Mandatory Disclosure and Financial Contagion*

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Abstract

This paper explores whether mandatory disclosure of bank balance sheet information can improve welfare. In our benchmark model, mandatory disclosure can raise welfare only when markets are frozen, i.e. when investors refuse to fund banks absent such information. Even then, intervention is only warranted if there is sufficient contagion across banks, in a sense we make precise within our model. If investors are willing to fund banks without balance sheet information, so markets aren't frozen, mandatory disclosure cannot raise welfare and it may be desirable to forbid banks from disclosing their financial positions. When we modify the model to allow banks to engage in moral hazard, mandatory disclosure can increase welfare in normal times. But the case for intervention still hinges on there being sufficient contagion. Finally, we argue disclosure represents a substitute to other financial reforms rather than complement them, in contrast to views expressed by some policymakers.

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Introduction

In trying to explain how the decline in U.S. house prices evolved into a full-blown financial crisis during which trade between financial intermediaries collapsed, economists have singled out the role of uncertainty about which entities incurred the bulk of the losses associated with the housing market. For example, in his early analysis of the crisis, Gorton (2008) argues

> "The ongoing Panic of 2007 is due to a loss of information about the location and size of risks of loss due to default on a number of interlinked securities, special purpose vehicles, and derivatives, all related to subprime mortgages... The introduction of the ABX index revealed that the values of subprime bonds (of the 2006 and 2007 vintage) were falling rapidly in value. But, it was not possible to know where the risk resided and without this information market participants rationally worried about the solvency of their trading counterparties. This led to a general freeze of intra-bank markets, write-downs, and a spiral downwards of the prices of structured products as banks were forced to dump assets."[1]

Policymakers seem to have adopted this view as well, as evidenced by the Federal Reserve's decision to release the results of its stress tests of large US banks. These tests required banks to report their expected losses under stress scenarios and thus the losses banks were vulnerable to. In contrast to the confidentiality usually accorded to bank examinations, these results were made public. Bernanke (2013a) captures the view that disclosing this information played an important role in stabilizing financial markets:

> "In retrospect, the [Supervisory Capital Assessment Program] stands out for me as one of the critical turning points in the financial crisis. It provided anxious investors with something they craved: credible information about prospective losses at banks. Supervisors' public disclosure of the stress test results helped restore confidence in the banking system and enabled its successful recapitalization."

In fact, the disclosure of stress-test results was viewed so favorably that policymakers subsequently began to argue for conducting these tests routinely and releasing their results. For example, Bernanke (2013b) argues

> "The disclosure of stress-test results, which increased investor confidence during the crisis, can also strengthen market discipline in normal times."

[1] Similar views were voiced by non-academics. In February 24, 2007, before the crisis unfolded, the *Wall Street Journal* attributed the following to former Salomon Brothers vice chairman Lewis Ranieri, the "godfather" of mortgage finance: "The problem ... is that in the past few years the business has changed so much that if the U.S. housing market takes another lurch downward, no one will know where all the bodies are buried. 'I don't know how to understand the ripple effects through the system today,' he said during a recent seminar."

This paper investigates whether forcing banks to disclose their balance sheet information is indeed desirable in both crisis and normal times. One question that motivates our analysis is why intervention is necessary at all: If disclosure is so useful, why don't banks hire auditors or directly release the information they provide bank examiners? Although in the recent crisis banks may have found it difficult to rely on private auditors given the tarnished reputations of rating agencies, such incentive problems could presumably be resolved privately. An argument for ongoing intervention requires explaining why banks fail to release information even though doing so enhances welfare.

We show that there may be scope for mandatory disclosure if there is a possibility of financial contagion in which shocks to certain banks lead to indirect losses at other banks. For example, during the recent crisis banks with minimal exposure to subprime mortgages still appeared to be vulnerable to potential losses because of the actions of banks more heavily invested in subprime. When contagion is severe, in a sense we can make precise, requiring banks to disclose information can improve welfare. Intuitively, contagion implies information about individual banks is systemically important given that one bank's performance matters for the health of other banks. Since banks do not take into account the systemic value of the information they reveal about their own balance sheets, they tend to disclose less than is socially optimal.

At the same time, our model does not imply disclosure is always desirable, even in the presence of contagion. To the contrary, in our benchmark model not only is disclosure sometimes undesirable, but it may be optimal to force banks to keep information hidden. This is because secrecy can sustain a socially beneficial insurance arrangement between banks. The notion that opacity is desirable for sustaining insurance dates back to Hirshleifer (1971), and has been recently applied to explain the tendency towards secrecy in the banking sector by Goldstein and Leitner (2014) and Dang, Gorton, Holmström and Ordoñez (2014). Our model shares features with the latter papers. In fact, in our benchmark model we find that mandatory disclosure cannot improve welfare in normal times, in contrast to the view advocated in Bernanke (2013b),

To be fair, the argument for mandatory disclosure during normal times that appears in Bernanke's speech rests on a need for market discipline, a feature absent from our benchmark model. We therefore modify our model to allow banks to engage in moral hazard. In this case, mandatory disclosure can raise welfare in normal times, not by stimulating trade but by preventing trade with insolvent banks that results in waste. However, in this case contagion is still necessary for disclosure to raise welfare. Essentially, when agents accrue the gains from revealing information, they have a strong incentive to disclose on their own. If they choose not to, it must be because they find the costs of disclosure exceed its benefits, and forcing them to disclose reduces welfare.

While our discussion is focused on stress tests and banks, our analysis would extend to any setting in which contagion creates a situation where information that agents can release is systemically important. One example is sovereign debt crises in which default by one sovereign prompt runs on debt issued by others. The analog to our results concerning the release of stress tests would be international agreements to enforce reporting standards on fiscal authorities.[2]

The remainder of the paper is organized as follows. In the rest of this section we discuss some of the related literature. We then lay out the information structure and the economic environment of our model in Sections 1 and 2, respectively. In Section 3, we analyze strategic interaction in our model, i.e. how bank disclosure decisions affect others. Our key results are in Section 4, where we characterize when no information is disclosed in equilibrium and yet disclosure can improve welfare. We introduce moral hazard in Section 5 to show that mandatory disclosure may be welfare enhancing in normal times, but only with sufficient contagion. Since we model contagion in a reduced-form way, we discuss a model of balance-sheet contagion in Section 6 that corresponds to our reduced form model. We use this setup to show how our measure of contagion can be shaped by economic forces. Section 7 concludes.

Related Literature

Our paper is related to several literatures, specifically work on (i) financial contagion and networks, (ii) disclosure, (iii) market freezes, and (iv) stress tests.

The literature on financial contagion is quite extensive. We refer the read to Allen and Babus (2009) for a survey. Most of our analysis relies on a reduced-form model for contagion, while most papers in the literature focus on specific channels for contagion. However, we discuss in some detail an example based on balance sheet contagion that occurs when banks that suffer shocks to their balance sheet default on other banks. This idea was originally developed in Kiyotaki and Moore (1997), Allen and Gale (2000), and Eisenberg and Noe (2001) and was explored more recently by Acemoglu, Ozdaglar, and Tahbaz-Salehi (2014), and Elliott, Golub, and Jackson (2014). These papers were largely concerned with how the pattern of obligations across banks affects the extent of contagion, and whether certain network structures can reduce the extent of contagion. We instead focus on how disclosure policies can be used to mitigate the fallout from contagion for what is essentially a fixed network structure.

Our model is also closely related to the work on disclosure. Verrecchia (2001) and Beyer et al (2010) provide good surveys of this literature. A key result in this literature, established by Milgrom (1981) and Grossman (1981), is an "unravelling principle" which holds that all private information will be disclosed because agents with favorable information want to avoid being pooled with inferior types and receive worse terms of trade. Beyer et al (2010) summarize the various conditions subsequent research has established as necessary for this unravelling result to hold: (1) disclosure must be costless; (2) outsiders know the firm has private information; (3) all outsiders interpret disclosure identically, i.e.

[2]For an early survey on contagion and sovereign debt, see Kaminsky, Rienhart, and Végh (2003). On the lack of transparency by fiscal authorities, see Koen and van den Noord (2005).

outsiders have no private information (4) information can be credibly disclosed, i.e. information is verifiable; and (5) agents cannot commit to a disclosure policy ex-ante before observing the relevant information. Violating any one of these conditions can result in equilibria where not all relevant information is conveyed. In our model, non-disclosure can be an equilibrium outcome even when all of these conditions are satisfied. We thus highlight a distinct reason for the failure of the unravelling principle that is due to informational spillovers.

Ours is not the first paper to explore disclosure in the presence of informational spillovers. One important predecessor is Admati and Pfleiderer (2000). Their setup also allows for informational spillovers and gives rise to non-disclosure equilibria, although these equilibria rely crucially on disclosure being costly. When disclosure is costless in their model, all information will be disclosed. Our framework allows for non-disclosure even when disclosure is costless because it allows for informational complementarities that are not present in their model. Specifically, in our model is determining a bank's equity requires information about other banks, a feature that has no analog in their model. However, Admati and Pfleiderer (2000) are similar to us in showing that informational spillovers can make mandatory disclosure welfare-improving.[3] Another difference between our model and theirs is that they assume agents commit to disclosing information before learning it, while in our model banks know their losses and then choose to disclose it.

Beyond the papers that explicit discuss disclosure, there is also a literature on the social value of information in the presence of externalities, e.g. Angeletos and Pavan (2007). However, such papers are less directly related to ours, not only because they abstract from disclosure but also because they assume recipients of information wish to coordinate their actions, a feature missing in our framework.

Our paper is also related to the literature on market freezes. The existing literature emphasizes the role of informational frictions. Some papers emphasize private information in which agents are reluctant to trade with others for fear of being exploited by more informed agents. Examples include Rocheteau (2011), Guerrieri, Shimer and Wright (2010), Guerrieri and Shimer (2012), Camargo and Lester (2011), and Kurlat (2013). Others have focused on uncertainty concerning each agent's own liquidity needs and the liquidity needs of others which discourages trade. Examples include Caballero and Krishnamurthy (2008) and Gale and Yorulmazer (2013). Our framework combines private information about a bank's own balance sheet with uncertainty about the health of other banks. However, the key difference is that we assume information is verifiable and can be disclosed, a feature existing papers ignore.

Finally, there is an emerging literature on stress tests. On the empirical front, Peristian, Morgan, and Savino (2010), Bischof and Daske (2012), Ellahie (2012), and Greenlaw et al (2012) have looked at how the release of stress-test results in the US and Europe affected bank stock prices. These results

[3]Foster (1980) and Easterbrook and Fischel (1984) also argue that spillovers may justify mandatory disclosure, although these papers do not develop formal models to study this.

are complementary to our analysis, which is more concerned with normative questions regarding the desirability of releasing stress-test results. There are also several recent theoretical papers on stress tests, e.g. Goldstein and Sapra (2013), Shapiro and Skeie (2012), Spargoli (2012), Bouvard, Chaigneau, and de Motta (2013), and Goldstein and Leitner (2014). A common feature of these papers is that they ignore the notion that banks can disclose information on their own. Thus, these papers sidestep the main question we are after as to whether banks choose not to disclose and yet forcing banks to disclose is desirable.

1 Information Structure

Our model features a banking system with n banks indexed by $i = 1, \ldots, n$. We begin by describing the information structure of our economy, i.e. what each of the n banks knows. In the next section, we describe the strategic choices banks make and how banks interact with other agents in the model.[4]

Each bank can be one of two types, good and bad. For now, we do not need to assign an economic interpretation to what these types mean. Eventually, we will assume that a bank's type reflects its previous investment decisions, so a good bank is one whose investments proved profitable *ex-post*.

Let Ω denote the set of all possible type profiles for the n banks. Since each bank can assume two types, $|\Omega| = 2^n$, i.e. Ω contains 2^n distinct elements. For any state $\omega \in \Omega$, we can summarize each bank's type with an n-dimensional vector $S(\omega)$ where $S_i(\omega) = 0$ if bank i is bad in state ω and $S_i(\omega) = 1$ if bank i is good in state ω. In what follows, we will sometimes suppress the explicit reference to ω and simply refer to S and S_i as if S were the state. Let $\pi(\omega)$ denote the probability of state ω, i.e. $\pi(\omega) \geq 0$ and $\sum_{\omega \in \Omega} \pi(\omega) = 1$. This probability distribution is common knowledge across all agents.

Each bank i knows its own type but not the types of any of the other $n-1$ banks in the system. That is, when the true state is $\omega \in \Omega$, each bank i knows the true state belongs to the set $\Omega_i(\omega) \subset \Omega$ where

$$\Omega_i(\omega) \equiv \{x \in \Omega \mid S_i(x) = S_i(\omega)\} \tag{1}$$

Note that Ω_i contains 2^{n-1} elements, although some of these may be assigned zero probability under $\pi(\omega)$. If all banks were to reveal their information, i.e. if each bank i announced that the true ω lies in the set $\Omega_i(\omega)$, the state of the banking system would be revealed, since for any $\omega \in \Omega$,

$$\bigcap_{i=1}^{n} \Omega_i(\omega) = \{\omega\}$$

[4] In distinguishing the information structure and the strategic game agents play, we follow Gossner (2000), Lehrer, Rosenberg, and Shmaya (2010, 2013), and Bergemann and Morris (2013, 2014). These papers study how changes in the information structure affect the set of equilibria in the game agents subsequently play. While these questions are related to those we explore, our setup assumes agents first choose what to disclose to others, so the key strategic interaction in our model occurs before the information structure changes rather than after.

Put another way, since bank i knows the i-th element of an n-dimensional vector, the information of all n banks perfectly reveals the underlying state ω. However, if bank i knew only that the true ω was confined to the set Ω_i, it would assign 0 probability to any $\omega \notin \Omega_i$, while to states $\omega \in \Omega_i$ it would assign probability

$$\Pr(\omega \mid \omega \in \Omega_i) = \frac{\pi(\omega)}{\sum_{x \in \Omega_i} \pi(x)}$$

It is worth comparing this information structure to the global games literature associated with Carlsson and van Damme (1993) and Morris and Shin (1998). Those papers assume there is an aggregate state ω and that each agent i observes a private signal $s_i \equiv \omega + \varepsilon_i$ where ε_i are i.i.d. across agents and independent of ω. In both this formulation and ours, collecting the signals of all agents reveals ω, although in the former this requires the number of agents to tend to infinity. In addition, in both frameworks agents receive a mix of idiosyncratic and aggregate information. This is clear in the global games literature, where signals combine aggregate and idiosyncratic terms. In our setup, it might seem as if agents receive a purely idiosyncratic signal about their own type. However, each agent can deduce from his respective signal that the aggregate state $\omega \notin \Omega \backslash \Omega_i(\omega)$. Depending on the distribution $\pi(\omega)$, beliefs about ω may be quite different if $S_i = 0$ and $S_i = 1$. Thus, the signal agents receive in our model can be informative about ω. At the same time, our information structure differs in an important way from the global games setup. Specifically, in our setup agents can tell whether their idiosyncratic term is high or low, while in the global games setup individuals observe the sum $\omega + \varepsilon_i$ and have no idea whether their respective ε_i is high or low. This is important, since in our setup agents who know their idiosyncratic term is high may want to communicate this fact to others, an issue that does not arise in the global games setup.

So far, we have imposed no restrictions on the distribution $\pi(\omega)$. However, for analytical tractability we prefer to work with a symmetric environment. To impose symmetry, we first need some terminology. We will say that vector S' is a *permutation* of vector S if there exists a one-to-one mapping $\phi : \{1, \ldots, n\} \to \{1, \ldots, n\}$ such that $S'_i = S_{\phi(i)}$ for all $i \in \{1, \ldots, n\}$. The symmetry condition we invoke holds that any two states whose vector representations are permutations must be equally likely:

A1. Symmetric likelihood: Given two states ω and ω', if $S(\omega)$ is a permutation of $S(\omega')$, then $\pi(\omega) = \pi(\omega')$.

To help interpret A1, let $B(\omega)$ denote the number of bad banks in state ω, i.e. $B(\omega) \equiv \sum_{i=1}^{n} (1 - S_i(\omega))$. Since all elements of $S(\omega)$ are either 0 or 1, $S(\omega)$ is a permutation of $S(\omega')$ if and only if $B(\omega) = B(\omega')$, i.e. if the number of bad banks is the same in ω and ω'. Hence, A1 implies that if we knew there were exactly b bad banks, any collection of b banks is equally likely to be those that are bad. But A1 imposes no restrictions on the distribution of bad banks, as formalized in the following Lemma:

Lemma 1: A1 holds if and only if there exist numbers $\{q_b\}_{b=0}^{n}$ where $q_b \geq 0$ and $\sum_{b=0}^{n} q_b = 1$ such that $\Pr(B(\omega) = b) = q_b$ and

$$\pi(\omega) = p_{B(\omega)} \binom{n}{B(\omega)}^{-1} \tag{2}$$

To rule out uninteresting cases, we henceforth assume $q_0 < 1$, i.e. at least one bank can be bad. But we impose no other restrictions on $\pi(\omega)$.

We conclude our discussion of the information structure by observing how disclosure of bank j's information would alter beliefs about bank $i \neq j$. In particular, let us compare the unconditional probability that bank i is good with the probability bank i is good given bank $j \neq i$ is good. Under A1, neither the conditional nor the unconditional probability depends on which i and j we choose. Hence, this measure captures informational spillovers between any pair of banks. The unconditional probability that bank i is good is given by

$$\Pr(S_i = 1) = \sum_{\{x | S_i(x) = 1\}} \pi(x) \tag{3}$$

while the probability that bank i is good given bank j is good is given by

$$\Pr(S_i = 1 | S_j = 1) = \frac{\displaystyle\sum_{\{x | S_i(x)=1 \,\cap\, S_j(x)=1\}} \pi(x)}{\displaystyle\sum_{\{x | S_j(x)=1\}} \pi(x)} \tag{4}$$

The next few examples show A1 does not restrict how (3) and (4) can be ranked.

Example 1: Suppose $\pi(\omega) = (1-q)^{B(\omega)} q^{n-B(\omega)}$ for some $q \in (0,1)$. This corresponds to the case where bank types are independent and each bank is good with probability q, i.e. $\Pr(S_i = 1) = q$. Learning that bank j is good has no effect on beliefs about bank i, and (3) and (4) will be identical. □

Example 2: Suppose

$$\pi(\omega) = \frac{1}{2}(1-q_L)^{B(\omega)} q_L^{n-B(\omega)} + \frac{1}{2}(1-q_H)^{B(\omega)} q_H^{n-B(\omega)}$$

where $0 \leq q_L < q_H \leq 1$. This distribution corresponds to the case where bank types are independent as in Example 1, but the probability q that any bank is good is now a random variable, equally likely to be $q_L$or q_H. We now have

$$\Pr(S_i = 1) = \frac{1}{2} q_L + \frac{1}{2} q_H < \Pr(S_i = 1 | S_j = 1)$$

Intuitively, learning that one bank is good raises the odds that $q = q_H$, and thus increases the odds that each of the remaining banks is good. □

Example 3: Pick some $b \in \{1, ..., n-1\}$ and suppose $\pi(\omega) = \binom{n}{b}^{-1}$ for any ω such that $B(\omega) = b$ and $\pi(\omega) = 0$ otherwise. This corresponds to the case where there are exactly b bad banks with certainty, but their identity is uncertain. In this case, $\Pr(S_i = 1) = 1 - \frac{b}{n}$, which is larger than $\Pr(S_i = 1 | S_j = 1) = 1 - \frac{b}{n-1}$. Intuitively, learning that one bank is good implies that the b bad banks are concentrated among fewer banks, reducing the likelihood that each is good. □

The above examples compare the effect of revealing bank j is good to a benchmark of no information about any banks in the system. More generally, we will want to consider how disclosing information affects beliefs about other banks for different prior beliefs. Consider an observer who knows the true state ω lies in some set $\Omega_0 \subset \Omega$ where $\Pr(\omega \in \Omega_0 \cap S_j(\omega) = 1) > 0$, i.e. the fact that the true ω lies in Ω_0 is compatible with bank j being good.[5] We will say there are *positive informational spillovers* if, for any set Ω_0 such that $\Pr(\omega \in \Omega_0 \cap S_j(\omega) = 1) > 0$,

$$\Pr(S_i = 1 | S_j = 1 \cap \omega \in \Omega_0) \geq \Pr(S_i = 1 | \omega \in \Omega_0)$$

and there exists at least one set Ω_0 for which this inequality is strict. Likewise, we will say that there are *negative informational spillovers* if for any set Ω_0 such that $\Pr(S_j = 1 \cap \omega \in \Omega_0) > 0$,

$$\Pr(S_i = 1 | S_j = 1 \cap \omega \in \Omega_0) \leq \Pr(S_i = 1 | \omega \in \Omega_0)$$

with strict inequality for some set Ω_0. Finally, we will say that there are *no informational spillovers* if for any set Ω_0 such that $\Pr(\omega \in \Omega_0 \cap S_j(\omega) = 1) > 0$,

$$\Pr(S_i = 1 | S_j = 1 \cap \omega \in \Omega_0) = \Pr(S_i = 1 | \omega \in \Omega_0)$$

Technically, our definitions correspond to *global* spillovers since they require the direction of spillovers be the same for all relevant information sets Ω_0. We will omit the term global for the sake of brevity, although A1 allows informational spillovers to be positive for some Ω_0 and negative for others.

As Examples 1, 2, and 3 suggest, our framework is compatible with positive, negative, and no informational spillovers. We alert the reader that we will introduce a separate spillover in Section 2 that reflects real linkages between banks rather than information. Unlike informational spillovers, this spillover will work in an unambiguous direction: A bank will be (weakly) worse off the more bad banks there are in the system. Thus, our model implies each bank will be better off when there are more good banks in the system, but it need not be better off if more banks announce they are good. For example, with negative informational spillovers as in Example 3, outside observers may be more inclined to believe bank i is bad when other banks announce they are good, making bank i worse off. In essence, informational spillovers govern how the number of banks that revealed themselves as good affects beliefs about other banks' types, while the real spillovers we introduce below govern how the actual number of good banks in the system affects other banks' outcomes.

[5]Note that under A1, if $\Pr(\omega \in \Omega_0 \cap S_j = 1) > 0$ for some j, it is positive for all j. Thus, which j we choose to verify this condition is irrelevant.

2 Economic Environment

We now turn to the strategic aspects of our economy. We begin with an overview. Per our earlier comments, we assume that what distinguishes good and bad banks is that the latter suffer losses on their past investments. However, we want to allow for the possibility that good banks can still incur losses because of their exposure to bad banks, a phenomenon known as *contagion*. This is the spillover across banks we alluded to above. In what follows, we model contagion in a reduced-form way by assuming good banks are endowed with less equity whenever certain other banks in the system are bad. We then show this setup can capture specific channels by which contagion may occur.

The reason a bank's equity endowment matters in our model is that we assume banks face a debt overhang problem as in Myers (1977). That is, all n banks, regardless of their equity, can undertake profitable projects that require external finance. However, banks' existing liabilities cannot be renegotiated and must be senior to any new debt obligations of banks. As a result, the investors that banks need to finance projects may be reluctant to trade, knowing that if a bank is bad or is exposed to bad banks it must surrender any returns from the project to senior debt holders. Good banks thus have an incentive to reveal their type and mitigate concerns about their equity. However, we assume disclosure is costly, so banks may not always be willing to incur the cost of disclosure. Moreover, if good banks can be exposed to bad banks, unilateral disclosure may not be enough to induce outsiders to invest. It is therefore possible that no bank opts to disclose its type, and that in the absence of information outsiders invest in none of the banks, i.e. markets freeze.

Formally, our economy operates as follows. First, nature chooses $\omega \in \Omega$ according to the distribution $\pi(\omega)$. Each bank i then learns its type $S_i(\omega)$. Next, banks participate in a disclosure game in which they simultaneously choose whether to reveal their types, and any bank that discloses its type incurs a cost. After banks make their disclosure decisions, outside investors observe whatever information was disclosed and choose which banks to invest in, if any. Banks that succeed in raising funds undertake their projects and distribute their earnings. The remainder of this section fleshes this timeline in more detail.

2.1 Equity Endowments and Contagion

The key attribute of a bank is its equity endowment. Let $e_i(\omega)$ denote bank i's equity in state ω. In general, $e_i(\omega)$ can be positive or negative, where a negative equity value implies the bank's existing liabilities exceed the value of its assets. We will sometimes refer to $e_i(S_i, S_{-i})$, where $S_{-i} = \{S_j\}_{j \neq i}$, to emphasize that a bank's equity can depend on both its own type and the types of the remaining $n-1$ banks. The dependence on a bank's own type captures the idea that bad banks undertook failed projects in the past and so will tend to have less equity. The fact that a bank's equity can depend on other banks' types captures the idea of contagion. It might seem unsatisfactory to assert that bank i's equity will be low when some bank j is bad without an explicit reason as to why. However, our results do not depend on the exact reason for contagion, and below we show that our model can be understood as the reduced form of models in which the channel for contagion is explicit. Note that a bank initially knows its type but not its equity, since the latter depends on ω.

We now impose some assumptions on how the equity of diffrent banks $e_i(\omega)$ varies with the state ω. The first restriction involves symmetry. In principle, we can appeal to an analogous condition to A1, i.e. assume that the equity of bank i depends on the total number of other banks that are bad. However, this form of symmetry implies banks must be equally exposed to all banks.[6] We therefore appeal to the following weaker notion of symmetry:

A2. Symmetric equity: e_i is such that for each pair (i, j) from $\{1, ..., n\}$, there exists a one-to-one mapping $T_{ij} : \Omega \to \Omega$ where, for each $\omega \in \Omega$,

i. $S(T_{ij}(\omega))$ is a permutation of $S(\omega)$

ii. $S_j(T_{ij}(\omega)) = S_i(\omega)$

iii. $e_i(S_i(\omega), S_{-i}(\omega)) = e_j(S_j(T_{ij}(\omega)), S_{-j}(T_{ij}(\omega)))$

In words, A2 requires that for each state $\omega \in \Omega$, we can find a corresponding state $\omega' = T_{ij}(\omega)$ such that (1) the number of bad banks is the same in states ω and ω', i.e. $B(\omega) = B(\omega')$; (2) bank j's type in state ω' is the same as bank i's type in state ω'; and (3) bank j's equity in state ω' is the same as bank i's equity in state ω. The requirement that T_{ij} be one-to-one implies that each ω will map into a distinct ω'. As an illustration of how we can satisfy A2, consider the case where e_i exhibit *rotational symmetry.*[7] That it, for any state $(S_1, ..., S_n) \in \{0, 1\}^n$, bank 1's equity when the state is $(S_1, S_2, ..., S_{n-1}, S_n)$ is the same as bank 2's equity when the state is $(S_n, S_1, S_2, ..., S_{n-1})$, the same as bank 3's equity when the state is $(S_{n-1}, S_n, S_1, ..., S_{n-2})$, and so on. This allows bank i's equity to depend on exactly which banks are bad.[8]

Since A2 requires the number of bad banks to be the same in states ω and $T_{ij}(\omega)$, it follows from A1 that ω and $T_{ij}(\omega)$ are equally likely. This implies that the distribution of equity at banks i and j, either unconditionally or conditional on its own realization, must be the same, as summarized in the next lemma.

[6]Essentially, A1 is akin to an anonymity condition in that it implies a bank's identity does not affect what its type is likely to be. By contrast, we want identities to potentially matter for equity, e.g. a bank may be more exposed to certain banks than to others, while still insisting that banks be sufficiently similar that they are similarly exposed to the rest of the system.

[7]Rotational symmetry is stronger than A2, i.e. rotational symmetry implies A2 while A2 need not imply rotational symmetry.

[8]Beside allowing for differential exposure to different banks, our weaker notion of symmetry allows the equity of good banks to depend on how bad banks are distributed within the system when there are multiple bad banks. For example, our specification allows the equity of good banks to depend on whether bad banks are close to each other or far apart (for a suitable notion of distance). An implication of this is that even when the number of bad banks is fixed, the number of affected good banks and thus aggregate equity can still be stochastic.

Lemma 2: Suppose A1 and A2 hold. Then for any x, $\Pr(e_i = x)$ and $\Pr(e_i = x|S_i)$ must be the same for all i.

Our next assumption on $e_i(\omega)$ stipulates that bad banks have significantly negative equity. In particular, further below we will allow banks to undertake a project with a gross return of R. We assume that the equity of bad banks is sufficiently negative that investing in the project could not restore them to positive equity even if they could retain all of the returns from the project, i.e.,

A3. Negative equity at bad banks: $e_i(0, S_{-i}) \leq -R$ for all S_{-i}.

Next, we assume the equity of good banks satisfies a weak monotonicity condition with respect to the number of good banks in the system:

A4. Monotonicity: For all S_i, if $S'_{-i} > S_{-i}$, then $e_i\left(S_i, S'_{-i}\right) \geq e_i(S_i, S_{-i})$.

A4 is the assumption that allows for contagion, since if the inequality were ever strict, bank i's equity would fall when some *other* bank is bad.[9] While this approach assumes contagion without explaining it, we now show that our model can be understood as a reduced form of existing models of contagion that specify what actions bad banks take that harms good banks. The first example are models of fire sales. In these models, bad banks sell their assets to marginal buyers who value these asset less (and hence did not own them originally). Good banks that hold the same assets then suffer. Shleifer and Vishny (1992) were among the first to offer a formal analysis of this narrative.[10] The next example shows how a model of fire sales can be captures within our framework.

Example 4 (Fire Sales): Consider the following adaptation of the Greenwood, Landier, and Thesmar (2014) model of fire sales. At some initial date, banks borrow to purchase assets. Banks can buy two types of assets. Bad banks own both types and good banks own one. If the realized return on the asset uniquely held by bad banks were sufficiently negative, bad banks would have to liquidate their assets. Greewood, Landier, and Thesmar assume the price of an asset declines linearly in the amount of the asset bad banks sell. But we could equally assume the price falls only when the amount sold exceeds a threshold. The non-linearity captures the idea that there must be enough good banks to absorb the assets sold by bad banks for the price not to fall. We can represent this in our framework by setting $e_i(0, S_{-i}) \leq -R$ for all S_{-i} and

$$e_i(1, S_{-i}) = \begin{cases} \overline{e} & \text{if } B(\omega) \leq b^* \\ \underline{e} & \text{if } B(\omega) > b^* \end{cases}$$

where $\overline{e} > 0$ and $\underline{e} \leq -R$, and b^* represents the threshold number of bad banks before the valuation of the marginal buyer falls. □

[9] In principle, a good bank's equity could be higher when other banks are bad. For example, a good bank may attract more business if its rivals suffer. In imposing A4, we are implicitly assuming these considerations are not enough to overturn the underlying force of contagion.

[10] Shleifer and Vishny (1992) argue that when agents in distress sell their assets, other agents who would naturally buy their assets may be unlikely to do so because of debt overhang. As will become clear in our description, debt overhang features prominently in our model as well.

The next example involves models of balance-sheet contagion. In these models, banks are financially linked. If bad banks have obligations to good banks, their inability to pay harms bad banks. Kiyotaki and Moore (1997), Allen and Gale (2000), and Eisenberg and Noe (2001) were among the first to develop these models. The next example shows our setup capture these models as well.

Example 5 (Balance Sheet Contagion): Consider the Caballero and Simsek (2012) model of balance sheet contagion. In their model, n banks are organized along a circle and each bank is obligated to the bank above it modulo n. They assume a single bad bank that defaults on its obligation, triggering a domino effect in which the next k banks are also unable to pay their obligations in full. Our model can capture this if we set $\pi(\omega) = 1/n$ when $B(\omega) = 1$ and 0 otherwise, and if we set $e_i(0, S_{-i}) \leq -R$ for all S_{-i} and

$$e_i(1, S_{-i}) = \begin{cases} \overline{e} & \text{if } S_j = 1 \text{ for all } j \text{ s.t. } (i-j) \bmod n \in \{1, 2, ..., k\} \\ \underline{e}(S_{-i}) & \text{if } S_j = 0 \text{ for some } j \text{ s.t. } (i-j) \bmod n \in \{1, 2, ..., k\} \end{cases}$$

where $\overline{e} > 0$ and $\underline{e}(S_{-i}) \leq -R$ for all S_{-i}. The reason $\underline{e}$ depends on S_{-i} is that bank i's equity position depends on how far away it is from the bad bank. □

Finally, we impose one last assumption on $e_i(S_i, S_{-i})$, although this assumption is meant for convenience rather than because it is essential. Specifically, we require that no bank be marginal in the sense that its ability to meet obligations depends on whether it undertakes a project that promises a return of R we introduce later. That is, either a bank has enough equity to discharge all its obligations or its equity is sufficiently negative that it can never earn enough from operating a project would not be enough to render it solvent. Formally:

A5. No Marginal Banks: Either $e_i(S_i, S_{-i}) \geq 0$ or $e_i(S_i, S_{-i}) \leq -R$ for all (S_i, S_{-i}).

The advantage of A5 is that it allows us to analyze each bank without knowing whether other banks were funded. This is because bank i's equity will not be affected by whether bank j is able to attract funds, since whether bank j pays bank i or not depends only on bank j's initial equity. While this sidesteps interesting questions about whether encouraging outsiders to invest in banks can mitigate the extent of contagion, these questions will not matter for our main results that concern extreme degrees of contagion. In Section 5, we show that we can replace A5 with a different assumption that is equally tractable to work with but does not require the distribution of equity to feature a gap.

2.2 Trade between Outsiders and Banks

Now that we described how equity at each bank depends on the state ω, we can turn to the why equity is relevant. The idea is that equity is essential for inducing outside investors to trade with banks. Thus, a bank's equity will matter, as will the beliefs outsiders have about each bank's equity.

Suppose each bank regardless of its type can earn a gross rate of return of $R > 1$ if it invests a single unit of resources, although banks lack the resources to invest. At the same time, there is a group of outside investors who can earn a gross return of $\underline{r} < R$ on their own and who collectively own more resources than banks can invest. Thus, there is scope for gains from trade between outside investors and banks. However, we assume contracting frictions that restrict a bank's ability to trade with outsiders. Specifically, we assume that any outstanding liabilities a bank is endowed with must be senior to payments banks can promise outsiders. If outsiders believe a bank's initial equity is negative, they would be reluctant to invest in it, knowing that the returns to the project would go to others. This is the debt overhang problem introduced by Myers (1977). Although we do not explicitly model why original debt holders were given seniority that cannot be renegotiated, previous work such as Hart and Moore (1995) offers conditions in which such a contract can be optimal even though it may lead to debt overhang.

In what follows, we restrict outsiders and banks who wish to trade to using debt contracts, i.e. investors provide bank i with 1 unit of resources in exchange for a fixed repayment of r_i. Since equity contracts are junior to debt obligations, allowing these would not matter for our results. We assume all investors simultaneously offer each bank a debt contract, and banks choose from the contracts offered. Investors thus engage in Bertrand competition. Although in principle we should specify what contracts investors offer, we will mostly refer to the equilibrium terms without explicitly referring to what investors do. For example, if outsiders knew ω, they would know the initial equity of each bank. In this case, in equilibrium only banks with positive equity would receive funding, and they would each be charged $\underline{r}$. If outsiders knew nothing about ω other than the prior $\pi(\omega)$, symmetry would imply that in equilibrium all banks generically receive the same terms. If outsiders had partial information about ω, banks would generally receive different terms. In what follows, we let information about ω emerge endogenously by letting banks chose whether to reveal their types in a disclosure game before outsiders make their investment decisions.[11] We now describe the disclosure game and then the investment decisions of outsiders.

2.3 The Disclosure Game

We begin with the disclosure game. Each bank i must decide whether to disclose its type after observing S_i but before observing S_{-i}. Disclosure involves hard information, i.e. announcements are verifiable and bank can only report truthfully. We also assume that disclosure is costly. This can be because information is either costly to produce or costly to communicate. For simplicity, we model disclosure costs as as a utility cost $c > 0$ that is only incurred if S_i is disclosed. This specification equates the private and social costs of disclosure.

[11] Other papers have also studied disclosure games that precede decisions, e.g. Alonso, Dessein, and Matouschek (2008), Hagenbach and Koessler (2010, 2011), and Galeotti, Ghiglino, and Squintani (2013). However, these papers are interested in economies where agents want to coordinate with one another but disagree on what action to coordinate on. By contrast, in our model agents communicate to an outside party rather than to other players, are not trying to coordinate. Players care what others disclose only because of spillovers.

This is not too restrictive, since differences between the two can be folded into the payoff to disclosure, since the cost of disclosure can equally be represented as a benefit from non-disclosure. Indeed, for reasons that will become clear below, the private and social benefits from disclosure will differ in our model.[12]

Bank i's strategy can be summarized as a rule $\sigma_i : S_i \to [0,1]$ that assigns probability of disclosure σ_i when its type is S_i. The outcome of the disclosure game is a vector of announcements $A = \{A_1, \ldots, A_n\}$, where $A_i = S_i$ if bank i announces its state and $A_i = \varnothing$ if bank i fails to announce. Given the true state ω and the strategy profile $\sigma = (\sigma_1, \ldots, \sigma_n)$, we can determine the distribution of announcements A that will be observed in that state, $\Pr(A|\omega, \sigma)$. Outsiders, however, face the opposite problem: Given a vector of announcements and knowing the equilibrium strategy profile σ, they assign probabilities over the true state ω, $\Pr(\omega|A, \sigma)$.[13] It will be convenient to also introduce a vector that summarizes which banks disclosed, as distinct from what they disclosed. Thus, define $\alpha = (\alpha_1, \ldots, \alpha_n)$ where $\alpha_i = 1$ if $A_i \in \{0, 1\}$ and $\alpha_i = 0$ if $A_i = \varnothing$. Since disclosure is costly, we must subtract $c\alpha_i$ from bank i's payoff.

2.4 Investment Decisions

After banks decide whether to disclose their type or not, outside investors observe A and decide whether to make offers to any of the banks and at what terms. Let $I_i(A)$ be a variable that is equal to 1 if bank i obtains funding from some investor and 0 otherwise, and let $r_i(A)$ denote the rate bank i is charged. We will refer to $\{I_i(A), r_i(A)\}$ as the equilibrium terms bank i receives given A.

Our first observation is that $r_i(A)$ cannot exceed R, since if it did bank i would never agree to borrow. To solve for $r_i(A)$, note that under A5, initial equity at each bank is either nonnegative or below $-R$. Since $r_i \leq R$, a bank will repay its debt in full if $e_i(S_i, S_{-i}) \geq 0$ and nothing if $e_i(S_i, S_{-i}) \leq -R$. The expected payoff to an outside investor in bank i in equilibrium will equal

$$\Pr(e_i \geq 0 \,|A, \sigma)\, r_i(A) \tag{5}$$

Outsiders will agree to finance bank i if (5) is at least $\underline{r}$. Competition among outsiders ensures the equilibrium expected return from lending is $\underline{r}$, i.e.

$$r_i(A) = \frac{\underline{r}}{\Pr(e_i \geq 0 \,|A, \sigma)} \tag{6}$$

[12] In general, the private net benefit from disclosure can be higher or lower than the social net benefit. On the one hand, if banks fear that disclosing information will erode the rents they can earn, the private cost of disclosure will exceed the social cost. In the opposite direction, as we discussed in the Introduction disclosure can prevent insurance arrangements, so its social cost may exceed its private cost. Our model features the latter but not the former.

[13] Since σ represents the strategies outsiders believe banks use, these expressions are undefined if $\Pr(A|\sigma) = 0$, i.e. when outsiders observe announcements they should not given their beliefs. We define equilibrium in Section 4 in a way that restricts beliefs in these cases.

Since $r_i(A)$ cannot exceed R, then we know that after observing the announcements A, outsiders will not finance a bank i for which

$$\Pr(e_i \geq 0 \,|A, \sigma) < \frac{r}{R}$$

Hence, equilibrium investment $I_i(A)$ will be given by

$$I_i(A) = \begin{cases} 1 & \text{if } \Pr(e_i \geq 0 \,|A, \sigma) \geq \dfrac{r}{R} \\ \\ 0 & \text{if } \Pr(e_i \geq 0 \,|A, \sigma) < \dfrac{r}{R} \end{cases} \tag{7}$$

Equations (6) and (7) together fully characterize the terms each bank would receive given vector of announcements A. Since bank i retains the equity that remains after discharging its debts, and incurs a cost c if it chooses to disclose, its payoff in state ω if banks announce A is equal to

$$\max\{0, e_i(\omega) + [R - r_i(A)]\, I_i(A)\} - c\alpha_i \tag{8}$$

This completes the description of the disclosure game. Essentially, good banks compare the cost of disclosure c to the expected benefit of revealing their type and potentially improving their terms of trade. Bank i knows that $\omega \in \Omega_i$, and so given the strategy profile σ_{-i}, if it chooses σ_i, it assigns probability $\Pr(\omega, A|\Omega_i, \sigma)$ that the true state is ω and that A will be announced. Using these probabilities to weigh the payoffs in (8), a bank can compute its optimal response to other banks disclosure rules. Although we proceed as if banks play a static disclosure game, our setup is in fact a dynamic game of incomplete information in which investors make offers, and the equilibrium terms $\{I_i(A), r_i(A)\}$ banks accept represent the outcome of the continuation game after banks choose whether to announce their types. Since the equilibrium of the continuation game is standard, we find it easier to analyze our model as if it were a static game.

2.5 Implications of Debt Overhang in our Model

Before we analyze the disclosure game we just described, we offer some perspective on the broader resource allocation problem that underlies our model. Since each bank in our model has fixed capacity, any bank that fails to raise funds represents a lost opportunity for society to earn a higher return R. An unconstrained planner would thus want all banks to obtain funding. But due to contractual frictions, outsiders may not be willing to go along with this.[14]

In principle, a planner could attempt to directly overcome the contractual friction by transferring resources to banks that cannot raise funds, then redistributing the resources they earn. Philippon and Schnabl (2013) pursue this line

[14]The optimal policy prescription is thus similar to what optimal policy would dictate if banks were merely illiquid, even though in the model banks with negative equity are insolvent under A5. The reason it is optimal to keep insolvent banks operating as if they were merely illiquid is because banks in the model are uniquely able to create surplus.

in a general equilibrium model of debt overhang closely related to the model we study. They allow the planner to tax agents and then transfer the resources collected to banks. They find such an intervention can increase welfare.[15]

In what follows, we consider interventions that only involve the disclosure of information rather than resource transfers. This allows us to analyze the virtue of disclosing information on banks separately from the role of this information in determining which banks ought to receive capital injections. If the goal of stress-tests was simply to lay the groundwork for capital injections, there would be no need to publicly disclose this information once it was collected. Yet as we discussed in the Introduction, policymakers have argued that public disclosure is beneficial in and of itself, and we aim to investigate this proposition.

How can a planner use information to maximize the number of banks that receive funding? The answer depends on what would happen when banks disclose no information. If outsiders refuse to invest in the absence of disclosure, revealing some information will allow outsiders to identify some banks that are worth investing in, thus improving welfare. But if outsiders are already willing to fund banks even without disclosure, revealing information may discourage outsiders from investing in banks revealed to have negative equity. Disclosure is thus sometimes desirable and sometimes not. We will confirm this intuition below and elaborate on when intervention may be desirable.

3 Strategic Interaction in the Disclosure Game

Before we examine the equilibria of our game and deduce what information is disclosed and whether intervention is warranted, we explore how strategic interaction operates in our model to better appreciate its features. We focus on two questions. First, would one bank's decision to disclose encourage outsiders to trade with other banks? This question concerns externalities, i.e. whether disclosure by one bank benefits other banks by facilitating trade. Second, would one bank's decision to disclose encourage other banks to also disclose their types? This question concerns the possibility of strategic complementarities, i.e. whether multiple equilibria and asymmetric equilibria are possible.

We begin with an observation that helps to simplify our discussion:

Lemma 2: If $S_i = 0$ so bank i is bad, not disclosing is a dominant strategy.

Intuitively, a bad bank gains nothing from disclosure: Given Assumption A3, no outsider would want to invest in a bank knowing it was bad. If disclosure is at all costly, a bad bank would be better off not disclosing. The implication

[15]The simplest way to enact this transfer is a cash injection to banks, coupled with a lump-sum tax on banks that is senior to all other claims. Philippon and Schnabl (2013) discuss various transfer schemes that have been used in practice, e.g. capital injections in which a bank promises to pay dividends in exchange for the resources it receives; asset purchases where the bank sells its assets; and loan guarantees, where a bank is assessed a fee based on how much the bank borrows and in turn new borrowers are guaranteed to be repaid.

of Lemma 2 is that we can reduce the strategy of a bank to the single number $\sigma_j \equiv \sigma_j(1)$, the probability that bank j discloses its type if it learns it is good. It also implies that in equilibrium we would observe $A_j \in \{\varnothing, 1\}$ but not $A_j = 0$.

In studying strategic interaction between banks, it will be useful to distinguish between the effect of a bank's *announcement*, i.e. the effect of observing $A_j = 1$ and learning that bank j is good, and the effect of a bank's *strategy*, i.e. the effect of knowing that bank j chose to disclose its type if good with probability σ_j. The two are obviously related: A higher σ_j increases the odds that we observe $A_j = 1$. But a higher σ_j also changes the informational content of observing $A_j = \varnothing$. In particular, a higher σ_j leads outsiders to assign higher probability that bank j is bad if they observe $A_j = \varnothing$. While a higher σ_j is a greater commitment by bank j to disclose its type when good, it is better to interpret a higher σ_j as a more informative signal about bank j regardless of its type. The latter interpretation will help in understanding some of our results.

3.1 The Effect of Announcements

We begin with the effect of announcements: How does news that $A_j = 1$ affect bank i where $j \neq i$? Let us refer in this section to the investment in bank i as $I_i(A_i; A_j)$ as opposed $I_i(A_1, \ldots, A_n)$, even though investment depends on all bank announcements. This notation highlights our focus on the effect of bank j's announcement holding the announcements of any remaining banks fixed.

Not surprisingly, the effect of announcements is closely related to the notion of informational spillovers we introduced in Section 1. We begin with the case where informational spillovers are positive or absent. In this case, news that one bank is good makes it easier for remaining banks to raise funds. Intuitively, if outsiders observe that $A_j = 1$, they would assign a higher probability that both bank i and any banks that bank i is exposed to are good, making them more likely to invest in bank i. This is confirmed in the following proposition:

Proposition 1: Suppose informational spillovers are positive or absent. Then $I_i(\varnothing; 1) \geq I_i(\varnothing; \varnothing)$ and $I_i(1; 1) \geq I_i(1; \varnothing)$, i.e. news that bank j is good will encourage outsiders to fund bank i.

Proposition 1 implies banks will be better off at news that some other bank is good. However, it does not tell us whether such news will encourage or discourage a bank to disclose its own type. To some extent, this question is moot given our setup: By the time A_j is revealed, banks would have already made their disclosure decisions. However, banks choose their disclosure strategy anticipating what announcements will be made and what would be optimal to disclose in that case. The question of what a bank's preferred action would be if certain other banks announced they were good is therefore relevant.

It turns out that whether news that some other bank is good encourages or discourages other banks to disclose their own types depends on whether a bank can raise funds when its type is uncertain. Our next result shows that if a bank expects not to raise funds when it does not reveal its type, news that other banks are good implies would encourage it to disclose.

Proposition 2: Suppose informational spillovers are positive or absent. If $I_i(\varnothing;\varnothing) = I_i(\varnothing;1) = 0$, then the gain to bank i from disclosure is weakly higher when $A_j = 1$ than when $A_j = \varnothing$.

Intuitively, if a bank cannot raise funds when outsiders are uncertain as to its type, the benefit of disclosure is that it may be able to raise funds and earn profits. Since news that some other bank is good makes outsiders more optimistic about all banks, including the banks a good bank would still be exposed to, a good bank that reveals its type will be able to earn higher profits.

Proposition 2 tells us that if bank i cannot raise funds when its type is not known to outsiders, news that more banks are good would encourage bank i to disclose. However, Proposition 1 implies that as more banks are revealed to be good, bank i is more likely to obtain funding even if outsiders do not know its type. We now show that if bank i can raise funds even without revealing its type, news that more banks are good discourages bank i from disclosing.

Proposition 3: Suppose informational spillovers are positive or absent. If $I_i(\varnothing;\varnothing) = I_i(\varnothing;1) = 1$, then the gain to bank i from disclosure is weakly lower when $A_j = 1$ than when $A_j = \varnothing$.

Intuitively, when a bank can raise funds even without disclosing its type, the gain from disclosure comes from reducing the interest it pays outside investors. But with positive informational spillovers, when others announce they are good, banks are charged lower rates. As more banks announce they are good, the interest charges a bank can save by disclosing it is good fall.

Propositions 2 and 3 suggest that banks might prefer to disclose their type when only a few banks are known to be good but not to disclose when a large number of banks are known to be good.[16] This suggests disclosure decisions in our model cannot be neatly characterized as either strategic complements or substitutes. While the effect of announcements are not equivalent to the effect of strategies, we confirm below that disclosure decisions in our model indeed cannot be generically described as either substitutes or complements.

Next, we turn to the case where informational spillovers are negative. In this case, there is no analog to Propositions 1 and 2. To see why, consider Example 5 above. In this case, news that a bank in $\{n-k+1, n-k+2, ..., n\}$ is good will encourage outsiders to invest in bank 1, while news that a bank in $\{2, 3, ..., n-k\}$ is good will discourage them. Similarly, whether news that other banks are good make bank 1 wish it had disclosed its own type when it is unable to raise funds otherwise depends on which bank is revealed as good. This reflects an important difference between negative and nonnegative informational

[16] Propositions 2 and 3 omit case which $I_i^*(\varnothing;\varnothing) = 0$ and $I_i^*(\varnothing;1) = 1$ (the opposite case is ruled out by Proposition 1). We show in the Appendix that in this case the gain to disclosure rises by no more than it would if $I_i^*(\varnothing;1) = 0$ and falls no more than it would if $I_i^*(\varnothing;\varnothing) = 1$.

spillovers. With nonnegative spillovers, news that some bank is good will be beneficial for bank i regardless of which bank it is: Outside investors will raise their assessment that both bank i and and any banks that bank i is exposed to are good. With negative informational spillovers, which bank reveals itself to be good matters. The upshot is that there is no general sense in which announcements that banks are good is beneficial for other banks, nor do they generally encourage or discourage other banks from disclosing their types.

However, we can still establish an analogous result to Proposition 3 when informational spillovers are negative, i.e. news that a bank is good will have unambiguous implications when a bank of unknown type can raise funds. The effect is now the opposite of what we found for nonnegative information spillovers, i.e. news that other banks are good encourages other banks to disclose:

Proposition 4: Suppose that informational spillovers are negative. If we have $I_i(\varnothing;\varnothing) = I_i(\varnothing;1) = 1$ then the gain to bank i from disclosure is weakly higher when $A_2 = 1$ than when $A_2 = \varnothing$.

Intuitively, if a bank can raise funds without disclosing its type, the gain from disclosure comes from reducing the interest it has to pay. With negative spillovers, news that another bank is good will make outsiders more concerned that bank i is good. If they are still willing to invest in bank i, they will charge it a higher rate, and so the bank stands to gain more from disclosure.

3.2 The Effect of Disclosure Strategies

So far, we have described the effect of news that a bank is good on other banks. This is a natural way to frame the discussion of bank interaction given it involves observables. But strategic interaction among banks involves a more subtle object, the probability σ_j that bank j announces its type if it is good. It might seem as if the effect of increasing σ_j is similar to the effect of news that $A_j = 1$, since a high σ_j is just a promise to replace $A_j = \varnothing$ with $A_j = 1$ when bank j is good. However, the two are not the same, since a commitment to a higher σ_j also changes the informativeness of a bank making no announcement.

We begin with the question of whether a higher σ_j encourages outsiders to trade with banks $i \neq j$. Above we showed that news that some bank j is good will encourage outsiders to trade with bank $i \neq j$ when informational spillovers are nonnegative, but it may discourage outsiders from trading when informational spillovers are negative. We might therefore expect increasing the probability a good bank discloses its type will encourage trade only when informational spillovers are nonnegative. But we now show that increasing σ_j encourages trade with banks regardless of the nature of informational spillovers.

To see this, let us define $\mathcal{G}_i(\sigma)$ as the ex-ante gains from trade with bank i that outsiders expect given the strategies $\sigma \equiv (\sigma_1, \ldots, \sigma_n)$, i.e. before A is revealed. Since competition ensures outsiders earn no more than their outside option $\underline{r}$, we cannot learn much about the propensity of outsiders to trade with bank i based on their expected returns. However, the gains from trade that outsiders expect tells us whether outsiders view trade as more rewarding, even if they ultimately do not reap those rewards. Thus, $\mathcal{G}_i(\sigma)$ is a good indicator of the incentive to trade. To compute it, note that if outsiders invest in bank i, they together with bank i would earn a return of R instead of $\underline{r}$ if bank i has positive equity, but a return of 0 if bank i's equity is negative. Hence,

$$\begin{aligned}\mathcal{G}_i(\sigma) &= E[(\Pr(e_i \geq 0)R - \underline{r})I_i|\sigma] \\ &= \sum_{A\in\{\varnothing,1\}^n} (\Pr(e_i \geq 0|A,\sigma)R - \underline{r})I_i(A)\Pr(A|\sigma) \qquad (9)\end{aligned}$$

Our next result shows that outsiders expect there to be more gains from trade with bank i when σ_j is higher, regardless of the nature of informational spillovers.

Proposition 5: $\mathcal{G}_i(\sigma)$ is weakly increasing in σ_j for all $j \neq i$, i.e. the gains from trade outsiders expect to achieve with outsiders is always weakly increasing in the probability that banks $j \neq i$ discloses its type.

The fact that Proposition 5 holds even when informational spillovers are negative may seem surprising at first. If news that some other bank is good causes outsiders to infer that bank i is more likely to be bad, why wouldn't an increase in the probability that bank j announces it is good similarly discourage trade? The reason is that even though an announcement of $A_j = 1$ may discourage outsiders from trading with bank i, a higher σ_j will cause an announcement of $A_j = \varnothing$ to encourage trade. Essentially, outsiders only want to trade with bank i only when its equity is positive, and a higher σ_j helps outsiders identify when bank i is likely to be able to repay and when it is not.

Next, we consider the effect of a bank's disclosure on the incentives for other banks to disclose. Recall that our results for announcement effects suggested disclosure is not in general a strategic substitute or complement. We now show by numerical example bank i's incentive to disclose are non-monotonic in σ^*, even more dramatically than suggested by our results on announcement effects.

Example 6: Consider the fire-sale channel for contagion in Example 4 above, with $n = 10$ banks. Suppose types are independent and that each bank is good with probability 0.9. We set $b^* = 0$, meaning a good bank defaults if even one bank in the system is bad. The returns to outsiders and banks are $\underline{r} = 1$ and $R = 2.55$, respectively. These parameters ensure that if no bank discloses its type, outsiders would not trade with a bank even if it disclosed its type, since $p_g R = (0.9)^9 \times 2.55 = 0.99 < \underline{r}$. At the same time, if all other banks disclose, outsiders would trade with a bank of uncertain type since $0.9 \times 2.55 = 2.30 > \underline{r}$. We also set $c = 0.5$, although it plays no role in our analysis.

Since we assume independent types, there are no informational spillovers. Our results for announcement effects suggest that in this case, disclosure by some banks would initially encourage disclosure, but that this should eventually taper off. To examine this, suppose all banks other than bank i disclose with the same probability σ^*. Figure 1 plots the gain to bank i from revealing it is good as a function of σ^*. Bank i indeed gains less from disclosure when $\sigma^* = 0$ than when $\sigma^* = 1$. Moreover, the gains to disclosure seem to generally rise faster with σ^* at low values of σ^*. But as σ^* ranges from 0 to 1, the gains to disclosure rise and fall multiple times. This is because as we increase σ^*, outsiders grow more reluctant to invest in bank i unless enough other banks announce they are good. The threshold number of banks that must announce depends on whether bank i reveals it is good or not. The local minima in Figure 1 occur at values of σ^* for which the threshold number jumps when bank i fails to disclose. □

To recap, we find that disclosure by one bank benefits other good banks by increasing their scope for achieving gains from trade with outsiders. To anticipate some of our results below, note that the increase in gains from trade is in part because outsiders can avoid investing in banks with negative equity. While this is privately optimal for outsiders and a good bank, it may not be socially optimal given the goal of funding all banks. Hence, this positive externality may not always make more disclosure desirable. We also find that disclosure decisions are not inherently complementary. Thus, it will not generally be the case that good banks will agree to disclose if they could only coordinate among themselves. These insights will prove useful later for interpreting our results.

4 Equilibrium of the Disclosure Game

We now characterize the equilibrium of our economy. We first need to discuss what notion of equilibrium is appropriate for our model. We use the notion of sequential equilibria in Kreps and Wilson (1982). That is, in a sequential equilibrium each player's strategy is optimal given the others' strategies – including investors, even though in our discussion we don't explicitly describe their strategies, and off-equilibrium beliefs coincide with the limit of beliefs from a sequence in which players choose all strategies with positive probability but the weight on suboptimal actions tends to zero. This restriction on beliefs rules out off-equilibrium path beliefs that are arguably implausible. For example, suppose bank i sets $\sigma_i = 0$ in equilibrium. If it were to deviate, outsiders could believe anything after observing bank i show it was good, including that the state of the world is one for which $\pi(\omega) = 0$. Outsiders might also change their beliefs about banks $j \neq i$ even though bank i knew nothing about these types.[17] By contrast, sequential equilibria require beliefs to conform with objective features of the state space and information structure we assume. Although we focus our attention on disclosure, recall that outsiders should also be viewed as strategic players, and so the requirement that strategies are optimal encompasses them.

Given Lemma 2, we know that we can reduce the disclosure game to each bank choosing a probability σ_i of revealing its type if it were good. To confirm that a strategy profile σ constitutes an equilibrium, we need to verify that each σ_i is optimal given what the strategies σ_{-i} imply about the distribution

[17] This is referred to by Fudenberg and Tirole (1991) as signalling something you don't know, and show that ruling it out requires weaker notions than sequential equilibria.

of announcements $A = \{A_1, \ldots, A_n\}$. Since the contract terms $\{I_i(A), r_i(A)\}$ bank i receives are functions of A, we can compute the expected payoffs to bank i if it discloses its type and if it did not.

In principle, our model may admit multiple equilibria. Given our interest in mandatory disclosure, though, we are mainly interested in equilibria where disclosure is incomplete, since only at these equilibria is there scope to compel banks to reveal more information than they would on their own. While full disclosure may also be an equilibrium for this economy, focusing on an equilibrium with no disclosure is still informative about what a policymaker should do if the economy ever gets stuck in an equilibrium without disclosure. We therefore focus on equilibria in which $\sigma_i = 0$ for all i. We first derive conditions under which no disclosure is an equilibrium, and then ask whether forcing all banks to disclose their types in this case raises welfare. We then return to the possibility of equilibria with disclosure, i.e. where $\sigma_i > 0$ for some i.

4.1 Existence of a Non-Disclosure Equilibrium

We begin with conditions for non-disclosure to be an equilibrium. We need to verify that if bank i expects banks $j \neq i$ not to disclose, it would also be willing not to disclose its type. By symmetry, the choice of i is irrelevant: If we find this result holds for one bank, it will be true for all banks. We now show that for a non-disclosure equilibrium to exist, either the cost of disclosure or the degree of contagion must be large, in a way we make precise.

To determine whether bank i will agree not to disclose its type if it is good when $\sigma_j = 0$ for $j \neq i$, we must first know what outsiders would do if bank i did and did not disclose its type when good. Suppose bank i disclosed its type. Since information is verifiable, outsiders know bank i is good. In addition, given our restriction to sequential equilibria, even if outsiders expected bank i not to disclose, if bank i did reveal its type it would have no effect on what outsiders believe about other banks. Hence, the probability outsiders assign to bank i repaying them when only bank i discloses its type is just

$$p_g \equiv \Pr(e_i \geq 0 | S_i = 1) \tag{10}$$

As will soon become clear, p_g plays an important role in our analysis. It can be readily calculated given $\pi(\omega)$ and $e_i(\omega)$. For example, in Example 4 we have $p_g = \sum_{b=0}^{b^*} \binom{n}{b} q^{n-b}(1-q)^b$ while in Example 5 we have $p_g = 1 - \frac{k}{n-1}$.[18] Outsiders will invest in bank i only if

$$p_g \geq \frac{\underline{r}}{R}$$

This is because if p_g were lower than $\frac{\underline{r}}{R}$, outsiders would have to charge bank i more than R to ensure a return of $\underline{r}$. Hence, when only bank i discloses, we have $I_i(A) = 0$ if $p_g < \frac{\underline{r}}{R}$ and $I_i(A) = 1$ if $p_g \geq \frac{\underline{r}}{R}$.

[18]Barlevy and Nagaraja (2015) analyze circular networks as in Example 5 under more general conditions , e.g. when the number of banks can exceed 1 and are allowed to be random. Their results can be used to obtain expressions for p_g in those cases as well.

Next, suppose bank i opts not to disclose its type. The beliefs of outsiders will now depend on what they believe bank i's strategy to be. From Lemma 2, we know a bank will not disclose if it is bad. Hence, if outsiders observe $A_i = \varnothing$, the likelihood they assign that bank i would be able to repay them is given by

$$\Pr(e_i \geq 0 | A_i = \varnothing) = \frac{\Pr(e_i \geq 0 | S_i = 1) \Pr(S_i = 1)(1 - \sigma_i)}{\Pr(S_i = 0) + \Pr(S_i = 1)(1 - \sigma_i)} \tag{11}$$

This expression is maximized when $\sigma_i = 0$, in which case when the probability above is equal to $p_g \Pr(S_i = 1)$.[19] It follows that if

$$p_g < \frac{1}{\Pr(S_i = 1)} \frac{\underline{r}}{R},$$

bank i will not be able to raise funds when neither it nor any of the other banks disclose their types, i.e. $I_i(\varnothing, \ldots, \varnothing) = 0$ when $p_g < \frac{1}{\Pr(S_i=1)} \frac{\underline{r}}{R}$.

In short, when no other bank discloses its type, bank i's ability to trade with outsiders depends on the value of p_g in (10) and whether bank i discloses its type or not. When $p_g < \frac{\underline{r}}{R}$, outsiders refuse to invest in bank i whether it discloses its type or not. When $\frac{\underline{r}}{R} < p_g < \frac{1}{\Pr(S_1=1)} \frac{\underline{r}}{R}$, outsiders will invest if bank i discloses its type but not if it does not. When $p_g > \frac{1}{\Pr(S_1=1)} \frac{\underline{r}}{R}$, outsiders will invest in bank i if it discloses its type, and may invest if it does not. We can use these insights to determine when non-disclosure is an equilibrium.

If $p_g < \frac{\underline{r}}{R}$, non-disclosure is an equilibrium for any $c > 0$. This is because disclosure does not induce outsiders to invest but is still costly. Non-disclosure is also an equilibrium if $c = 0$, when banks are indifferent about disclosing.

If $\frac{\underline{r}}{R} < p_g < \frac{1}{\Pr(S_1=1)} \frac{\underline{r}}{R}$, since bank i will be able to raise funds from outsiders but only if it discloses it type, non-disclosure will be an equilibrium only if a bank cannot expect to gain from revealing its type. Revealing its type would secure the bank an expected profit of $p_g R - \underline{r}$ and incurs a cost of c. Hence, non-disclosure is an equilibrium only if $c > p_g R - \underline{r}$.

For $p_g > \frac{1}{\Pr(S_i=1)} \frac{\underline{r}}{R}$, bank i will be able to raise funds if it discloses its type. If it did not reveal its type, whether outsiders trade with bank i will depend on their beliefs about bank i's strategy σ_i. However, in a non-disclosure equilibrium, outsiders would have to correctly anticipate that $\sigma_i = 0$. In this case, outsiders would invest in bank i even if they did not know its type. For non-disclosure to be an equilibrium, bank i must not expect to gain from revealing its type. By disclosing its type, it reduces the interest rate r_i it pays from $\frac{\underline{r}}{p_g \Pr(S_i=1)}$ to $\frac{\underline{r}}{p_g}$. Since it borrows one unit of resources, and since it only earns profits with probability p_g, the gains can be simplified to $\frac{\Pr(S_i=0)}{\Pr(S_i=1)}\underline{r}$. These must be less than the cost c for bank i to be willing not to disclose its type.

We summarize these results as a Proposition:

[19] Note that $\Pr(S_i = 1) = \sum_{b=0}^{n} (1 - \frac{b}{n}) \Pr(B(\omega) = b) = \sum_{b=0}^{n} (1 - \frac{b}{n}) q_b$ per Lemma 1.

Proposition 6: A non-disclosure equilibrium exists if and only if one of the following conditions is satisfied:

1. $p_g < \frac{r}{R}$
2. $p_g \in \left[\frac{r}{R}, \frac{1}{\Pr(S_i=1)}\frac{r}{R}\right]$ and $c > p_g R - \underline{r}$
3. $p_g > \frac{1}{\Pr(S_i=1)}\frac{r}{R}$ and $c > \frac{\Pr(S_i=0)}{\Pr(S_i=1)}\underline{r}$

In cases (1) and (2) no bank is funded in the non-disclosure equilibrium, while in case (3) all banks are funded.

Figure 2 shows the region in (p_g, c)-space in which a non-disclosure equilibrium exists. Non-disclosure is an equilibrium when either c is large or p_g is small.[20] The fact that a non-disclosure equilibrium exists when disclosure is costly, i.e. when c is large, is not surprising. The more novel finding is the connection between non-disclosure and p_g, which is most naturally interpreted as a measure of contagion. This is because $p_g = \Pr(e_i \geq 0 | S_i = 1)$ represents the likelihood that bank i will be able to avoid default despite its exposure to other banks. When $p_g \to 1$ contagion is insignificant, since a good bank will be able to repay almost regardless of what happens at other banks. When $p_g \to 0$ contagion is severe, since a good bank will default in most states of the world. Proposition 6 therefore states that non-disclosure is an equilibrium if either disclosure is costly or if contagion is severe.[21]

4.2 Mandatory Disclosure and Welfare

We next ask whether when a a non-disclosure equilibrium exists, forcing all banks to disclose their type can improve welfare relative to these equilibria outcomes. We are not claiming that this policy is optimal. However, showing that mandatory disclosure can improve welfare is a sufficient condition to justify intervention. We focus on a policy of forcing all banks to disclose both because it is easier to analyze and because it has been used in practice.

We begin with the case where $p_g < \frac{r}{R}$. From Proposition 6, we know that in this case, no disclosure is an equilibrium for any value of c and that no bank receives funding in equilibrium. If all banks were instead forced to disclose their type, all banks that were proven to have positive equity would raise funds while

[20] When $c = 0$, our model satisfies all of the conditions Beyer et al (2010) identify under which equilibrium should involve full disclosure, highlighting the novelty of our explanation. Costless disclosure also implies unravelling in Admati and Pfleiderer (2000). Our result is probably closest to Example 4 in Okuno-Fujiwara, Postlewaite, and Suzumura (1990), in which agents are at a corner and so disclosure has no effect on actions.

[21] Empirically, one could try to deduce p_g from default premia or spreads on credit default swaps for banks that are known not to have made bad investments. The idea of measuring contagion with conditional distributions is reminiscent of the CoVaR measure proposed by Adrian and Brunnermeier (2011). However, they consider bank outcomes conditional on other banks being in distress, while we condition on those banks having avoided bad investments.

the rest would not. The unconditional probability that each bank will be able to raise funds is just $\Pr(e_i \geq 0)$, and so the expected surplus that we could generate by forcing all banks to disclose their types is

$$n\left[\Pr(e_i \geq 0)(R - \underline{r}) - c\right] \tag{12}$$

Using the fact that $\Pr(e_i \geq 0) = p_g \Pr(S_i = 1)$, we infer that (12) is positive iff

$$c \leq p_g \Pr(S_i = 1)(R - \underline{r})$$

Hence, as long as disclosure isn't too costly, forcing disclosure can raise welfare.

Next, suppose $p_g > \frac{1}{\Pr(S_i=1)} \frac{\underline{r}}{R}$. From Proposition 6 we know that a non-disclosure equilibrium exists only if $c > \frac{\Pr(S_i=0)}{\Pr(S_i=1)} \underline{r}$ and that all banks are funded in equilibrium. Mandatory disclosure is then strictly welfare reducing, since it incurs disclosure costs cn but if anything only reduces the number of banks that undertake projects by revealing which banks have negative equity.[22]

The remaining case is when $p_g \in \left[\frac{\underline{r}}{R}, \frac{1}{\Pr(S_i=1)} \frac{\underline{r}}{R}\right]$. From Proposition 6, we know that a non-disclosure equilibrium exists only if $c \geq p_g R - \underline{r}$, and that in this equilibrium no bank will receive funding. The expected gain from forcing all banks to disclose their types is thus equal to (12), which is positive only if $c \leq p_g \Pr(S_i = 1)(R - \underline{r})$. In the Appendix, we analyze when these two conditions are compatible. The results can be summarized as follows:

Theorem 1: Suppose a non-disclosure equilibrium exists. Then

(i) $\exists p_g^* \in \left(\frac{\underline{r}}{R}, 1\right)$ such that for all $p_g \in (0, p_g^*)$, forced disclosure improves welfare relative to the non-disclosure equilibrium if c is not too large.

(ii) If $p_g > \frac{1}{\Pr(S_i=1)} \frac{\underline{r}}{R}$ so all banks can raise funds when no information is revealed, mandatory disclosure cannot increase welfare for any $c \geq 0$.

Figure 3 provides a graphical interpretation of Theorem 1 in (p_g, c)-space. The region in which a non-disclosure equilibrium exists, depicted in light gray, is the same as in Figure 2. The region in which mandatory disclosure is superior to no trade is depicted in dark gray. The intersection of the two regions, depicted in blue, corresponds to parameter values for which it will be possible to improve upon a non-disclosure equilibrium. For severe degrees of contagion, intervention is warranted as long as disclosure costs are not too high. For intermediate degrees of contagion, intervention is warranted when disclosure costs are neither too high nor too low, since at low costs non-disclosure cannot be an equilibrium.

Theorem 1 is the key result in our paper. Part (i) establishes that severe contagion is a necessary condition for intervention to be beneficial. Intuitively, with contagion each bank's information is useful for inferring ω and facilitating

[22] Our results when $p_g > \frac{1}{\Pr(S_i=1)} \frac{\underline{r}}{R}$ are reminiscent of Jovanovic (1984) and Haggerty and Fishman (1989). They also consider a case where agents reap private gains from disclosure, but disclosure is costly and yields no social surplus and thus not desirable.

trade between banks and outside investors, a spillover banks fail to internalize in contemplating disclosure. By contrast, when the degree of contagion is small, banks internalize most of the benefits of their disclosure. If a bank chose not to disclose its type in that case, the cost of disclosure must have exceeded the benefit, and forcing it to disclose would make it worse off.[23]

Part (ii) of Theorem 1 establishes that mandatory disclosure is only warranted when markets are frozen, i.e. when outsiders fail to invest in any of the banks. If instead banks are able to raise funds without disclosing their types, mandating disclosure can only make things worse. This is related to our discussion in Section 3, where we argued that if disclosure encourages outsiders to trade with banks by helping outsiders avoid funding banks with negative equity, as must be true if banks can already secure funding, then mandatory disclosure will benefit good banks but will reduce total surplus. Our result contradicts the argument cited in the Introduction releasing stress test results routinely rather than only during crises. To be sure, our model abstracts from various forces that may favor disclosure in normal times. Indeed, we show in Section 5 that when we modify the model to allow for moral hazard concerns, mandatory disclosure can be desirable in normal times. However, even in this case, contagion is a necessary condition for mandatory disclosure to be desirable. Without contagion, if disclosure was worth undertaking, banks would do so on its own.

4.3 Equilibria with Disclosure

So far, we have ignored equilibria with disclosure. We now explore this possibility, focusing our discussion around two questions. First, if the conditions we identify in Proposition 6 for a non-disclosure equilibrium to exist fail, will there exist an equilibrium with disclosure, and if so can we say anything about it? Second, if a non-disclosure equilibrium exists and can be improved upon, must there equilibria with disclosure also exist? To put it another way, is intervention simply shifting to an equilibrium that agents could have coordinated on? Although these questions concern equilibria with disclosure, our answers shed light on what is driving our key welfare result when there is no disclosure.

Consider first the case where non-disclosure equilibria do not exist. Per Proposition 6, this occurs when $p_g > \frac{r}{R}$ and c is sufficiently small. In this case, good banks will be too tempted to reveal their type and improve their terms of trade. Standard existence results ensure that our game always admits an equilibrium. Hence, there must be an equilibrium with some disclosure, i.e. $\sigma_i > 0$ for some i. We now show that it may be possible to improve upon this equilibrium. However, the relevant intervention in this case is not to force more information out, but to prevent information from being disclosed.

[23] The reason mandatory disclosure cannot improve welfare for even small degrees of contagion, i.e. when p_g is close to but less than 1, is that mandatory disclosure forces both good and bad banks to disclose their types. While the private cost to a good bank of disclosing its type is c, expected disclosure costs per good bank exceeds c under mandatory disclosure.

To see this, suppose

$$p_g \Pr(S_i = 1) > \frac{r}{R} \tag{13}$$

As we showed in Proposition 6, this condition ensures that if no bank discloses its type, outsiders would to invest in all banks. We write the condition in a way that highlights that neither p_g nor $\Pr(S_i = 1)$ can be too small, i.e. contagion must be low and banks are likely to be good. We establish the following result:

Proposition 7: If $p_g > \frac{1}{\Pr(S_i=1)} \frac{r}{R}$ and $c < \frac{\Pr(S_i=0)}{\Pr(S_i=1)} \underline{r}$, there will be at least one bank in equilibrium that discloses its type with positive probability. In this case, forcing all banks to set $\sigma_i = 0$ will weakly improve welfare.

Note the difference between Proposition 7 and Theorem 1. The latter states that under (13), if we start with an equilibrium in which no information is disclosed, forcing banks to release information cannot improve welfare. Proposition 7 states that under (13), if we start with an equilibrium in which some information is disclosed, forcing banks to remain secretive can raise welfare. Theorem 1 argues that under certain conditions there will be no justification for mandatory disclosure, while Proposition 7 argues that it will be better to enforce opacity.

Proposition 7 reaffirms that even though our model offers a justification for mandatory disclosure, our model does not imply that disclosure is always inherently desirable. We view this as an advantage, since empirically banks have had a long tradition of secrecy. For example, Gorton and Tallman (2014) document that prior to the establishment of the Federal Reserve, bank clearing houses went to great lengths to restrict what information was available about their member banks. The tendency to secrecy has continued into the modern era. For example, Prescott (2008) observes that banks that do well on regulatory exams are forbidden from releasing their results. He argues this custom can be optimal if banks have discretion on what to report to regulators, since disclosure may lead banks to volunteer less information. In our model, the virtue of opacity is instead due to the fact that it allows for insurance across banks. In particular, if banks collude to hide their types, outsiders will fund all banks. Effectively, high equity banks insure low equity banks by paying higher rates. This channel is similar to recent work Goldstein and Leitner (2014) and Dang, Gorton, Holmström and Ordoñez (2014) on opacity in banking.

So far, our results have concerned situations in which non-disclosure equilibria do not exist. We now consider the case where such equilibria exist. Specifically, if such equilibria can be improved upon, we want to know whether equilibria in which banks disclose information must also exist. That is, is mandatory disclosure pushing banks to an outcome they can achieve on their own?

In some cases, the conditions that ensure mandatory disclosure can improve upon a non-disclosure equilibrium imply that additional equilibria must exist. For example, consider the case in which disclosure is costless, i.e. $c \to 0$. In this case, a good bank never suffers from revealing its type. The only reason not to disclose is that it achieves nothing, e.g. if $p_g < \frac{r}{R}$ and no other bank choose to reveal. In this case, banks could coordinate on their own to disclose, since none is ever made worse off disclosing. However, our next example shows that this is not true more generally. When $c > 0$, non-disclosure can be the unique equilibrium, and yet mandatory disclosure still improves welfare.

Example 7: Consider an environment similar to Example 6 in which bank types are independent and good bank will default if even one bank is bad. Set the number of banks n now to 3. As before, each bank is good with probability 0.9 and $b^* = 0$. The returns to outsiders and banks are given by $\underline{r} = 1$ and $R = 1.22$, respectively. These parameters ensure that if the other two banks choose $\sigma_i = 0$,, a good bank cannot raise funds by announcing its type, since

$$p_g R = (0.9)^2 \times 1.22 = 0.99 < 1 = \underline{r}$$

Finally, we set $c = 0.16$. This ensures mandatory disclosure is preferable to non-disclosure, since

$$(0.9)^3 \times 3(1.22 - 1) = 0.481 > 0.48 = 3c$$

We can verify numerically that a bank will be better off not disclosing its type for all values of σ_{-i}. As an illustration, consider bank 1's best response when $(\sigma_2, \sigma_3) \in \{(0,0), (0,1), (1,1)\}$. When neither bank commits to disclosure, bank 1 has no reason to disclose since by design revealing its type will not convince outsiders to invest. When the two other banks both commit to disclose if good, bank 1 will be able to raise funds even without disclosing when both other banks are good, but the cost of disclosure exceeds the gain from better terms. When only one bank commits to disclosure, bank 1 must disclose its type to attract investment, but given the odds it prefers not to disclose. The gain from disclosure for any pair $(\sigma_2, \sigma_3) \in [0,1]^2$ will be even lower. □

The fact that our argument for mandatory disclosure does not reduce to a coordination failure accords with our result in Section 3 that disclosure decisions are not generally complementary. It is therefore not surprising that the argument for disclosure does not rely on multiple equilibria. Rather, the case for intervention relies on the fact that disclosure helps other good banks by encouraging outsiders to trade with them. The problem is not coordination *per se* but that externalities lead to too little disclosure of information by banks.

5 Adding Moral Hazard

The model we presented up to now offers a stark conclusion: Mandatory disclosure can be desirable when markets are frozen but not when banks can raise funds when no information is revealed. However, our model abstracts from various frictions that might justify intervention even when markets operate normally. The quote we cite in the Introduction for routinely disclosing stress tests alludes to this, citing the need for market discipline which suggests some friction we ignore. We now explore this idea by allowing banks to engage in a particular form of moral hazard. Our chief insight is that mandatory disclosure can be desirable in normal times, but again only when there is sufficient contagion.

We introduce moral hazard by assuming banks can divert new funds they raise to achieve a purely private benefit. Diversion is meant to stand in for various actions banks can undertake that are not in the interest of investors. At the same time, we assume banks cannot divert the assets they already own, and that these can be seized. Initial equity can mitigate moral hazard problems, since banks that engage in moral hazard can be deprived of their equity.

We modify the model in Section 2 as follows. As before, nature chooses ω, each bank i observes $S_i(\omega)$, and then banks play a simultaneous-move disclosure game. Investors observe the outcome $A \equiv \{A_1, \ldots, A_n\}$ and offer terms to the different banks. If outsiders invest in banks, they give up the option to earn $\underline{r}$, i.e. there is a time limit on when they can exercise their outside option. After all investments are made, banks learn their equity $e_i(\omega)$ if they haven't yet learned it from A. Define $\overline{e}$ as the highest possible equity a bank can have, i.e.

$$\overline{e} = \max_{\{\omega:\pi(\omega)>0\}} e_i(\omega) \tag{14}$$

An equity value of $e_i(1, S_{-i}) < \overline{e}$ can be interpreted as contagion, since A4 implies bank i's equity can only fall if some bank $j \neq i$ is bad.

Once banks learn their equity, they can decide whether to invest in a project with a return $R > \underline{r}$ or divert the funds and earn a private benefit of size v that outsiders cannot seize. We assume v is neither too big nor too small:

A6. Binding Moral Hazard: The value of private benefits v satisfies

$$R - \underline{r} < v < R - \max\{\underline{r} - \overline{e}, 0\} \tag{15}$$

The first inequality in (15) implies a bank that knows it has zero or negative equity would prefer to divert funds, since the most they can earn from initiating their project is $R - \underline{r}$. The second inequality implies that if a bank knows its equity is $\overline{e}$, it would prefer to initiate the project than to divert. Since the interest rates banks are charged in equilibrium depend on A, each bank will have a threshold level of equity $e_i^*(A) \in (0, \overline{e})$ as a funciton of A above which it prefer the project and below which it would divert. Note that we can interpret the model in Section 2 as a special case of this model in which $v = -\infty$.

Returning to the timeline of the model, after banks decide what to do with any funds they raised, payoffs are realized and banks pay their obligations. Outsiders who invested in banks but are not paid back can go after the equity banks have. We continue to assume that outsiders claims are junior to any of the bank's other outstanding liabilities. Thus, if a bank has negative equity, outside investors will not be able to recover anything from banks.

As in the previous section, we can ask when non-disclosure is an equilibrium and when mandatory disclosure can improve welfare relative to a non-disclosure equilibrium. Outsiders investors will expect a bank they fund to default with probability $\Pr(e_i > e_i^*(A)|A, \sigma)$, although if $0 < e_i(\omega) < e_i^*(A)$ outsiders can still seize some of the bank's remaining equity. Bank i knows that if $e_i(\omega) < e_i^*(A)$, its utility will not be equal to (8) but

$$e_i(\omega) + [v + \max\{-r_i(A), -e_i(A)\}] I_i(A) - c\alpha_i \tag{16}$$

For brevity, we will not repeat our analysis for this new payoff function in full. We instead begin with an example that illustrates our claim that with moral hazard there may be a role for mandatory disclosure when markets operate normally, i.e. when outsiders invest in all banks when no information is disclosed.

Example 8: Consider the case where the equity $e_i(\omega)$ for different states of the world is such that $e_i(0, S_{-i}) < 0$ for all S_{-i} and $e_i(1, S_{-i})$ is either equal to $\overline{e}$ or negative. This condition is analogous to A5 in that it implies banks either repay outsiders in full or default in full, regardless of which other banks are funded. It follows that the probability outsiders will paid back if bank i is good still corresponds to p_g as defined in (10), i.e. $p_g = \Pr(e_i \geq 0|S_i = 1)$. By the same logic as in Section 4, we can show that if $p_g > \frac{1}{\Pr(S_i=1)} \frac{\underline{r}}{R}$, if no other banks disclose their type, bank i will be able to attract funds whether it discloses its type or not. Hence, non-disclosure is an equilibrium as long as the cost of disclosure exceeds the reduction in interest charges a bank could obtain from disclosure. This is the same as when $v = -\infty$ we already studied, i.e.

$$c > \frac{\Pr(S_i = 0)}{\Pr(S_i = 1)} \underline{r} \tag{17}$$

Turning to the benefit of requiring all banks to disclose their type, an important difference from our previous analysis is that now disclosure prevents outsiders from investing in banks that end up diverting resources. When $v < \underline{r}$, diversion is wasteful: Society would have been richer exercising the alternative option available to outside investors that financing insolvent banks. This interpretation hinges on outsiders giving up the right to exercise their outside option when they invest with outsiders, since it precludes banks renegotiating with outsiders after learning the equity values. Since the expected fraction of banks that will divert is equal to $1 - p_g \Pr(S_i = 1)$, mandatory disclosure is preferable to the non-disclosure equilibrium where all banks raise funds if

$$(1 - p_g \Pr(S_i = 1))(\underline{r} - v) > c \tag{18}$$

Combining (17) and (18) implies that we can satisfy both conditions whenever $p_g < \frac{1}{\Pr(S_i=1)} \left(1 - \frac{\Pr(S_i=0)}{\Pr(S_i=1)} \frac{\underline{r}}{\underline{r}-v}\right)$. If this threshold value exceeds $\frac{1}{\Pr(S_i=1)} \frac{\underline{r}}{R}$, we can be sure that there exists a nonempty set of values of p_g for which a non-disclosure equilibrium exists in which all banks are funded and yet mandatory disclosure can make agents better off. This condition can be rewritten as

$$\frac{\Pr(S_i = 0)}{\Pr(S_i = 1)} < \left(1 - \frac{\underline{r}}{R}\right) \left(1 - \frac{v}{\underline{r}}\right) \tag{19}$$

Hence, contrary to our results for the case with no moral hazard, we now find it can be desirable to force all banks to disclose when markets operate normally. Intuitively, when $v < \underline{r}$ a planner would want to avoid resources from going to low equity banks. Since good banks do not internalize the value the information they disclose has in revealing the equity position of others, equilibrium disclosure will generally be too low and forcing disclosure can improve welfare. □

Example 8 shows that if we modify the model so that it is no longer desirable to keep insolvent banks operating, mandatory disclosure can turn desirable even when markets aren't frozen. However, the purpose of disclosure changes: Rather than stimulating trade, which is the justification for disclosure when markets are frozen, the purpose of disclosure now is discouraging investment that will prove to be socially wasteful. Our specification for moral hazard can be viewed as a way of capturing the market disciplining role of disclosure Bernanke (2013) alludes to in advocating for routine stress test disclosures. However, the next result shows that the case for intervention still hinges on contagion:

Theorem 2: Suppose A1 - A4, and A6 hold. If a non-disclosure equilibrium exists, then

(i) There exists a cutoff $e^* \in (0, \overline{e})$ such that if $\Pr(e_i > e^* \mid S_i = 1)$ is sufficiently close to 0 but strictly positive, mandatory disclosure can increase welfare relative to the non-disclosure equilibrium as $c \to 0$.

(ii) If $\Pr(e_i = \overline{e} \mid S_i = 1)$ is sufficiently close to 1, mandatory disclosure cannot improve welfare relative to the non-disclosure equilibrium for any $c \geq 0$.

In comparing this result to Theorem 1 from the previous section, recall that we can interpret the model without moral hazard as a special case of the model with moral hazard but where $v = -\infty$. Imposing A6 allows us to drop A5, i.e. we no longer need to assume that the distribution of equity across values of ω exhibits a gap. This helps to focus attention on contagion, since Theorem 2 makes clear that the condition which rules out a role for mandatory disclosure is if a good bank is not very vulnerable to other banks and is likely to have its equity at the maximum level $\overline{e}$. The necessity of contagion is due to the fact that when information is valuable, banks have an incentive to reveal it. Contagion ensures that a bank's information is systemically important, so that banks fail to fully take the benefits of disclosure into account. This is not to argue that contagion is the only type of spillover that can justify disclosure. However, it is noteworthy that even when there is little contagion in the sense of Theorem 2, disclosure can still exhibit informational spillovers as we defined them earlier, i.e. affecting beliefs about other banks. Theorem 2 thus establishes that the mere existence of any spillover cannot on its own justify mandatory disclosure.

6 Balance Sheet Contagion

Up to now, we have assumed contagion operates through endowments $e_i(\omega)$. While we demonstrated that our setup represents a reduced form of certain models in which contagion emerges endogenously, our formulation makes it easy to lose sight that contagion depends on the underlying model. We therefore conclude our discussion with an extended example to highlight that the measure of contagion that drives our our results depends on the underlying economic environment. A practical implication of our example is that the justification for mandatory disclosure may vanish once other financial reforms are instituted. This runs counter to conventional wisdom sometimes expressed by policymakers that mandatory disclosure complements other proposed financial reforms.

Our example relies on a model of balance sheet contagion in which contagion arises when banks whose balance sheets are impaired default on other banks whose balance sheets are not directly affected. Our formulation follows Eisenberg and Noe (2001). Each bank is endowed with $\overline{e} > 0$ worth of assets as well as a series of claims and obligations to other banks. Formally, we let Λ denote the $n \times n$ matrix of obligations between any pair of banks, so that Λ_{ij} corresponds to the amount bank i owes bank j. The diagonal terms are all zero. As in Example 5, we assume each bank has zero net position, i.e.

$$\sum_{j \neq i} \Lambda_{ij} = \sum_{j \neq i} \Lambda_{ji} \tag{20}$$

for each $i \in \{1, \ldots, n\}$. We take these claims as given, although in principle banks would choose the claims the obligations they want to enter in; this line is explored in Zawadowski (2013). To satisfy A2, we would need to assume the links implied by Λ represent a symmetric network. However, we will not need to impose this assumption for the result we wish to emphasize.

Of the n banks in the network, we assume a random number B are bad. To satisfy A1, we would need to assume that given $B = b$, each of the $\binom{n}{b}$ groups of b banks are equally likely to those that incur losses. Again, though, we will not require this assumption for our result. What distinguishes a bad bank in this setup is that it incurs a loss of magnitude $\phi > \overline{e}$. The simplest interpretation for ϕ is that it represents an obligation to a senior claimant that has priority over any of the banks. Let S_i be a variable equal to 1 if bank 1 is bad and equal to 0 otherwise. The state of the network is given by $S = (S_1, \ldots, S_n)$.

Ignoring transfers between banks, a bad bank would see its equity position fall to a negative $\overline{e} - \phi$. However, the final equity position of a bank will depend on payments to and from other banks. Let $x_{ij}(S)$ denote the amount bank i pays bank j in state S. Following Eisenberg and Noe (2001), we define an equilibrium clearing payment as a set of payments $x_{ij}(S)$ in which each bank i pays all of his obligations ϕ and Λ_{ij} in full or else pays claims according to prescribed priority, and pays those with equal priority on a pro-rata basis, i.e. in proportion to its obligations to each of the banks.

Formally, define Λ_i as the total obligations of bank i to other banks, i.e.

$$\Lambda_i = \sum_{j=1}^{n} \Lambda_{ij}$$

The equilibrium payments $x_{ij}(S)$ in state S will solve the system of equations

$$x_{ij}(S) = \frac{\Lambda_{ij}}{\Lambda_i} \max\left\{0, \min\left\{\Lambda_i, \overline{e} - \phi S_i + \sum_{k=1}^{n} x_{ki}(S)\right\}\right\} \tag{21}$$

Hence, the equity position of each bank, before it attracts funds from outside investors, which the bank may not know, is given by

$$e_i(S) = \overline{e} - \phi S_i + \sum_{k=1}^{n} x_{ki}(S) - \sum_{j=1}^{n} x_{ij}(S) \tag{22}$$

The expression in (22) confirms that this model gives rise to a reduced form in which we can assign an equity endowment to each bank in each state of the world. However, under this the value of $e_i(S)$ is not fixed but depends on certain parameters that influence how contagion works. In particular, the amount of equity each bank has depends on its own type S_i as well the size of the loss ϕ bad banks incur and the matrix of obligations Λ_{ij}. To describe this dependence more formally, let us index the matrix of obligations across banks Λ by a scale factor λ so that $\Lambda(\lambda) = \lambda\Lambda(1)$. That is, the scalar multiplies each entry of the baseline matrix $\Lambda(1)$, so a higher λ increases obligations between all banks proportionately. The next proposition formalizes the way in which equity $e_i(S)$ depends on the magnitude of losses at bad banks ϕ and the magnitude of debt obligations across banks as scaled by λ.

Proposition 8: For every $x \in [0, \overline{e}]$, $\Pr(e_i \leq x | S_i = 1)$ is weakly increasing in ϕ and λ, i.e the distribution of equity is stochastically decreasing in ϕ and λ.

As Theorem 2 makes clear, the relevant measure of contagion that matters for whether mandatory disclosure is desirable or not is the distribution of equity at good banks. Proposition 8 thus reveals what features exacerbate contagion and may create a role for policy intervention. In particular, contagion will be more severe the larger the losses ϕ of bad banks, as well as the greater the debt obligations λ between banks. The latter implies that restrictions on leverage that place limits on λ may reduce the need for mandatory disclosure. Under this view, reforms such as leverage restrictions may obviate the justification for mandatory disclosure rules rather than complement these rules.

7 Conclusion

One of main lessons policymakers drew from the recent financial crisis is the usefulness of mandatory disclosure. Specifically, a consensus view has emerged that the release of stress test results for large banks played an important role in stabilizing financial markets in the US. Although the release of stress test results for European did not seem to have the same salutatory effect, for a variety of reasons, policymakers have continued conducting these tests and dutifully releasing their result. Even as crisis conditions mitigated in the US, policymakers continued advocating for mandatory disclosure, citing it as a useful tool that naturally complements existing regulatory policy and which ought to used routinely rather than only during crisis periods.

In this paper, we tackled the question of why it might be necessary to compel banks to disclose information rather than rely on them to disclose the information on their own. We argue that there may be a role for mandatory disclosure when there is a possibility of financial contagion. Indeed, within our framework, contagion is a necessary condition to justify intervention. This, rather than markets being frozen or moral hazard problems that can arise with incomplete information, proves to be the decisive factor for whether such a policy can work. At the same time, even with contagion, our model does not imply that mandatory disclosure is always and everywhere desirable.

We conclude with a few comments and caveats about our analysis. First and foremost, we wish to stress that our model does not imply mandatory disclosure constitutes an optimal policy. Our focus is only in establishing conditions under which mandatory disclosure can increase welfare. Once this is established, one can proceed to ask what the optimal way of disclosing information might be. Goldstein and Leitner (2014) provide analysis along this line, studying how top optimally release information. Their results suggest that even in disclosing information, some opacity might be optimal. This coincides with the fact that historically, the private sector solution implemented by clearinghouses often involved less transparency during crises, providing just enough information about the system as a whole to encourage investment in banks without revealing too much about individual banks. Questions about what type of information bank examiners should collect and what they should release are just as important as the question of when there might be a need to compel information that isn't already being provided privately.

Second, in our quest for analytical tractability, we have ignored various practical issues involved with the design of disclosure policy. For example, we invoked symmetry restrictions to simplify the analysis. But in practice banks differ in important ways, which raises the question of which banks should be forced to disclose information. Our analysis suggests that banks whose information is the most systemically important in terms of affecting other banks are those that are most likely to disclose too little. But demonstrating this requires working with asymmetric environments. Still another question is what type of information should be collected. Our specification assumes that the only relevant information are bank balance sheets, since once we know each bank's type the equity position of each bank is known. In practice, though, the linkages between banks may also be private information, raising the question of what optimal disclosure might be when information on both bank types and how banks are linked is initially private but might be elicited and made public.

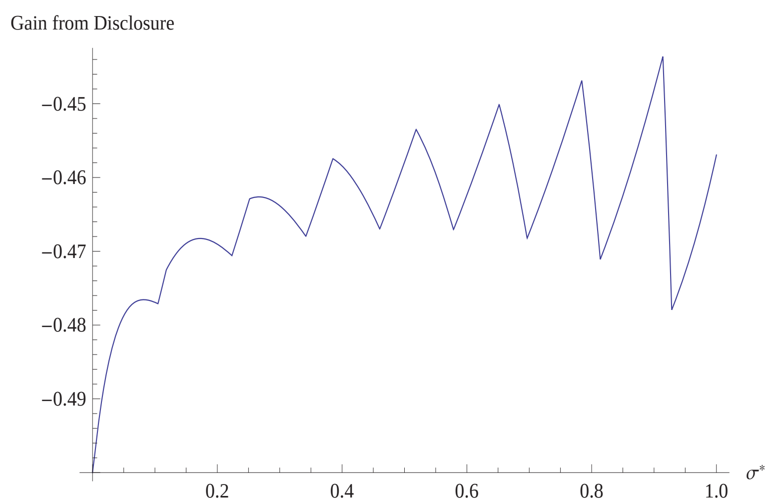

Figure 1: Gains from disclosure as a function of σ^* chosen by other banks

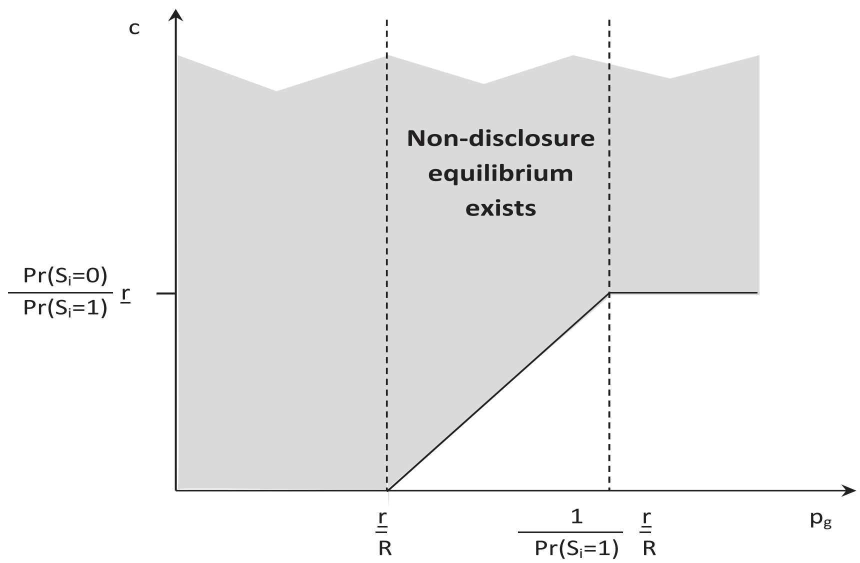

Figure 2: Values of (p_g, c) for which non-disclosure is an equilibrium

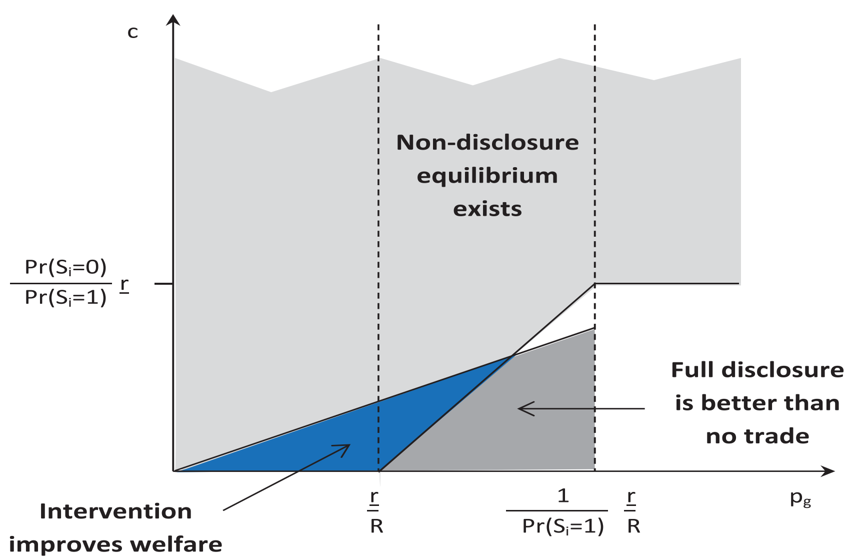

Figure 3: Values of (p_g, c) in which mandatory disclosure improves welfare

Proofs

Proof of Lemma 1: Suppose $\pi(\omega)$ is given by (2). In this case, it easy to verify that A1 is satisfied, since for any pair ω and ω' where $B(\omega) = B(\omega')$will have $\pi(\omega) = \pi(\omega')$. In the opposite direction, suppose A1 holds. For a given state ω, define $\pi^* = \pi(\omega)$. A1 implies that $\pi(\omega'^*$ for any ω' where $B(\omega') = B(\omega)$. Since $\Pr(B = b) = \sum_{\{\omega|B(\omega)=b\}} \pi(\omega)$, it follows that $\pi^* = \binom{n}{b}^{-1} \Pr(B = b)$, which pins down the value of $\pi(\omega)$.

Proof of Lemma 2: The Lemma follows from the fact that the expected benefit from disclosure for a bad bank under A3 is negative.

Lemma 3: The scenarios where informational spillovers and positive and absent exhibit the following properties:

(i) When informational spillovers are positive, then for all $k = 1, \ldots, n$ and all vectors $(s_{k+1}, \ldots, s_n) \in \{0, 1\}^{n-k}$ for which $\Pr(S_{k+1} = s_{k+1}, \ldots, S_n = s_n) > 0$, it must be the case that

$$\Pr(S_1 = 1, S_2 = 1, \ldots, S_k = 1 | S_{k+1} = s_k, \ldots, S_n = s_n) > 0$$

i.e. regardless of what types banks $k+1$ through n are, there is a strictly positive probability that the remaining banks 1 through k are all good.

(ii) Informational spillovers are absent iff S_i and S_j are independent for all i and j, i.e. $\Pr(S_i + s_i | S_j = s_j) = \Pr(S_i = s_i)$.

Proof of Lemma 3: Part (i): Suppose not, i.e.

$$\Pr(S_1 = 1, S_2 = 1, \ldots, S_k = 1 | S_{k+1} = s_{k+1}, \ldots, S_n = s_n) = 0$$

We will show that this implies $\Pr(S_1 = 1 | S_{k+1} = s_{k+1}, \ldots, S_n = s_n) = 0$, i.e. if the condition holds, then no bank can be good. To show this, we first argue that $\Pr(S_1 = 1, \ldots, S_{k-1} = 1 | S_{k+1} = s_{k+1}, \ldots, S_n = s_n) = 0$, i.e. if there is zero probability that all banks 1 through k are good, then there also zero probability that banks 1 through $k-1$ are all good.

Let us write $\Pr(\cdot | S_{k+1}, \ldots, S_n)$ for $\Pr(\cdot | S_{k+1} = s_{k+1}, \ldots, S_n = s_n)$ to conserve on notation. Then we can write $\Pr(S_1 = 1, \ldots, S_{k-1} = 1 | S_{k+1}, \ldots, S_n)$ as a sum of probabilties that are conditional on the realized value of S_k:

$$\Pr(S_1 = 1, \ldots, S_{k-1} = 1 | S_k = 1, S_{k+1}, \ldots, S_n) \Pr(S_k = 1 | S_{k+1}, \ldots, S_n) +$$
$$\Pr(S_1 = 1, \ldots, S_{k-1} = 1 | S_k = 0, S_{k+1}, \ldots, S_n) \Pr(S_k = 0 | S_{k+1}, \ldots, S_n)$$

Since we supposed that $\Pr(S_1 = 1, S_2 = 1, \ldots, S_k = 1 | S_{k+1}, \ldots, S_n) = 0$, then $\Pr(S_1 = 1, \ldots, S_{k-1} = 1 | S_k = 1, S_{k+1}, \ldots, S_n)$ must also equal zero. Hence, the first term above is zero, and $\Pr(S_1 = 1, \ldots, S_{k-1} = 1 | S_{k+1}, \ldots, S_n)$ equals

$$\Pr(S_1 = 1, \ldots, S_{k-1} = 1 | S_k = 0, S_{k+1}, \ldots, S_n) \Pr(S_k = 0 | S_{k+1}, \ldots, S_n)$$

Consider the first term, $\Pr(S_1 = 1, \ldots, S_{k-1} = 1 | S_k = 0, S_{k+1}, \ldots, S_n)$. With positive spillovers, we know that $\Pr(S_1 = 1, \ldots, S_{k-1} = 1 | S_k = 0, S_{k+1}, \ldots, S_n)$ is less than or equal to $\Pr(S_1 = 1, \ldots, S_{k-1} = 1 | S_k = 1, S_{k+1}, \ldots, S_n)$ provided

$$\Pr(S_k = 1, S_{k+1}, \ldots, S_n) > 0 \tag{23}$$

Suppose first that $\Pr(S_k = 1, S_{k+1}, \ldots, S_n) > 0$. Since we just argued above that $\Pr(S_1 = 1, \ldots, S_{k-1} = 1 | S_k = 1, S_{k+1}, \ldots, S_n)$ must be zero, it follows that

$$\Pr(S_1 = 1, \ldots, S_{k-1} = 1 | S_k = 0, S_{k+1}, \ldots, S_n) = 0$$

and so it follows that $\Pr(S_1 = 1, \ldots, S_{k-1} = 1 | S_{k+1}, \ldots, S_n) = 0$ as claimed. Next, suppose $\Pr(S_k = 1, S_{k+1}, \ldots, S_n) = 0$. This implies

$$\Pr(S_k = 1 | S_{k+1}, \ldots, S_n) = 0.$$

But A1 implies that $\Pr(S_j = 1 | S_{k+1}, \ldots, S_n) = 0$ for all $j \in \{1, ..., k\}$, meaning that $\Pr(S_1 = 1, \ldots, S_{k-1} = 1 | S_{k+1}, \ldots, S_n) = 0$. This confirms that when informational spillovers are positive, if it is not possible for banks 1 through k to all be good, then it is impossible for just banks 1 through $k-1$ to all be good.

We can proceed inductively to show that it is also not possible for banks 1 through $k-2$ to all be good, for banks 1 through $k-2$ to all be good, and so on, until eventually we can establish that $\Pr(S_1 = 1 | S_{k+1}, \ldots, S_n) = 0$. By the symmetry condition A1, it follows that $\Pr(S_j = 1 | S_{k+1}, \ldots, S_n) = 0$ for all $j \in \{1, \ldots, k\}$. In other words, if $\Pr(S_1 = 1, S_2 = 1, \ldots, S_k = 1 | S_{k+1}, \ldots, S_n) = 0$ as we suppose, then no bank can be good given $(S_{k+1}, \ldots, S_n)$.

We now argue that $\Pr(S_j = 1 | S_{k+1}, \ldots, S_n) = 0$ for all $j \in \{1, \ldots, k\}$ is incompatible with positive informational spillovers. Without loss of generality, we can assume that $S_j = 1$ for all $j \in \{k+1, \ldots, k^*\}$ and $S_j = 0$ for $j \in \{k^* + 1, \ldots, n\}$, where k^* is some number between k and n. Since $\Pr(S_{k+1}, \ldots, S_n) > 0$ by assumption, positive informational spillovers imply

$$\Pr(S_1 = 1 | S_{k+1} = 1, \ldots, S_n) \geq \Pr(S_1 = 1 | S_{k+2}, \ldots, S_n)$$

Hence $\Pr(S_1 = 1 | S_{k+2}, \ldots, S_n) = 0$. We can continue this process through k^* until we conclude that $\Pr(S_1 = 1 | S_{k^*+1}, \ldots, S_n) = 0$. Since $S_{k^*+1}, \ldots, S_n$ are all equal to zero, we can conclude that $\Pr(S_1 = 1) = 0$. In words, we showed that if S_1 had to equal 0 when a subset of banks are zero, then no bank could ever be good.

Finally , we reached a contradiction: If all banks are bad with probability 1, then we cannot have positive informational spillovers, since that requires that there exists some set Ω_0 for which the inequality $\Pr(S_i = 1|S_j = 1, \Omega_0) > \Pr(S_i = 1|\Omega_0)$ is strict.

Proof of part (ii): If we set $\Omega_0 = \{S_j = 1\}$ for each $j \neq i$, we can immediately deduce that S_i and S_j are independent. Note that under assumption A1, S_i and S_j are not only independent but also identically distributed.

Lemma 4: Define

$$\Omega_0 \equiv \left\{ \{\sigma_j(S_j)\}_{j=1}^n, \{A_j\}_{j\neq 2} \right\} \tag{24}$$

where $\sigma_j(0) = 0$, $\sigma_j(1) \in [0,1]$ and $A_j \in \{0,1\}$ for all j. If informational spillovers are positive, then

$$\Pr(e_1 \geq 0 \,|S_2 = 1, \Omega_0) \geq \Pr(e_1 \geq 0 \,|A_2 = \varnothing, \Omega_0) \tag{25}$$

This condition also holds when informational spillovers are absent, as long as we impose that $\Pr(S_2 = 1|\Omega_0) > 0$.

Proof of Lemma 4: Since (25) is undefined when either (i) $\Pr(A_2 = \varnothing, \Omega_0) = 0$ or (ii) $\Pr(A_2 = 1, \Omega_0) = 0$, we need to verify these probabilities are both positive for the condition to be meaningful. To establish (i), recall that we restrict $\pi(\omega)$ to be symmetric and to ensure that $\Pr(B(\omega) = 0) < 1$. Hence, any bank can be bad with positive probability. Given $\sigma_j(0) = 0$, it follows that $\Pr(A_2 = \varnothing, \Omega_0) > 0$. As for (ii), we will show below that positive informational spillovers directly implies (ii). When informational spillovers are absent, the condition is ensured by our requirement that $\Pr(S_2 = 1|\Omega_0) > 0$.

In a slight abuse of notation, we will refer to distributions conditional on the event $\{S_2 = 1, \Omega_0\}$ as being conditional on the event $\{A_2 = 1, \Omega_0\}$. The two events are obviously related: $\{S_2 = 1\}$ corresponds to knowing that bank 2 is good, while $\{A_2 = 1\}$ corresponds to observing bank 2 announce it is good. Moreover, A_2 is independent of $\{S_j\}_{j\neq 2}$, so observing it equal to 1 teaches us nothing else about the underlying state. Formally, $\{A_2 = 1\} \Rightarrow \{S_2 = 1\}$. However, if $\sigma_2(1) = 0$, it will not be possible to observe $A_2 = 1$, so conditioning on $\{A_2 = 1\}$ does not yield a well-defined probability even though we can still condition on $\{S_2 = 1\}$. Our results would then hold conditional on $\{S_2 = 1\}$. Intuitively, our result shows what would happen if agents were given external information that bank 2 is good. However, since our interest in (25) is motivated by questions about how bank 2's disclosure might impact bank 1, it is more natural to frame our results as if bank 2 was the source of the information.

The LHS of (25), $\Pr(e_1 \geq 0|A_2 = 1, \Omega_0)$, can be written as

$$\Pr(e_1 \geq 0|S_1 = 1, A_2 = 1, \Omega_0) \Pr(S_1 = 1|A_2 = 1, \Omega_0) + \\ \Pr(e_1 \geq 0|S_1 = 0, A_2 = 1, \Omega_0) \Pr(S_1 = 0|A_2 = 1, \Omega_0)$$

Assumption A3 implies $\Pr(e_1 \geq 0 \,|S_1 = 0, \Omega_0) = 0$, and so we have

$$\Pr(e_1 \geq 0 \,|A_2 = 1, \Omega_0) = \Pr(e_1 \geq 0|S_1 = 1, A_2 = 1, \Omega_0) \Pr(S_1 = 1|A_2 = 1, \Omega_0)$$

By the same logic,

$$\Pr\left(e_1 \geq 0\,|A_2 = \varnothing, \Omega_0\,\right) = \Pr\left(e_1 \geq 0|S_1 = 1, A_2 = \varnothing, \Omega_0\right)\Pr\left(S_1 = 1|A_2 = \varnothing, \Omega_0\right)$$

Consider first the case with no informational spillovers. From part (ii) of Lemma 3, we know that the S_j are independent. Since $\sigma_i(S_i)$ is independent of $\{S_j\}_{j\neq i}$, then $\Pr\left(S_1 = 1|A_2 = 1, \Omega_0\right) = \Pr\left(S_1 = 1|A_2 = \varnothing, \Omega_0\right) = \Pr(S_1 = 1)$. Since we are requiring that $\Pr(S_2 = 1|\Omega_0) > 0$, it follows that $\Pr(S_2 = 1) > 0$. Symmetry then implies $\Pr(S_1 = 1) > 0$. Substituting our expressions into (25) and cancelling $\Pr(S_1 = 1)$ allows us to rewrite (25) as

$$\Pr\left(e_1 \geq 0|S_1 = 1, A_2 = 1, \Omega_0\right) \geq \Pr\left(e_1 \geq 0|S_1 = 1, A_2 = \varnothing, \Omega_0\right) \tag{26}$$

Define K as the set of banks in $\{3, \ldots, n\}$ that fail to disclose their type, i.e. $K = \{k \in \{3, \ldots, n\} : A_k = \varnothing\}$. We can write $\Pr\left(e_1 \geq 0|S_1 = 1, A_2 = 1, \Omega_0\right)$ as a sum over all possible realizations $(s_3, \ldots, s_n) \in \{0, 1\}^{n-2}$:

$$\sum_{(s_3,\ldots,s_n)} \mathbb{I}_{\{e_1(1,1,s_3,\ldots,s_n)\geq 0\}} \prod_{k\in K} \Pr\left(S_k = s_k\,|\Omega_0\,\right) \tag{27}$$

Similarly, we can write $\Pr\left(e_1 \geq 0|S_1 = 1, A_2 = \varnothing, \Omega_0\right)$ as

$$\begin{aligned}&\sum_{(s_3,\ldots,s_n)} \mathbb{I}_{\{e_1(1,1,s_3,\ldots,s_n)\geq 0\}} \cdot \Pr\left(S_2 = 1\right) \prod_{k\in K} \Pr\left(S_k = s_k|\Omega_0\right) + \\ &\sum_{(s_3,\ldots,s_n)} \mathbb{I}_{\{e_1(1,0,s_3,\ldots,s_n)\geq 0\}} \cdot \Pr\left(S_2 = 0\right) \prod_{k\in K} \Pr\left(S_k = s_k|\Omega_0\right)\end{aligned} \tag{28}$$

where $\mathbb{I}_{\{e_1(s)\geq 0\}}$ is an indicator that is equal to 1 equity is positive when the state $S = s$ and 0 otherwise. Since assumption A4 implies $e_1\left(1, 1, s_3, ..., s_n\right) \geq e_1\left(1, 0, s_3, ..., s_n\right)$, it follows that for any vector $(s_3, \ldots, s_n)$, the expression $\mathbb{I}_{\{e(1,1,s_3,\ldots,s_n)\geq 0\}}$ is greater than or equal to

$$\Pr\left(S_2 = 1\right)\mathbb{I}_{\{e(1,1,s_3,\ldots,s_n)\geq 0\}} + \Pr\left(S_2 = 0\right)\mathbb{I}_{\{e(1,0,s_3,\ldots,s_n)\geq 0\}}$$

Hence, the expression multiplying $\prod_{k\in K} \Pr\left(S_k = s_k|\Omega_0\right)$ in (27) exceeds the expression multiplying this same term in (28). From this, it follows that (26) holds, which in turn implies condition (25).

We now move to the case of positive informational spillovers. We first need to verify that $\Pr(S_2 = 1, \Omega_0) > 0$, i.e. that it is even possible for bank 2 to be good. Here, observe that the event Ω_0 may reveal the types of some banks with certainty (which may include a bank being bad if $\sigma_j(1) = 1$ and $A_j = \varnothing$) and will assign a distribution over the types of remaining banks. But from Lemma 3 part (i), we know that $\Pr\left(S_1 = \cdots = S_k = 1|S_{k+1} = s_{k+1}, ..., S_n = s_n\right) > 0$ for any $(s_{k+1}, ..., s_n)$. Hence, $\Pr\left(S_1 = \cdots = S_k = 1|\Omega_0\right) > 0$, which requires that $\Pr\left(S_2 = 1|\Omega_0\right) > 0$, and which in turn implies $\Pr(S_2 = 1, \Omega_0) > 0$.

To establish the claim, recall that we can rewrite (25) as

$$\Pr(e_1 \geq 0|S_1 = 1, A_2 = 1, \Omega_0) \Pr(S_1 = 1|A_2 = 1, \Omega_0) \geq \\ \Pr(e_1 \geq 0|S_1 = 1, A_2 = \varnothing, \Omega_0) \Pr(S_1 = 1|A_2 = \varnothing, \Omega_0) \tag{29}$$

By the same argument as above, we can appeal to part (i) of Lemma 3 to argue that $\Pr(S_1 = 1|A_2 = 1, \Omega_0) > 0$ and $\Pr(S_1 = 1|A_2 = \varnothing, \Omega_0) > 0$. Hence, (25) follows if we can show that

$$\Pr(e_1 \geq 0|S_1 = 1, A_2 = 1, \Omega_0) \geq \Pr(e_1 \geq 0|S_1 = 1, A_2 = \varnothing, \Omega_0) \tag{30}$$

Once again, we can write $\Pr(e_1 \geq 0|S_1 = 1, A_2 = 1, \Omega_0)$ as a sum over all possible realizations $(s_3, \ldots, s_n) \in \{0,1\}^{n-2}$:

$$\sum_{(s_3,\ldots,s_n)} \mathbb{I}_{\{e(1,1,s,\ldots,s_n)\geq 0\}} \Pr(S_3 = s_3, \ldots, S_n = s_n|S_1 = 1, A_2 = 1, \Omega_0)$$

Similarly, we can write $\Pr(e_1 \geq 0|S_1 = 1, A_2 = \varnothing, \Omega_0)$ as

$$\sum_{(s_3,\ldots,s_n)} \mathbb{I}_{\{e_1(1,1,s_3,\ldots,s_n)\geq 0\}} \Pr(S_2 = 1, \ldots, S_n = s_n|S_1 = 1, A_2 = \varnothing, \Omega_0) + \\ \sum_{(s_3,\ldots,s_n)} \mathbb{I}_{\{e_1(1,0,s_3,\ldots,s_n)\geq 0\}} \Pr(S_2 = 0, \ldots, S_n = s_n|S_1 = 1, A_2 = \varnothing, \Omega_0)$$

To show that the first expression above is larger, consider state $\underline{S} = (1, 0, ..., 0)$. If $e_1(\underline{S}) \geq 0$, then per assumption A4, we can conclude that $e_1(S) \geq 0$ for all S for which $\Pr(S|S_1 = 1) > 0$. In this case, both expressions above are equal to 1 and condition (30) holds trivially.

Finally, if $e_1(\underline{S}) < 0$, then we claim that for each state S where $e_1(S) \geq 0$, it must be the case that

$$\Pr(S|S_1 = 1, A_2 = 1, \Omega_0) \geq \Pr(S|S_1 = 1, A_2 = \varnothing, \Omega_0)$$

To see this, observe that $e_1(S) \geq 0$ only if $S \geq \underline{S}$ per Assumption A4. Positive informational spillovers then implies that

$$\Pr(S|S_1 = 1, S_2 = 1, \Omega_0) \geq \Pr(S|S_1 = 1, S_2 = 0, \Omega_0) \tag{31}$$

as long as $\Pr(S_1 = 1, S_2 = 1, \Omega_0) > 0$, which follows from Lemma 3 part (i). But since $\Pr(S|S_1 = 1, A_2 = \varnothing, \cdot)$ is a weighted average of $\Pr(S|S_1 = 1, S_2 = 1, \cdot)$ and $\Pr(S|S_1 = 1, S_2 = 0, \cdot)$, it follows that

$$\Pr(S|S_1 = 1, S_2 = 1, \cdot) \geq \Pr(S|S_1 = 1, A_2 = \varnothing, \cdot)$$

The claim thus follows.

Proof of Proposition 1: Recall that $I_1(A_1, \ldots, A_n)$ is weakly increasing in $\Pr(e_1 \geq 0\,|A, \{\sigma_j\})$. But from Lemma A1, we know that

$$\Pr\left(e_1 \geq 0 \left| A_2 = 1, \Omega_0 \right.\right) \geq \Pr\left(e_1 \geq 0 \left| A_2 = \varnothing, \Omega_0 \right.\right)$$

for any $\Omega_0 = \left\{ \{\sigma_j(S_j)\}_{j=1}^n, \{A_j\}_{j \neq 2} \right\}$. Since we can substitute any vector of announcements, that would include vectors where $A_1 = \varnothing$ and $A_1 = 1$ (or, alternatively, $S_1 = 1$ if bank 1 never discloses it is good). From this, we can deduce that

$$\Pr\left(e_1 \geq 0 \left| A_1 = \varnothing, A_2 = 1, A, \{\sigma_j\} \right.\right) > \Pr\left(e_1 \geq 0 \left| A_1 = \varnothing, A_2 = \varnothing, A, \{\sigma_j\} \right.\right)$$

which implies $I_1(\varnothing, 1) \geq I_1(\varnothing, \varnothing)$, and

$$\Pr\left(e_1 \geq 0 \left| A_1 = 1, A_2 = 1, A, \{\sigma_j\} \right.\right) > \Pr\left(e_1 \geq 0 \left| A_1 = \varnothing, A_2 = \varnothing, A, \{\sigma_j\} \right.\right)$$

which implies $I_1(1, 1) \geq I_1(1, \varnothing)$, as claimed.

Proof of Proposition 2: Suppose $I_1^*(\varnothing; \varnothing) = I_1^*(\varnothing; 1) = 0$. If bank i disclosed it was good, its gain from disclosure would correspond to the expected profits is could retain using the funds it raises minus the cost of disclosure, i.e.

$$\left[\Pr\left(e_1 \geq 0 | A_1 = 1, A_2, \cdot\right) R - \underline{r}\right] I_1\left(A_1 = 1, A_2, \cdot\right) - c \tag{32}$$

By Lemma A1, we know that

$$\Pr\left(e_1 \geq 0 | A_2 = 1, A_2 = 1, \cdot\right) \geq \Pr\left(e_1 \geq 0 | A_2 = \varnothing, A_2 = \varnothing, \cdot\right)$$

In addition, Proposition 1 tells us that

$$I_1^*(1; 1) \geq I_1^*(1; \varnothing)$$

From these two inequalities, we can deduce that the gain (32) is higher conditional on $A_2 = 1$ than on $A_2 = \varnothing$.

Proof of Proposition 3: Suppose $I_1(\varnothing; \varnothing) = I_1(\varnothing; 1) = 1$. The gain in this case is the reduction in interest charges bank 1 would pay if it had equity net of the cost of disclosure c, i.e.

$$\Pr\left(e_1 \geq 0 | A_1 = 1, \Omega_0\right) \left[\frac{\underline{r}}{\Pr\left(e_1 \geq 0 | A_1 = \varnothing, \Omega_0\right)} - \frac{\underline{r}}{\Pr\left(e_1 \geq 0 | A_1 = 1, \Omega_0\right)} \right] - c$$

This reduces to

$$\left[\frac{\Pr\left(e_1 \geq 0 | A_1 = 1, \Omega_0\right)}{\Pr\left(e_1 \geq 0 | A_1 = \varnothing, \Omega_0\right)} - 1 \right] \underline{r} - c \tag{33}$$

which is equal to

$$\left[\frac{\Pr\left(e_1 \geq 0 | S_1 = 1, \Omega_0\right)}{\Pr\left(e_1 \geq 0 | A_1 = \varnothing, \Omega_0\right)} - 1 \right] \underline{r} - c \tag{34}$$

However, since A3 implies $\Pr(e_1 \geq 0|S_1 = 0, \Omega_0) = 0$, it follows that

$$\Pr(e_1 \geq 0|A_1 = \varnothing, \Omega_0) = \Pr(e_1 \geq 0|A_1 = \varnothing, \Omega_0)\Pr(S_1 = 1|\Omega_0)$$

Hence,

$$\frac{\Pr(e_1 \geq 0|S_1 = 1, \Omega_0)}{\Pr(e_1 \geq 0|A_1 = \varnothing, \Omega_0)} = \frac{1}{\Pr(S_1 = 1|\Omega_0)}$$

If informational spillovers are either positive or absent, then for any set Ω_0, we have $\Pr(S_1 = 1|S_2 = 1, \Omega_0) \geq \Pr(S_1 = 1|S_2 = 0, \Omega_0)$. From this, it follows that if $\Pr(S_1 = 1|A_2 = 1, \Omega_0)$ is well-defined, we can deduce that

$$\Pr(S_1 = 1|A_2 = 1, \Omega_0) \geq \Pr(S_1 = 1|A_2 = \varnothing, \Omega_0)$$

It follows that the gain from disclosure to bank 1 is lower when $A_2 = 1$ than when $A_2 = \varnothing$, as claimed.

Change in gain when $I_1(\varnothing; \varnothing) = 0$ and $I_1(\varnothing; 1) = 1$: From the proofs of Propositions 2 and 3, we can conclude that the gain from disclosure when $A_2 = \varnothing$ is equal to

$$[\Pr(e_1 \geq 0|A_1 = 1, A_2 = \varnothing, \Omega_0) R - \underline{r}] I_1(1, \varnothing) - c$$

and the gain from disclosure when $A_2 = 1$ is equal to

$$\left[\frac{\Pr(e_1 \geq 0|A_1 = 1, A_2 = 1, \Omega_0)}{\Pr(e_1 \geq 0|A_1 = \varnothing, A_2 = 1, \Omega_0)} - 1\right]\underline{r} - c$$

We can use these expressions to compute the change in the gain from disclosure between $A_2 = \varnothing$ and $A_2 = 1$. If if $I(1; \varnothing) = 0$, the change in gain is equal to

$$\left[\frac{\Pr(e_1 \geq 0|A_1 = 1, A_2 = 1, \Omega_0)}{\Pr(e_1 \geq 0|A_1 = \varnothing, A_2 = 1, \Omega_0)} - 1\right]\underline{r} \tag{35}$$

and if $I(1; \varnothing) = 1$ the change in gain is equal to

$$\frac{\Pr(e_1 \geq 0|A_1 = 1, A_2 = 1, \Omega_0)}{\Pr(e_1 \geq 0|A_1 = \varnothing, A_2 = 1, \Omega_0)}\underline{r} - \Pr(e_1 \geq 0|A_1 = 1, A_2 = \varnothing, \Omega_0) R \tag{36}$$

if $I(1; \varnothing) = 1$.

In the case of (35), we know from Lemma 4 that the expression is positive, i.e. in this case disclosure is a strategic complement.

In the case of (36), Recall that for $I_1(1; \varnothing) = 1$, it must be the case that

$$\Pr(e_1 \geq 0|A_1 = 1, A_2 = \varnothing, \Omega_0) \geq \frac{\underline{r}}{R}$$

Using this inequality, we can deduce that (36) is bounded below by

$$\frac{\Pr\left(e_1 \geq 0|A_1 = 1, A_2 = 1, \Omega_0\right)}{\Pr\left(e_1 \geq 0|A_1 = \varnothing, A_2 = 1, \Omega_0\right)}\underline{r} - \frac{\Pr\left(e_1 \geq 0|A_1 = 1, A_2 = \varnothing, \Omega_0\right)}{\Pr\left(e_1 \geq 0|A_1 = \varnothing, A_2 = \varnothing, \Omega_0\right)}\underline{r}$$

This is the change in gain if $I_1\left(\varnothing; \varnothing\right) = I_1\left(\varnothing; 1\right) = 1$. Next, since we are given that $I_1\left(\varnothing, 1\right) = 1$, it follows that

$$\Pr\left(e_1 \geq 0|A_1 = \varnothing, A_2 = 1, \Omega_0\right) \geq \frac{\underline{r}}{R}$$

From this, we can conclude that (36) is bounded above by

$$\Pr\left(e_1 \geq 0|A_1 = 1, A_2 = 1, \cdot\right) R - \Pr\left(e_1 \geq 0|A_1 = 1, A_2 = \varnothing, \cdot\right) R$$

which is the change in gain if $I(\varnothing; \varnothing) = I(\varnothing; 1) = 0$. Hence, in this case the change in gain is bounded by the two cases, and can in principle be either positive or negative.

Proof of Proposition 4: Suppose $I_1\left(\varnothing; \varnothing\right) = I_1\left(\varnothing; 1\right) = 1$. As in Proposition 3, the gain from disclosure is given by

$$\left[\frac{1}{\Pr\left(S_1 = 1|\Omega_0\right)} - 1\right]\underline{r} - c$$

Since informational spillovers are negative, we know that

$$\Pr\left(S_1 = 1|A_2 = 1, \Omega_0\right) \geq \Pr\left(S_1 = 1|A_2 = \varnothing, \Omega_0\right)$$

Hence, the expected gain from disclosure to bank 1 is lower when $A_2 = 1$ than when $A_2 = \varnothing$.

Proof of Proposition 5: Define $\Omega_g^i = \{\omega | e_i(\omega) \geq 0\}$, so Ω_g^i represents the set of states in which bank i is capable of paying back investors.

Consider a hypothetical decision maker who can either choose to invest in bank i or not. If she invests, she receives R if $\omega \in \Omega_g^i$ and 0 if $\omega \notin \Omega_g^i$, while if she does not invest she receives $\underline{r}$ regardless of ω.

The hypothetical decision maker observes a vector of signals A. We first consider the case where $A_i = 1$, i.e. where the hypothetical decision maker knows bank i is good. For $j \neq i$, the signal A_j is equal to 1 with probability σ_j if $S_j(\omega) = 1$ and is equal to $\varnothing$ otherwise, i.e. with probability 1 if $S_j(\omega) = 0$ and with probability $1 - \sigma_j$ if $S_j(\omega) = 1$. Let $I_i^D(A)$ denote the decision maker's investment decision after observing the signal A, i.e. $I_i^D(A)$ is equal to 1 if the decision maker invests.

Since a signal with a value σ_j' represents a garbled version of a signal whenever $\sigma_j > \sigma_j'$, by the Blackwell (1953) sufficiency condition, we know the hypothetical decision maker is weakly better off when σ_j is higher. Formally, if we define $\mathbb{I}_{\{\omega \in \Omega_g^i\}}$, then the expected payoff to the hypothetical decision maker is

$$E[I_i^D(A) R \; \mathbb{I}_{\{\omega \in \Omega_g^i\}} + (1 - I_i^D(A))\underline{r}]$$

is weakly increasing in σ_j. Note that the hypothetical decision maker will invest after observing A if and only if

$$E[\mathbb{I}_{\{\omega \in \Omega_g^i\}}|A]R = \Pr(\omega \in \Omega_i \ |A)R > \underline{r}$$

However, this is the same condition that determines whether in the decentralized market outsiders will be willing to trade. Hence, the payoff to the hypothetical decision maker is identically equal to the expected gains from trade $\mathcal{G}_i(\sigma)$.

Proof of Proposition 6: In text

Proof of Theorem 1: The cases where $p_g < \frac{\underline{r}}{R}$ and $p_g > \frac{1}{\Pr(S_i=1)}\frac{\underline{r}}{R}$ are described in the text. Note that part (ii) follows directly from the analysis fo the latter case supplied in the text. We therefore only need to consider the intermediate case where $p_g \in \left(\frac{\underline{r}}{R}, \frac{1}{\Pr(S_i=1)}\frac{\underline{r}}{R}\right)$. From Proposition 6, we know that in this case, a non-disclosure equilibrium involves no trade. The conditions for a non-disclosure equilibrium to exist and for mandatory disclosure to improve upon no trade can be summarized as

$$p_g \Pr(S_i = 1)(R - \underline{r}) \leq c \leq p_g R - \underline{r} \tag{37}$$

For this inequality to be valid, we need

$$p_g \Pr(S_i = 1)(R - \underline{r}) < p_g R - \underline{r}$$

which after rearranging implies

$$p_g \leq \frac{\underline{r}}{\Pr(S_i = 0)R + \Pr(S_i = 1)\underline{r}} \tag{38}$$

Define

$$p_g^* \equiv \min\left\{\frac{1}{\Pr(S_i = 1)}\frac{r}{R}, \frac{\underline{r}}{\Pr(S_i = 0)R + \Pr(S_i = 1)\underline{r}}\right\} \tag{39}$$

Note that both expressions on the RHS above are greater than $\frac{\underline{r}}{R}$, so $p_g^* > \frac{\underline{r}}{R}$ as claimed. If $p_g < p_g^*$, then either $p_g \leq \frac{\underline{r}}{R}$, in which case a non-disclosure equilibrium exists and can be improved upon for any $c \geq 0$, or else (37) can be satisfied for a nonempty interval of values for c.

Finally, we need to show that $p_g^* < 1$. Since $R > \underline{r}$, we know the second expression is less than 1, which implies the minimum of it and another expression must also be less than 1. This establishes the claim.

Proof of Proposition 7: We know from Kreps and Wilson (1982) that the dynamic incomplete information game in which banks choose offers must have a sequential equilibrium. Since Proposition 6 rules out the possibility of a non-disclosure equilibrium, it follows that there exists some i such that $\sigma_i > 0$.

To show that this equilibrium can be improved upon, note that if we force all banks to set $\sigma_j = 0$, we can ensure all banks raise funds. Hence, we maximize total resources. In addition, we reduce the utility cost associated with disclosure, since $\sigma_j = 0$ implies $\alpha_j = 0$. Hence, forcing all agents to hide their type allows us to make all agents at least as well off and some strictly better off than under the original equilibrium.

Proof of Theorem 2: To prove part (i), define

$$e^* = \min_A e_i^*(A) \tag{40}$$

If $e_i < e^*$, bank i will be unable to repay his debt regardless of A. As long as the probability of repayment is less than $\frac{\underline{r}}{R}$, there is no scope for trade between outsiders and banks. Thus, non-disclosure will be an equilibrium for any $c \geq 0$, and in this equilibrium no bank will attract funds. The condition for mandatory disclosure to improve upon no trade is given by

$$E[\Pr(e_i \geq e_i^*(S))](R - \underline{r}) \geq c \tag{41}$$

where $E[\Pr(e_i \geq e_i^*(S))]$ denotes the expected probability that bank i will have enough equity that outsiders will trust it to not divert funds. Since by assumption we have $\Pr(e_i > e^*|S_i = 1) > 0$, it follows that $\Pr(e_i > e^*) > 0$, i.e. there exists a vector of announcements A that can occur with positive probability such that $\Pr(e_i > e_i^*(A)) > 0$. But this in turn implies there must exist a state of the world S such that $\Pr(e_i > e_i^*(S)) > 0$. Hence, $E[\Pr(e_i \geq e_i^*(S))] > 0$, and so there exists a nonempty interval for c such that mandatory disclosure is preferable to no trade. This establishes the claim.

We now turn to part (ii). We consider three different cases, depending on whether none, all, or only some banks get funded in equilibrium.

Suppose first that in the non-disclosure equilibrium, outsiders invest in none of the banks. If $\Pr(e_i = \overline{e}|S_i = 1) \to 1$, then by disclosing its type bank i will be able to attract funds even if no other bank discloses its type. Hence, a non-disclosure equilibrium to exist with no investment, it must be the case that the cost of disclosure c exceeds the expected value from disclosing and attracting funds. The latter is equal to

$$\rho_1 R + (1 - \rho_1)v - \underline{r} \leq c \tag{42}$$

where $\rho_1 = \Pr(e_1 > e_1^*(1, \varnothing, \ldots, \varnothing)|S_1 = 1)$ is the probability that a good bank will not default given the interest rate it is charged when it is the only bank that reveals its type (by symmetry, this will be the same for all banks). In the limit as $\Pr(e_i = \overline{e}|S_i = 1) \to 1$, it must also be the case that $\rho_1 \to 1$.

Next, the condition for mandatory disclosure to improve upon no trade is given by

$$\rho_2 \Pr(S_i = 1)(R - \underline{r}) \geq c \tag{43}$$

where $\rho_2 = E[\Pr(e_1 > e_1^*(S)|S_1 = 1)]$ is the expected probability that a good bank can be trusted to undertake the project when all information is revealed.

In the limit as $\Pr(e_i = \overline{e}|S_i = 1) \to 1$, it must also be the case that $\rho_2 \to 1$. In the limit when $\rho_1 = \rho_2 = 1$ conditions (42) and (43) are in contradiction, since $\Pr(S_i = 1) < 1$ given our assumption that $q_0 < 1$. Hence, mandatory disclosure cannot improve upon a non-disclosure equilibrium in the limit, and by continuity it cannot improve upon a non-disclosure equilibrium when $\Pr(e_i < \overline{e}|S_i = 1)$ is close to but strictly less than 1.

Next, suppose that in the non-disclosure equilibrium, outsiders invest in all of the banks. In this case, a bank will get funded whether it discloses or not, and the only benefit of disclosing is to reduce the interest charges. In equilibrium, the cost of disclosure c must exceed the reduction in interest rates, i.e.

$$c \geq \frac{\Pr(S_i = 0)}{\Pr(S_i = 1)} \underline{r} \tag{44}$$

The condition for mandatory disclosure to improve welfare is given by

$$(1 - \rho_2 \Pr(S_i = 1))(\underline{r} - v) \leq c \tag{45}$$

where as before $\rho_2 = E[\Pr(e_1 > e_1^*(S)|S_1 = 1)]$ is the expected probability that a good bank can be trusted to undertake the project when all information is revealed. In the limit as $\Pr(e_i = \overline{e}|S_i = 1) \to 1$, we still have that $\rho_2 \to 1$, and so (45) reduces to $\Pr(S_i = 0)(\underline{r} - v) > c$. Since A6 implies $v > R - \underline{r} > 0$ and since $\Pr(S_i = 1) > 0$, in the limit (44) and (45) are contradictory. hence, once again mandatory disclosure cannot improve upon a non-disclosure equilibrium in the limit, nor by continuity when $\Pr(e_i < \overline{e}|S_i = 1)$ is close to but strictly less than 1.

Finally, suppose that in the non-disclosure equilibrium some banks receive funding and some banks don't, i.e. outside investors are exactly indifferent. Consider the deterministic case where n_0 banks receive no funding and n_1 banks do receive funding, where $n_0 + n_1 = n$, and banks know whether they will receive funding or not. The condition for mandatory disclose to improve welfare is now

$$n_0 \rho_2 \Pr(S_i = 1)(R - \underline{r}) + n_1(1 - \rho_2 \Pr(S_i = 1))(\underline{r} - v) \leq nc \tag{46}$$

The conditions for the two types of banks to be willing to not disclose are the same as before. Hence, in the limit as $\Pr(e_i = \overline{e}|S_i = 1) \to 1$, each component in the sum will have to exceed c , and so the condition cannot be satisfied. This same sort of averaging argument holds in the case where a bank will be funded with some probability, since then the condition for a non-disclosure equilibrium to exist is a mixture of (42) and (44).

Proof of Proposition 8: We first define the shortfall D_{ij} in state S as the difference between what bank i owes bank j and what it actually pays bank j:

$$D_{ij} = \Lambda_{ij} - x_{ij} \text{ for all } i, j \in \{0, ..., n-1\} \tag{47}$$

Note that the RHS of (21) can be interpreted as an operator that maps payments x_{ij} into payments. We can express this operator as an alternative operator

$F : \mathcal{D} \to \mathcal{D}$, where $\mathcal{D} \subset \mathbb{R}^n_+$ is the space of possible shortfalls given by $\mathcal{D} = \{D_{ij} \in [0\ ,\ \Lambda_{ij}]\ ,\ i,j \in \{0,...,n-1\}\}$. This operator is defined by

$$(F)_{ij}(D) \quad = \quad \frac{\Lambda_{ij}}{\Lambda_i} \max\left\{\min\left\{\Lambda_i, \sum_{m\neq i} D_{mi} - \overline{e} + (1-S_i)\phi\right\}, 0\right\} \qquad (48)$$

The set of fixed points of the shortfall operator corresponds to the set of fixed points of the operator defined over payments. Either of these can be used to derive equity, and hence the distribution of equity we wish to characterize.

Our proof now proceeds as follows. First, we show that for each S the shortfall $D(S)$ are weakly increasing in ϕ and in λ. Next we argue that this implies that the distribution of equity is stochastically decreasing with ϕ and in λ for each S. Then the result follows since the distribution of S does not depend on (ϕ, λ).

We use the notation $F_{\phi,\lambda}$ to emphasize the dependence of the operator on the parameters (ϕ, λ). It is easy to show that F is monotone, i.e. $F_{\phi,\lambda}(D') \geq F_{\phi,\lambda}(D)$ if $D' \geq D$, where the comparison is component by component. Thus, by Tarski's fixed point theorem, there exists a smallest fixed point, which is obtained as $D^*(\phi, \lambda) = \lim_{n\to\infty} F^n(0)$. Additionally, F is monotone on (ϕ, λ), i.e. for each $D \in \mathcal{D}$, $F_{\phi',\lambda'}(D) \geq F_{\phi,\lambda}(D)$, whenever $(\phi', \lambda') \geq (\phi, \lambda)$. Then it follows that the smallest fixed point $D^*(\phi, \lambda)$ is increasing in (ϕ, λ).

For any vector of shortfalls D, parameter (ϕ, λ) and state of the network S the implied equity of bank i is:

$$\begin{aligned}
e_i(S) \quad &= \quad \max\left\{\ 0,\ \pi - \phi S_i - \sum_{j=0}^{n-1} \Lambda_{ij} + \sum_{m=0}^{n-1} x_{mi}(S)\right\} \\
&= \quad \max\left\{\ 0,\ \pi - \phi S_i - \Lambda_i - \left(\sum_{m=0}^{n-1} D_{mi}(S) + \sum_{m=0}^{n-1} \Lambda_{mi}\right)\right\} \\
&= \quad \max\left\{\ 0, \pi - \phi S_i - \sum_{m=0}^{n-1} D_{mi}(S)\right\}
\end{aligned}$$

where the last equality follows from our assumption that $\Lambda_i = \sum_m \Lambda_{mi}$.

Consider the equity corresponding to $D = D^*(\phi, \lambda)$. Equity at bank i is given by

$$e_i(\phi, \lambda; S) \quad = \quad \max\left\{\ 0, \pi - \phi S_i - \sum_{m=0}^{n-1} D^*_{mi}(\phi, \lambda; S)\right\} \qquad (49)$$

where $D^*_{mi}(\phi, \lambda; S)$ is the amount bank m falls short on bank i for the smallest fixed point for the state S and parameters (ϕ, λ). Using the monotonicity of $D^*(\phi, \lambda)$ it is immediate that $e_i(\phi, \lambda; S)$ is weakly decreasing in (ϕ, λ) for each S. While we have used the smallest fixed point in the definition (49), by Theorem 1 in Eisenberg and Noe (2001) every fixed point of $F_{\phi,\lambda}$ has the same implied equity values for each bank. Hence, the comparative static of equity must be the same for any fixed point.

Finally, the conditional probability of interest is given by

$$\Pr\left(e_i \le x \mid S_i = 1\right) = \frac{\sum_{\{s\in\{0,1\}^n:\, s_i=1\}} \mathbb{I}\{e_i(\phi,\lambda;s) \le x\} \Pr(S=s)}{\sum_{\{s\in\{0,1\}^n: s_i=0\}} \Pr(S=s)} \tag{50}$$

Since $\Pr(S = s)$ is just constant for each s, it follows that $\Pr(e_i \le x|S_i = 0)$ is decreasing in (ϕ, λ).