^*$. Thus, using familiar arguments from previous sections, we show that a high-cost dealer cannot make positive profits after entering the market, regardless of the identities of other dealers in the market.³⁰

Second, we show that a high-cost dealer does not want to deviate and stay out of the market when the benchmark is not introduced and $X_\Delta > s$. By the remark above, high-cost dealers make strictly positive profits on the equilibrium path in that case.

Third, low-cost dealers cannot deviate by changing their entry decision because, by the specification of their strategy, they enter if and only if their expected profits are strictly positive.

Fourth, we show that any coalition of high-cost dealers does not want to deviate by voting in favor of the benchmark. By what we established above, if the benchmark is introduced, a high-cost dealer finds it optimal not to enter and hence earns no profits. Thus, this cannot be a strictly profitable deviation.

Fifth, we show that any coalition of low-cost dealers does not want to deviate by voting against the benchmark. Note that $L \geq 2$ is common knowledge among low-cost dealers. In equilibrium, the benchmark is introduced, high-cost dealers stay out, and the low cost dealer's expected profit is equal to

$$\frac{1 - \mu}{L} \frac{s}{1 - \bar{\alpha}_L} > 0,$$

which does not depend on Δ . If the benchmark is not introduced, slow traders believe with probability one that only high-cost dealers are present in the market. By taking Δ high enough we can make $X_\Delta - s$ arbitrarily small, so the equilibrium probability of entry by slow traders is arbitrarily small without the benchmark (see the analysis in Section 3.2). Because $L \geq 2$ the expected profits of low-cost dealers in this case converge to zero as the posterior probability of meeting a slow trader approaches zero. Because the profit on equilibrium path is bounded away from zero, low-cost dealers do not want to deviate in this way if Δ is above some cutoff level Δ_2^* .

We conclude the proof by defining $\Delta^* = \max\{\Delta_0^*, \Delta_1^*, \Delta_2^*\}$.

Note that Δ_1^* and Δ_2^* can be chosen so that $X_\Delta > s$ if Δ is close to $\max\{\Delta_1^*, \Delta_2^*\}$. If additionally s is sufficiently small, we can guarantee that $X_\Delta > s$ for all Δ in some right neighborhood of Δ^* .

²⁹This is shown in Janssen and Moraga-González (2004).

³⁰Note that off-equilibrium path traders may observe offers above their reservation price, something that never happens on equilibrium path. We specify off-equilibrium beliefs of traders by saying that this off-equilibrium event does not change the belief of any trader about the types of active dealers. This is consistent with a perfect Bayesian equilibrium.

E Supporting Content for [Section 6](#)

In this subsection we provide the supporting arguments for the analysis in [Section 6](#). For simplicity, we assume that $\gamma = 1$ throughout (the results can be generalized to the heterogeneous case in a straightforward way).

Specifying and solving an equilibrium model of manipulation, while desirable, is beyond the scope of this paper. Instead, we consider the following mechanism design problem.

Suppose that there exists a benchmark administrator who can design an arbitrary “benchmark announcement” mechanism with transfers. Here, a mechanism is a pair (M, g) , where $M = (M_1 \times \cdots \times M_N)$ is the product of the message spaces of the N respective dealers, and where $g : M \rightarrow [\underline{c}, \bar{c}] \times \mathbb{R}^N$. The function g maps the dealers’ messages (m_1, \dots, m_N) to an announced benchmark \tilde{c} and to transfers t_1, \dots, t_N from the respective dealers to the mechanism designer. Each mechanism induces a game in which dealers first submit messages. The second stage of the game is the trading game presented in [Section 2](#) of this paper, in which traders assume that the announced benchmark \tilde{c} is a truthful report of the actual cost c .

In this setting, “Nash implementability” means that there exists a mechanism whose associated game has a Nash equilibrium in which the announced benchmark \tilde{c} is the true cost c . “Full implementability” adds the requirement that this is the unique equilibrium of the mechanism-induced game.

Proposition 10. *Truthful revelation of c is Nash implementable, but is not fully Nash-implementable.*

Proof. The first part of the proposition follows trivially from the observation that the administrator can ask all dealers to report c and punish them (with a high enough transfer) if the reports disagree. The benchmark may be then made equal (for example) to the average of the reports. The second part follows from the fact that the choice rule to be implemented is not monotonic. (See [Maskin 1999](#) for the definition of monotonicity and the relevant result.) \square

The proposition states that each dealer wants to report a message supporting the announcement of a benchmark that is the true cost c , provided that he believes that all other dealers report in this manner. However, for the mechanism that we construct, there is also an equilibrium in which all dealers report the same, but false, common cost level. The second part of [Proposition 10](#) asserts that this cannot be avoided. That is, there does not exist a mechanism that leads to truthful revelation of c as a *unique* Nash equilibrium. Informally, this means that the benchmark is not robust to collusion.

Benchmark manipulation is studied by two recent papers in different settings. [Coulter and Shapiro \(2014\)](#) solve a mechanism design problem with transfers in a setting that incorporates

important incentives to manipulate that are absent from our model. They reach a similar conclusion in that it is possible to implement a truthful benchmark, but their mechanism can also be “rigged” for false reporting through collusion by dealers. [Duffie and Dworczak \(2014\)](#) consider a different model of benchmark design and manipulation, showing that, without transfers, an optimizing mechanism designer will not in general implement truthful reporting. Instead, considering a restricted class of mechanisms, they characterize a robust benchmark that minimizes the variance of the “garbling,” meaning the difference between the announced benchmark and the true cost level.