

Market-Based Probabilities: A Tool for Policymakers*

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1 Basics of Policy Choice under Uncertainty

Many policy decisions take the following form: A policymaker must choose today among a set of possible actions, but the costs and benefits of those actions materialize in the future. For example, if the Federal Reserve decides to ease monetary policy today, the impact of that easing decision on the economy is typically thought to materialize only over the next year or two.

Of course, the policymaker does not know today what will happen in the future. This uncertainty means that the policymaker has to consider the current decision's net benefits – the difference between its benefits and its costs - in more than one possible future. To return to the monetary policy example, the policymaker has to consider at least two possible futures. In one, the economy faces adverse shocks over the next year or two that leads to low inflationary pressures. Easing policy today is beneficial if that low inflation future materializes. But in another possible future, the economy experiences positive shocks that lead to high inflationary pressures. Easing policy today will lead to undue inflation if that other future materializes.

To make choices in these situations, policymakers need a systematic way to combine the net benefits of the possible choices across the various possible futures that could materialize. Thus, a monetary policymaker needs a way to combine the positive net benefits of easing when the economy faces adverse shocks with the negative net benefits of easing when the economy faces positive shocks.

*Federal Reserve Bank of Minneapolis: Ron Feldman, Ken Heinecke, Narayana Kocherlakota, Sam Schulhofer-Wohl, Tom Tallarini. The views expressed in this paper are those of the authors and not necessarily those of others in the Federal Reserve System. We thank a number of people for thoughtful and helpful comments.

The paper is still preliminary and comments are welcome. Address any comments to narayana.kocherlakota@mpls.frb.org.

We can summarize this discussion in the form of a basic question for policymakers:

BASIC QUESTION: In making a current decision, how should the policymaker weigh the net benefits of that decision in one possible future against the net benefits of that decision in another possible future?

To answer this question, we start from the following premise:

PREMISE: Policymakers should act in the interests of households.¹

Given that premise, our answer to this question is the following:

ANSWER: In a wide variety of circumstances, *market-based probabilities* of possible futures provide a useful benchmark weighting for policymakers.

The market-based probability of a possible future refers to a weight on that future that is imputable from financial market (especially option) prices.² Our main result provides conditions under which weights imputed from financial market prices are equal to the weights that households themselves would put on those futures.³ It follows that, under these conditions, policymakers can best make choices in the interests of society if they use market-based probabilities to weight futures.

Others before us have discussed how policymakers can find financial market prices to be a useful guide to decision-making.⁴ However, our emphasis is quite distinct from this prior literature. The earlier papers saw financial market prices as potentially useful as a source of information about underlying “true” or “objective” probabilities of possible futures. In contrast, we emphasize that market-based probabilities imputed from financial market data

¹Formally, maximize the weighted average of household utility functions.

²The market-based probability of an event X is the price of a unit of consumption in event X, relative to the price of a risk-free unit of consumption in that same time frame. Market-based probabilities are often termed “risk-neutral” probabilities in financial economics.

³For simplicity, we first prove the main result in the case in which asset markets are complete and households are homogeneous. However, we show that the result can be extended to allow for heterogeneity and (rather severe forms of) financial market incompleteness.

⁴See Hetzel (1992), Sumner (1995) and Bernanke and Woodford (1997), among others.

will typically be quite different from objective probabilities estimated using statistical models.

Given this difference between objective and market-based probabilities, why should policymakers rely on the latter as a guide to decision-making? We discuss a number of answers to this question in our paper, but we see the following as possibly the most important. Households typically assign a higher marginal value to resources in a future with adverse economic outcomes (like a recession) than resources in other possible futures with better economic outcomes. Objective probabilities, by their very construction, do not embed this aspect of household preferences. Financial market prices do embed this aspect of household preferences - and so market-based probabilities, by their very construction, also do. It follows that policymakers who want to be reflective of this important aspect of household preferences should be guided more by market-based probabilities rather than by objective probabilities.

More generally, we see our main result as extending the basic principles of *intertemporal* policy choice to policy choice under *uncertainty*. Economists generally agree policymakers should use one kind of financial market prices - interest rates - as a benchmark approach to weighting resources at different points in time. Our main result extends this perspective by showing that policymakers can maximize social welfare by using another aspect of financial market prices - market-based probabilities - as a benchmark weighting of resources in different possible futures.

The Minneapolis Fed reports the market-based probabilities of various events on its website, including changes in inflation, interest rates and other asset values. The website — which offers users the option of receiving updated data and commentary — can be found at <http://www.minneapolisfed.org/banking/mpd/>.

The rest of the paper explains our answer in increasing detail and increasing complexity. Section 2 provides largely verbal intuition for our answer to the basic question, and addresses some concerns that we often hear about this intuition. Section 3 presents the main result in the context of a simple abstract model. Section 4 discusses the robustness of the main result

along a variety of dimensions. Section 5 concludes.

2 Intuitive Justification and Possible Concerns

In this section, we develop an intuitive rationale for informing policymaker decisions with market-based probabilities. We then provide responses to common questions and concerns about the use of market-based probabilities. This latter discussion is, again, at an intuitive level. We defer a more technical analysis to the next two sections.

Our basic question asked how policymakers should weigh the net benefits of a decision in one possible future versus the net benefits in another possible future. In arriving at our recommendation that policymakers use market-based probabilities, we start from the following premise:

Policymakers should make decisions in the interests of households.

This premise implies that, on the margin, policymakers should weigh the net benefits of a given choice in one possible future versus the net benefits in another possible future just as households would. Put another way, a policymaker's marginal willingness to substitute resources in one possible future for resources in another possible future should mimic that of a typical household. We take this view because policymakers are agents of households and should make decisions that, in the view of households, make them better off.

How can we learn households' willingness to substitute resources in one future for resources in another future? The answer lies in financial market prices. Financial assets differ in terms of what their owner receives in different possible futures. For example, a risk-free U.S. Treasury bond makes the same dollar payments in all possible futures. In contrast, the buyer of a risky corporate bond will receive smaller payments if the future turns out to be one in which the bond issuer faces financial difficulties. Hence, the relative prices of these

two financial assets implicitly reveal the willingness of households to substitute resources between two possible futures: a future in which the corporate bond issuer is financially sound (proxied by the risk-free security) and a future in which the issuer is troubled (captured by the higher rates paid by risky securities).

We can readily generalize this basic idea. By using the prices of many financial assets (especially options), we can impute households' marginal willingness to substitute resources in one possible future for resources in a wide variety of other possible futures. When we use the term "market-based probabilities," we are referring exactly to the outcome of this imputation exercise. The policymaker can then make decisions in households' interests by weighting resources in different futures using market-based probabilities.

We now address some commonly expressed concerns about the use of market-based probabilities.

2.1 Market-based probabilities aren't "true" probabilities

We often receive the comment that policymakers should weight net benefits in a possible future using the "true" probability of that future's occurring. We find the use of the word "true" unclear, but we think it refers to estimates derived from a statistical forecasting model.⁵ There are two closely related reasons why policymakers should not use these estimated probabilities to answer the basic policy question.

First, and most important, households' willingness to substitute resources from one possible future to another depends on the relative scarcity of resources in those futures. Thus, a household may be willing to pay a lot for insurance against the possibility of job loss, even

⁵Different people often have different information and different pre-existing beliefs about the likely future evolution of a given variable of interest. For example, when assessing the odds that inflation will be high or low, different people will often rely on different price changes they have observed or different inflation rates they have experienced during their lives. It is natural for these different people to arrive at different assessments of the probability of various possible future events. There is no clear sense in which one of these assessments is more "true" than any other. In contrast, Ross (forthcoming) shows that there is a unique recovered distribution in a stationary world. Borovička, Hansen, and Scheinkman (2014) extend the analysis of Ross (forthcoming) and establish an additional condition to guarantee that the unique recovered distribution matches the subjective distribution used by investors.

if the household sees the outcome as highly unlikely. Policymakers must take this factor into account when answering the basic question if they hope to act so as to improve household well-being. Statistical models do not take household resource valuation into account, while market-based probabilities do.⁶ We view using statistical probabilities to weight resources in different states as being equivalent to ignoring discounting when weighting resources in different states.

Second, households' assessments of the likelihood of various outcomes will typically differ from that of a statistical modeler. These differences may be attributable to different information, different beliefs, or the use of unconventional probabilistic modeling. A policymaker who wants to act in the interests of households must take these differences into account when answering the basic question.

2.2 Market-based probabilities don't forecast the future well

Some critics of market-based probabilities point out that forecasts of the future based on the prices of financial assets don't perform all that well relative to forecasts based on statistical models. This criticism is closely related to the above discussion of market-based probabilities relative to statistically estimated probabilities. In our view, the weighting across possible futures embedded in statistical forecasts is an inappropriate benchmark for policymakers because that weighting is based on an inappropriate loss function for policymakers. Generally, statistical forecasts are formed and evaluated using a standard loss function such as mean squared error. But this loss function does not put more weight on a state of the world just because households are more willing to substitute resources toward that state of the world. Hence, this evaluation criterion does not seem particularly relevant for a policymaker who wants to act in the interests of a typical household.

In a similar vein, ex-post evaluations of policymaker performance should be grounded

⁶Kitsul and Wright (2013) construct market-based probabilities for inflation. By comparing these probabilities to those from a statistical model, they produce estimates of household resource valuation associated with different outcomes for inflation.

in household welfare, not mechanical applications of mean square error. For example, in the context of monetary policy, a given miss of the inflation target results in more utility loss to households in recessions. Hence, ex-post evaluations of monetary policy performance relative to an inflation target should be based on criteria that put more weight on deviations in recessions than deviations in booms.

In a related context, Ellison and Sargent (2012) account for the poor forecasting of the Federal Open Market Committee (FOMC) relative to the Federal Reserve staff's Greenbook forecasts by attributing a concern for robustness on the part of the FOMC. As in the present analysis, Ellison and Sargent rely on the fact that the policymaker is not explicitly trying to produce an optimal forecast, but rather to maximize social welfare.

2.3 Households are heterogeneous

In sketching the intuition behind the use of market-based probabilities, we implicitly assumed away any differences among households. Of course, households differ in a number of ways. How, then, can we use market based probabilities to inform policymakers about the views of such a wide variety of households?

The answer lies in the power of financial markets. If households are able to trade a set of assets, then in equilibrium, they are all equally willing, on the margin, to substitute any one of those assets for another asset. As long as the assets differ sufficiently in terms of their payoffs in different possible futures, we can conclude as well that households who participate in financial markets are all equally willing, on the margin, to substitute resources from one possible future to another possible future.

In this sense, even if households are quite different before trading, the act of trading will make them at least marginally identical in terms of how they weight resources in different possible futures. A policymaker who uses market-based probabilities is using that same weighting.

This discussion focuses on how financial markets serve to align households' preferences

on the margin. Of course, with heterogeneous households, policy choices can also affect the distribution of wealth across households. Households won't be aligned in their attitudes toward those effects.

In Section 4, when we discuss heterogeneity more formally, we assume that the policy-maker has sufficient additional levers to target a desired distribution of wealth. We then focus on how market-based probabilities can be used to evaluate policy choices, given that the targeted distribution of wealth is being achieved using these other levers.

2.4 Private sector information-gathering

Financial market prices are often seen as valuable because they reflect information that financial market participants have about the future course of the economy. Some observers have expressed the concern that, if policymakers' choices are highly sensitive to financial market prices, the private sector may not have much incentive to gather the relevant information.⁷ Roughly speaking, the intuition for this perspective is as follows. Suppose the central bank commits to do whatever is necessary to keep a market-based inflation forecast equal to a fixed number (like 2%). Then the private sector will not be able to make money by "out-forecasting" the market and the private sector will have no incentive to gather information about future inflation.

We don't find this observation to be a compelling criticism of our recommendation that policymakers rely on market-based probabilities. A main lesson from the past thirty years of asset pricing research is that little of the variability in financial market prices is actually due to variations in the forecasts of market participants about future asset payoffs.⁸ Accordingly, as we have emphasized above, we are not primarily interested in financial market prices because they allow us to extract households' *information* about objects like inflation. We see financial market prices as being useful because, like other prices, they provide useful

⁷See especially Bernanke and Woodford (1997). They also criticize the use of market-based inflation expectations because "they could easily . . . be contaminated by changes in the inflation risk premium." Our argument is that this contamination is exactly what makes market-based measures so useful.

⁸See Cochrane (2011).

information about households' *tastes* - in this case, their willingness to substitute between goods in different possible futures.

2.5 Non-participation and illiquidity

So far, we have not found the concerns raised about using market-based probabilities compelling. However, observers raise two other concerns that we see as potentially more relevant for policymakers who plan to use market-based probabilities to inform their decisions.

The first is the issue of non-participation in asset markets. In particular, we rely heavily on option prices to calculate market-based probabilities. But relatively few people trade in option markets. How do we know if market-based probabilities, calculated using option prices, reflect the input from households who do not trade these securities?

The short answer is that we don't. However, we are comforted by two observations. First, households could choose to trade in these markets if they so desire. Indeed, by buying and selling put options and call options, it is relatively easy for them to take either side of a given bet. The non-traders must perceive the gains from undertaking these trades to be small. This reasoning suggests that non-traders' marginal valuations of options relative to other assets are probably not all that different from the marginal valuations of people who are trading. Second, some option trading occurs at the behest of investors charged with operating as fiduciaries of households, even if the households themselves do not trade.

As we say, these observations are comforting. Nonetheless, we would suggest that policymakers gather information about the perceived and actual costs of trading options. This information, together with that embedded in financial market prices, could be used to draw valuable conclusions about the marginal resource valuations of households who do not trade in option markets.

The second issue is that of illiquidity in options markets, which can take two forms. Option trading on some assets is limited in the sense that we often observe meaningful gaps between bids and asks or few trades at out-of-the-money strike prices. As a result, the

prices of options may reflect factors beyond investor expectations. (For example, the price could include compensation to investors for holding a security that will be costly to sell.) A policymaker who uses market-based probabilities should take account of these gaps — perhaps by calculating corresponding upper and lower bounds for estimates of market-based probabilities.

Moreover, there is not much trading at all with options that can inform the tail of the market-based probability distribution. It is possible to address this limitation by extrapolating to complete the more extreme part of the distribution. Ensuring that extrapolation is robust and subject to review is critical for policy questions surrounding tail events.

Finally, we believe that even if nonparticipation and illiquidity make market prices imperfect indicators of households' resource valuations, there are still strong reasons for policymakers to rely heavily on the information in market prices. Market prices are directly observable. By contrast, we cannot directly observe the resource valuations of households that do not transact in markets. Any attempt to estimate these households' resource valuations would need to rely on a host of modeling assumptions that could be just as subject to challenge as our assumptions about liquidity and participation.

It is worth noting that, in nonfinancial contexts, such as the calculation of inflation and gross domestic product, policymakers routinely use market prices to value resources. But nonparticipation and illiquidity are arguably at least as severe in nonfinancial markets as in financial ones. Thus, the market for single-family homes is highly illiquid, and many families do not participate in it. Nonetheless, the Bureau of Economic Analysis still use the relative price of single-family homes to determine their weight in Gross Domestic Product.

2.6 Behavioral finance

In a classic survey article, Barberis and Thaler (2003) define behavioral finance as a modelling approach that allows households to update their beliefs in a non-Bayesian way and/or allows households' choices to differ from what would be implied by expected utility, given their

subjective beliefs. Barberis and Thaler argue that a large number of empirical phenomena in financial markets are best understood through this kind of modelling approach.

Our argument that policymakers should use market-based probabilities is broadly consistent with a behavioral finance perspective. In particular, the link between market-based probabilities and households' marginal willingness to substitute resources across different outcomes is based on a static logic. In this sense, it is largely independent of how households formulate their beliefs or how their choices conform with those beliefs.⁹

It is important to note, though, that behavioral considerations open up the possibility that households' choices in financial markets are not consistent with their underlying true willingness to substitute resources across outcomes. In this case, policymakers may find it useful to exploit other information (possibly in addition to market-based probabilities) to form assessments of households' marginal valuations of resources in different outcomes. Such an assessment seems like a challenging task to us, though. Along those lines, it is worth keeping in mind that policymakers and their staffs might well have their own behavioral biases that could interfere with these assessments.

3 Main Result

In this section, we examine a simple abstract model in which a policymaker makes a decision under uncertainty. Our key main result is that, in this setting, maximizing expected net benefits relative to market-based probabilities is equivalent to maximizing an average of households' ex-ante utility. The model assumes that households are homogeneous and that financial markets are complete. In the next section, we explore the consequences of relaxing these assumptions.

⁹We are glossing over one subtlety here: our argument does implicitly assume that households assign a positive marginal valuation to resources in every outcome. This means that, if households are (subjective) expected utility maximizers, they need to agree on what outcomes have zero probability. Technically, we are assuming that household subjective beliefs are always absolutely continuous with respect to one another.

3.1 Two Policy Games

We consider two policy games that are distinguished by the objective of the policymaker. We begin by describing the common elements of the two games and then describe the different policymaker objectives.

3.1.1 Common Elements

In both games, there are households and a policymaker and there are three periods: a *trading period*, a *planning period*, and a *realization period*. During the *trading period*, households trade (a complete set of) financial securities. Then, during the *planning period*, the policymaker chooses an action, a , today that affects outcomes in the *realization period*. In addition, the outcome of the action in the realization period depends on the realization of a random variable x , with N possible realizations $\{x_n\}_{n=1}^N$.

The action chosen by the policymaker has costs and benefits that depend on the realization of the state variable, x . Let $B(a, x)$ denote the net benefits (gross benefits minus costs) associated with action a in state x . Since $B(a, x)$ measures *net* benefits, its realization may be positive or negative. In addition, $B(a, x)$ is measured in units of the consumption good.¹⁰

One example of a policymaker's problem is that of an inflation-targeting central bank. In that case, we can represent the realization of inflation next period as $\pi = a + x$, where a denotes the level of accommodation provided by the central bank and x represents a shock to inflation such as oil prices or other exogenous events outside the control of the central bank. The central bank has a target for inflation of π^* that is known to all of the agents in the economy. The net benefit function in this case would be $B(a, x) = -(a + x - \pi^*)^2$, which

¹⁰This structure assumes that the net benefit function B depends on the policymaker's action a and random influences x that are wholly independent of a . This restriction is without loss of generality when x is continuous. In particular, suppose that B is a function of (z, a, x) , where F is the cumulative distribution function of z , conditional on x and a . In this formulation, B depends on some random influence z that is influenced by the policymaker's choices. However, we can create an entirely isomorphic model by defining $\hat{B}(u, a, x)$, where u is uniform $[0, 1]$, to be equal to $B(F^{-1}(u|a, x), a, x)$. (This is the same trick that underlies most Monte Carlo simulation experiments.) In this isomorphic model, the random influences on \hat{B} are independent of the policymaker's choice a .

measures the lost consumption associated with the central bank missing its inflation target. (See Woodford (2003).)

A somewhat more abstract example is that of a financial regulator. In this case, B again measures net benefits in terms of consumption goods, but now the regulator chooses the size of the capital distributions (dividend payments or share buybacks) that banks may undertake. The post-distribution capital positions of the banking system may affect the likelihood of financial instability in the economy next period. In this framework, the consumption implications of that potential instability are captured in B .

In the trading period, households are identical and their consumption in state n is given by $c(a, x_n) = y(x_n) + B(a, x_n)$, where $y(x)$ is an endogenous endowment that depends on the realization of the state x . Households' ex-ante utility, conditional on policymaker action a , is

$$V(a) = \sum_{n=1}^N p_n U(y(x_n) + B(a, x_n), x_n). \quad (1)$$

$U(c, x)$ is a possibly state-dependent utility function and is increasing and concave in consumption, and p_n is the households' positive weight¹¹ on the utility from state x_n .¹²

3.1.2 Difference Between the Games: Policymaker's Objective

The policy games differ in terms of the objective function of the policymaker in the planning period. In the *social welfare* game, the policymaker seeks to maximize social welfare:

$$\sum_{n=1}^N p_n U(y(x_n) + B(a, x_n), x_n)$$

(Note that this formulation of the objective function assumes that, as will be true in equilibrium, the identical households do not trade in the asset market.)

In the *market-based* game, policymakers maximize the market-based expectation of net

¹¹The weight p_n could be interpreted as the households' subjective probability (in the sense of Savage (1954) that the state next period will be x_n .

¹²Our results could be readily extended to allow for nonseparabilities in preferences over bundles of state-contingent consumption.

social benefits. Formally, the policymaker observes the outcome of the trading period. Let $q_n(\hat{a})$ denote the implied price today of goods in state n , conditional on households' common beliefs \hat{a} in the trading period about the policymaker's action choice in the planning period. Now, define

$$q_n^*(\hat{a}) = \frac{q_n(\hat{a})}{\sum_{n=1}^N q_n(\hat{a})}. \quad (2)$$

Since $q_n(\hat{a})$ is the price of goods in state n , $q_n(\hat{a}) \geq 0$ for all n . As a result, $q_n^*(\hat{a}) \geq 0$ for all n . In addition, $\sum_{n=1}^N q_n^*(\hat{a}) = 1$. Therefore, $\{q_n^*(\hat{a})\}_{n=1}^N$ is a probability measure over the states of the world. We will call this the *market-based* probability measure.¹³ Given the market-based probability measure, we can define a new expectations operator, E^* , over any random variable ϕ :

$$E^*[\phi|\hat{a}] = \sum_{n=1}^N q_n^*(\hat{a})\phi_n, \quad (3)$$

where ϕ_n is the realization of the random variable ϕ in state x_n . In the market-based game, the policymaker's objective function in the planning period, conditional on the households' common belief \hat{a} , is given by:

$$E^*[B(a, x)|\hat{a}]$$

3.1.3 Equilibrium Equivalence Result

In both of these games, households are identical and so they do not trade in equilibrium. The key aspect of an equilibrium outcome is the policymaker's action choice in the planning period.

In the social welfare game, any equilibrium outcome a_{SW}^* must satisfy the first order condition:

$$\sum_{n=1}^N p_n MUC_n(a_{SW}^*) \frac{\partial B}{\partial a}(a_{SW}^*, x_n) = 0, \quad (4)$$

where $MUC_n(a) = U_c(y(x_n) + B(a, x_n), x_n)$, the marginal utility of consumption in state n given that the policymaker makes choice a .

¹³The vector $\{q_n^*\}$ is often referred to as a *risk-neutral* probability measure, especially in finance.

In the market-based game, the policymaker's choice a_{MKT}^* is characterized by the first order condition:

$$\sum_{n=1}^N q_n^*(\hat{a}) \frac{\partial B}{\partial a}(a_{MKT}^*, x_n) = 0 \quad (5)$$

In equilibrium, the households in the trading period have correct beliefs about the action choice of the policymaker in the planning period, and:

$$\hat{a} = a_{MKT}^*$$

It follows that any equilibrium outcome in the market-based game is characterized by the first order condition:

$$\sum_{n=1}^N p_n MUC_n(a_{MKT}^*) \frac{\partial B}{\partial a}(a_{MKT}^*, x_n) = 0 \quad (6)$$

These two first order conditions (5) and (6) are the same. As long as this equation has a unique solution — which will typically be the case when there is curvature in the utility function or the net benefit function — the equilibrium outcomes in the two games will be the same. We can summarize as follows:

Main Result: Suppose that there is a unique solution a^* to the equation:

$$\sum_{n=1}^N p_n MUC_n(a^*) \frac{\partial B}{\partial a}(a^*, x_n) = 0$$

Then, a^* is the unique equilibrium policy choice in the social welfare game and in the market-based game.

We could have potentially considered a third game, in which the policymaker's objective is given by the "true" expectation of net benefits:

$$\sum_{n=1}^N p_n B(a, x_n)$$

However, the equilibrium outcome a^* in this game is characterized by the first-order condi-

tion,

$$0 = \sum_{n=1}^N p_n \frac{\partial B}{\partial a}(a^*, x_n), \quad (7)$$

is typically different from the first order conditions (5) and (6). So a^* would *not* be the same as a_{SW}^* or a_{MKT}^* .

To sum up, a policymaker who maximizes social welfare will make the same choices as a policymaker who maximizes the market-based expectation of net benefits. However, the policymaker can only solve the social welfare problem if he knows the net benefit function, $B(a, x)$, state-dependent marginal utilities, $\{MUC_n(a)\}$, and household weights, $\{p_n\}_{n=1}^N$. The policymaker who seeks to solve for the market-based expectation of next benefits only needs to know the net benefit function B and the state prices q .

Returning to the example of an inflation-targeting central bank in Section 3.1, $B(a, x) = -(a + x - \pi^*)^2$, where $a + x = \pi$, the observed rate of inflation. The central bank would choose a so that $E^*[\pi]$ is π^* . In other words, the central bank would set the market-based expectation of inflation (equivalently, the price of a zero coupon inflation swap) equal to the inflation target.

4 Robustness

The equivalence between maximizing net expected benefits relative to market-based probabilities and maximizing households' ex-ante utility is based on a particular model. This section discusses several theoretical and practical concerns about the robustness of the equivalence result derived above.

4.1 Heterogeneity

The analysis above assumes that all households are identical, both ex ante and ex post. However, policy actions may affect some households differently from others. For example, some households may benefit from a given policymaker's action, whereas some may be made

worse off. A common example is the reduction of trade barriers that benefits many households by means of lower prices for the liberalized goods, but results in adverse outcomes for those involved in domestic production of those goods. In this subsection, we show that we can extend the above equivalence result as long as the policymaker has access to a complete set of (non-contingent) transfers across agents. These transfers allow the policymaker to target a desired distribution of wealth, regardless of the choice of a .

Suppose that there are I households. Household i has ex-ante utility:

$$\sum_{n=1}^N p_n^i U^i(c^i(x_n), x_n)$$

over (random) consumption c^i . We assume that p_n^i is positive for all (n, i) . (If we interpret p_n^i as a subjective probability, then we are assuming that households' beliefs are absolutely continuous with respect to one another.) The net benefit to household i , of a policy choice a , is given by $B^i(a, x_n)$.

As before, we contemplate two different games with three periods (trading, planning, and realization). In the trading period, we assume that households trade a complete set of financial assets. (We relax this assumption in the next subsection.) The games are distinguished by the policymaker's objective in the planning period. In the social welfare game, the policymaker chooses the action a so as to maximize a weighted average of household utilities, where the weights are given by $(\omega_1, \dots, \omega_I)$. In the market-based game, the policymaker chooses the action a so as to maximize the market-based expectation of net benefits. However, there is a key change in both games relative to the earlier subsection: we assume that, in the planning period, the policymaker can re-allocate consumption, contingent on a given state s occurring, across all households so as to maximize the ω -weighted average of household utilities.

In equilibrium, the households correctly anticipate the policymaker's choice of a and household-contingent transfers τ . Hence, the equilibrium outcome (a_{SW}^*, τ_{SW}^*) in the social

welfare game is now characterized by the policymaker's first order conditions:

$$\begin{aligned} \sum_{i=1}^I \sum_{n=1}^N \omega_i p_n^i MUC_n^i(a_{SW}^*, \tau^{i*}) \frac{\partial B^i}{\partial a}(a_{SW}^*, x_n) &= 0 \\ \omega_i p_s^i MUC_s^i(a_{SW}^*, \tau^{i*}) &= \omega_j p_s^j MUC_s^j(a_{SW}^*, \tau^{j*}) \text{ for all } i, j \\ \sum_{i=1}^I \tau_{SW}^{i*} &= 0 \end{aligned}$$

where $MUC_n^i(a, \tau^i)$ is household i 's marginal utility of consumption in state n given the policymaker's choice a and the transfer τ^i . In the market-based game, each household i has a multiplier $\xi_i(a_{MKT}^*, \tau_{MKT}^*)$ such that the equilibrium outcome $(a_{MKT}^*, \tau_{MKT}^*)$ is characterized by the first order conditions:

$$\sum_{i=1}^I \sum_{n=1}^N q_n^*(a_{MKT}^*, \tau_{MKT}^*) \frac{\partial B^i}{\partial a}(a_{MKT}^*, x_n) = 0 \quad (8)$$

$$q_n^*(a_{MKT}^*, \tau_{MKT}^*) = \frac{p_n^i MUC_n^i(a_{MKT}^*, \tau_{MKT}^{i*}) / \xi_i(a_{MKT}^*, \tau_{MKT}^*)}{\sum_{j=1}^I p_n^j MUC_n^j(a_{MKT}^*, \tau_{MKT}^{j*}) / \xi_j(a_{MKT}^*, \tau_{MKT}^*)} \quad (9)$$

$$\omega_i p_s^i MUC_s^i(a_{MKT}^*, \tau_{MKT}^{i*}) = \omega_j p_s^j MUC_s^j(a_{MKT}^*, \tau_{MKT}^{j*}) \text{ for all } i, j \quad (10)$$

$$\sum_{i=1}^I \tau_{MKT}^{i*} = 0 \quad (11)$$

where q^* is the vector of market-based probabilities.

It is straightforward to conclude from (9) and (10) that $\xi_i(a_{MKT}^*, \tau_{MKT}^*) \omega_i^{-1}$ is the same for all households. It follows then that (given sufficient curvature in U and B) that the equilibrium outcomes in the two games $((a_{MKT}^*, \tau_{MKT}^*)$ and (a_{SW}^*, τ_{SW}^*)) are equivalent because they both satisfy:

$$\begin{aligned} \sum_{i=1}^I \sum_{n=1}^N \omega_i p_n^i MUC_n^i(a^*, \tau^{i*}) \frac{\partial B^i}{\partial a}(a^*, x_n) &= 0 \\ \omega_i p_s^i MUC_s^i(a^*, \tau^{i*}) &= \omega_j p_s^j MUC_s^j(a^*, \tau^{j*}) \text{ for all } i, j \\ \sum_{i=1}^I \tau^{i*} &= 0 \end{aligned}$$

So the main result holds even when households are heterogeneous, as long as the policymaker has sufficient tools to target a desired distribution of wealth across households.

The household-contingent transfers τ allow the policymaker to ensure that households' relative marginal utilities of wealth are independent of a . Note that the optimal choice of τ requires the policymaker to have information beyond what is available in the market-based probabilities q^* . We can think of the choice of τ as being the responsibility of a fiscal authority who is charged with ensuring that the distribution of resources is socially optimal. That fiscal authority needs a considerable amount of information to be effective. As before, though, the policymaker who chooses a does not need any information beyond that embedded in market-based probabilities.

4.2 Robustness to Asset Trading Frictions

In the prior subsection, we assumed that all households can trade a complete set of contingent securities in the trading period. More generally, households may face restrictions on the assets that they can trade. We now turn to a discussion of the robustness of the main result to these kinds of frictions.

We begin with a generalization of the main equivalence result. Suppose that in the trading period, households trade an incomplete set of assets, subject to a variety of constraints. We make two assumptions about this set of assets and the trading restrictions. First, we assume that some (but not necessarily all) households can trade all available assets without constraints. This ensures that, in equilibrium, there is a unique market-based probability vector that is spanned by those assets. Second, we assume that there is some asset U (for unconstrained) which the households can trade without restrictions, and that the policymaker can redistribute that asset in the planning period. This last assumption allows the policymaker to undo the distributional effects of the policy choice a .

Now, consider the random marginal net benefit for household i associated with a policy

choice a :

$$\Delta^i(a) = \left(\frac{\partial B^i}{\partial a}(a, x_n)\right)_{n=1}^N$$

Let $(c^{i*}(a, \tau))$ be household i 's equilibrium consumption, given that all households believe that the policymaker will choose (a, τ) . Suppose that household i 's consumption $(c^{i*}(a, \tau) + \phi^i \Delta^i(a))$ is in household i 's consumption set for all ϕ^i sufficiently close to zero. Then, we say that the household i 's trading opportunities are **marginally policy-invariant** given (a, τ) . Marginal policy-invariance means that household i could have formed a portfolio of the traded assets to replicate the marginal impact of a change in a on its consumption,

The main equivalence result is readily generalized in the case in which all households' trading opportunities are marginally policy invariant. More formally, suppose that households' trading opportunities are marginally policy-invariant, given $(a_{MKT}^*, \tau_{MKT}^*)$ or given (a_{SW}^*, τ_{SW}^*) . Then, as long as B^i and/or U^i have sufficient curvature, $(a_{MKT}^*, \tau_{MKT}^*) = (a_{SW}^*, \tau_{SW}^*)$.

The proof is notationally intensive and so is in an appendix.¹⁴ However, the intuition behind the result is simple. Under marginal policy invariance, the random marginal net benefit $(\frac{\partial B^i}{\partial a})$ to any household i is spanned by the set of tradeable assets. Just as in the proof of the main result, the marginal impact of a perturbation in a on household i 's welfare (as weighted by the policymaker) is given by the market-based expectation of $(\frac{\partial B^i}{\partial a})$. It follows that the policymaker's optimal choice of a must set the market-based expectation value of the cumulative net benefits $(\sum_{i=1}^I \frac{\partial B^i}{\partial a})$ equal to zero.

We can now use this basic generalization to think through a variety of possible asset trading frictions.

4.2.1 Incompleteness of Asset Markets

Our baseline analysis of the main equivalence result assumes that asset markets are complete with respect to the relevant risks. We have seen in the prior subsection that this assumption

¹⁴That is available on request.

can be relaxed substantially: all that is needed is that the marginal net benefit for each household to be spanned by the set of available assets. However, if the marginal net benefit for household i is not spanned by the set of available assets, then the equivalence result typically does not apply. In this case, the policymaker's choice of a is expanding or shrinking the risk-sharing opportunities available to households. Financial market prices no longer serve as a good guide to the net benefit of the policymaker's policy choice.

If households are identical, so that risk-sharing is not an issue, it is possible to obtain a weaker version of the equivalence result in the form of a necessary condition for social optimality. Suppose that the set of (observed) financial assets does not span the random net marginal benefit $\frac{\partial B}{\partial a}(a^*)$. This means that there is a set S of market-based expectations of the marginal net benefits associated with that action. We can show that, if it is common knowledge that policymaker will choose the action that maximizes social welfare, then 0 lies in the set S . Put another way, if no element of S is zero, then we know that households believe that the policymaker is not going to make a socially optimal choice. Intuitively, one element of S is defined by the marginal utilities of consumption of the typical household and that element generates the first order conditions of the social welfare maximization problem.

4.2.2 Borrowing Limits and Short Sales Constraints

Many households seem to face significant restrictions on their ability to borrow or otherwise short-sell financial assets. The impact of these restrictions on the main equivalence result depends on whether the policy choice a allows households to circumvent these restrictions.

Suppose that, in the trading period, household i faces a binding short sale constraint on consumption in state s , and that:

$$\frac{\partial B_s^i}{\partial a}(a^*, \tau^*) > 0$$

In this case, the policymaker can relax the short-sales constraint by lowering a . This is an

example of a case in which household trading opportunities are not marginally invariant to the choice of policy, because:

$$c_s^{i*}(a^*, \tau^*) + \phi^i \frac{\partial B_s^i}{\partial a}(a^*, \tau^*)$$

is not in household i 's consumption set for ϕ^i less than zero. Market-based expectations are then typically not a reliable guide to policy choice.

Alternatively, the state-dependent short-sale constraint may arise from deeper economic considerations like private information or limited enforcement. In such instances, the policymaker will not be able to relax the short-sales constraint simply by varying a . It follows that:

$$\frac{\partial B_s^i}{\partial a}(a^*, \tau^*) = 0$$

and household i 's trading opportunities can still be marginally policy-invariant.

4.2.3 Limited Participation

We now examine the sensitivity of our main result to a particular kind of asset trading friction: limited participation in options markets. As we noted in section 2, relatively few households trade directly in the options markets used to compute market-based probabilities. Some attribute this lack of participation to these households facing such enormous (and ongoing) fixed costs that they can be seen as being essentially barred from participation (as modelled by Guvenen (2009), among others). This kind of trading restriction implies that households' trading opportunities are not marginally policy-invariant with respect to many policies. For example, suppose some households are unable to trade in inflation-based options. Then, the monetary policymaker's choice of accommodation affects the relative exposure of households to inflation risk. Maximizing market-based expectations of net benefits ignores these key effects.

Our own view is that it is implausible that some households are simply barred from trad-

ing in options markets. It is more likely that some households have decided not to participate, given the associated costs. Our current assessment is that the costs of participating (at least indirectly) in options markets are small, and so the non-participants' relative valuations of resources across future states is likely to be similar to that of the participants. But this preliminary assessment should be better informed by measures of the various participation costs.¹⁵

4.3 Policy Uncertainty

Up until now, we have assumed that the policymaker and households are symmetrically informed. This assumption implies that, during the trading period, households know what the policymaker will choose in the planning period. In other words, households face no policy uncertainty. But what if the policymaker has information available in the planning period that the households do not know in the trading period? In this subsection, we sketch an argument that the main equivalence result will generalize, as long as asset markets are sufficiently complete.

More specifically, return to the original case in which all households are identical. Suppose that the households' endowment, the net benefit function, and the utility function depend not only on a and x but also on a second random variable, Z . Suppose too that Z is realized after households trade contingent claims but before the policymaker acts. Because U , y , or B depends on the realization z of Z , the policymaker's optimal action for any given x will depend on z . Thus, in the trading period, the households cannot be certain how the policymaker will act for any given realization of x .

This uncertainty does not affect our results if markets are complete. Under complete markets, households trade claims in the trading period that are contingent on the realization of both x and z . Again, consider two games distinguished by the policymaker's objective.

¹⁵Unlike much of the earlier work in asset pricing, we are imposing limited a priori restrictions on preferences and subjective probabilities. Without those restrictions, we cannot reach clear conclusions about the magnitude of the barriers to participation, without knowledge of the actual costs themselves.

In the social welfare game, for each realization z of Z , the policymaker's strategy $a_{SW}^*(z)$ solves the problem

$$\max_a \sum_{n=1}^N p_n(z) U(y(x_n, z) + B(a, x_n, z), x_n, z),$$

where $p_n(z)$ is the positive weight on state x_n utility, conditional on Z equaling z . Suppose that in the market-based game, at the time of trading, the households believe that the policymaker's strategy is $\hat{a}(z)$ (as a function of the (as yet unrealized) random variable z .) Then, the market-based probability $q_n^*(z; \hat{a})$ of x_n , conditional on Z equaling z , is given by

$$q_n^*(z; \hat{a}) = \frac{p_n(z) MUC_n(\hat{a}(z); z)}{\sum_{m=1}^N p_m(z) MUC_m(\hat{a}(z); z)}, \quad (12)$$

where $MUC_n(a; z) = U_c(y(x_n, z) + B(a, x_n, z), x_n, z)$. Given the household beliefs \hat{a} , for any z , the policymaker chooses $a_{MKT}^*(z)$ so as to solve the following problem:

$$\max_a \sum_{n=1}^N q_n^*(z; \hat{a}) B(a, x_n, z)$$

Given these assumptions, any equilibrium strategy a_{SW}^* in the social welfare game satisfies:

$$\sum_{n=1}^N p_n(z) MUC_n(a_{SW}^*(z)) \frac{\partial B}{\partial a}(a_{SW}^*(z), x_n) = 0 \text{ for all } z \quad (13)$$

In an equilibrium to the market-based game, the household beliefs \hat{a} are equal to the policymaker's strategy a_{MKT}^* . It therefore satisfies:

$$\sum_{n=1}^N p_n(z) MUC_n(a_{SW}^*(z)) \frac{\partial B}{\partial a}(a_{SW}^*(z), x_n) = 0 \text{ for all } z$$

By using these first-order conditions, we can obtain the same equivalence result: the equilibrium outcome in the social welfare game is the same as in the market-based game.

As we noted above, we can obtain a weaker version of our equivalence result even if markets are incomplete with respect to z . For example, suppose that households can trade a

complete set of claims contingent on x , but no claims contingent on z . Then, there is a set of market-based probability measures that are consistent with the prices of the traded claims. We can show that, if the policymaker uses the socially optimal strategy a_{SW}^* , then there is some market-based probability measure that implies that the market-based expectation of the marginal net benefit of a is equal to zero. Again, the intuition is that some market-based probability measure is associated with the marginal utility of consumption of the households.

This last result — the socially optimal strategy maximizes net benefits relative to some set of market-based probabilities — is more informative when markets are more complete and there is less policy uncertainty. In the limiting cases of no policy uncertainty (Z has only one possible realization) or complete markets (claims are contingent on both x_n and z), only one market-based probability measure is consistent with the observed prices, and hence maximizing net benefits relative to that measure is not only necessary but also sufficient for optimality. At the other extreme, if there are no contingent claims on either x or z , any set of nonnegative numbers that sums to one is consistent with the observed prices, and no restriction comes from limiting ourselves to market-based probabilities. Thus, our claim that policymakers should set policy so as to maximize expected benefits relative to market-based probabilities is more useful when there is less policy uncertainty.¹⁶

5 Conclusion

Policy decisions affect the economy with a lag. Hence, policymakers need some way to gauge the relative likelihoods of future events. For an inflation-targeting central bank, the likelihoods of deflation on the one hand or high inflation on the other are important inputs to the policy-setting process. For a financial regulator, the likelihood of significant financial instability is needed to assess the risks associated with bank capital distributions. However, our analysis suggests that the typical approach of trying to discern the “true” probability

¹⁶From a normative perspective, our simple model certainly implies that policymakers should credibly reveal z if that can be done.

of events is typically inappropriate. Instead, policymakers should base their decisions on market-based probabilities or their equivalents. These probabilities encode households' ex ante marginal valuations of resources in different states of the world as well as their subjective likelihood of those states. An increase in the market-based probability of an outcome such as deflation could indicate that households consider it as more likely, or it could indicate that the costs associated with deflation have risen. Both of these changes should matter for a policymaker. And they do if the policymaker maximizes expected benefits relative to those market-based probabilities.

Our main equivalence result is that maximizing market-based expected social benefits is equivalent to maximizing social welfare. Our baseline derivation relies on relatively strong homogeneity and complete markets assumptions. We show, however, that the main equivalence result can be extended to cases in which households are heterogeneous and face substantial trading restrictions.

We describe cases in which the market-based expectations may be unreliable, because they cannot account for any beneficial (or adverse) impacts of policy on the risk-sharing opportunities available to households. Even in these situations, though, we see no reason for policymakers to turn to purely statistical forecasts as a guide to policy-making. Statistical expectations, based on mean square error minimization, ignore the relative scarcity of resources in different outcomes. But this relative scarcity does affect households' marginal valuations of resources in different outcomes - and so it should also affect policymakers' choices.

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