

Equilibrium Credit Spreads and the Macroeconomy*

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ABSTRACT

This paper develops a novel framework that brings together core ideas from asset pricing, capital structure, and macroeconomics within a tractable quantitative general equilibrium model with heterogeneous firms making optimal investment and financing decisions under uncertainty. Because credit risk premia is an important component of the cost of capital, movements in bond markets are propagated into the real economy generating a strong correlation between spreads and macro aggregates and producing large and skewed business cycles, with sharper, more pronounced, recessions.

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1 Introduction

The Great Recession of 2008-09 offers a primary example of the important role that fluctuations in credit risk play in the aggregate economy. Unfortunately these developments also exposed the current need for new state of the art models suitable to understand the joint behavior of credit risk, financial prices, and the key macroeconomic aggregates.

Attempts to provide an integrated discussion of these issues in a modern setting have gathered speed recently but most papers avoid some essential features of the data. Many general equilibrium models used in macroeconomics attempt to explain the cyclical behavior of credit markets quantities and their correlation with macroeconomic aggregates but largely abstract from variations in risk premia and asset prices.¹ By contrast, macro models that exploit the role of asset prices and risk premia explicitly, generally ignore the role of credit markets entirely.² Parallel efforts by financial economists have instead focused on explaining the magnitude of credit risk by linking it, initially, with the financing decisions of firms and, more recently, with exogenous movements in risk premia and aggregate factors.³

This paper offers a novel framework that brings together many of the core ideas from asset pricing, capital structure, and macroeconomics within a tractable general equilibrium model with heterogeneous firms that make optimal investment and financing decisions under uncertainty. Macroeconomic quantities are obtained by aggregating across the optimal decisions of each firm and required to be consistent with consumption and savings decisions of a representative household/investor. By integrating corporate investment and capital structure decisions into a modern asset pricing model, we endogenously link movements in aggregate quantities such as investment and output to plausible movements in the prices of stocks and bonds.

The joint endogeneity of financial prices and macro quantities implies that both the intertemporal elasticity of substitution and risk aversion matter separately for aggregate quantities, a result

¹Classic examples include Carlstrom and Fuerst (1997), Kyotaki and Moore (1997) and Bernanke, Gertler, and Gilchrist (1999). Some recent examples are Lorenzoni and Walentin (2007) and Philippon (2008).

²Some examples are Jermann (1998), Tallarini (2000), Boldrin, Christiano and Fischer (2001), Uhlig (2006), Guvenen (2009) and Loechster and Kaltenbrunner (2010).

³Building on Leland (1994) recent quantitatively successful contributions include Hackbarth, Miao, Morellec (2006), Chen, Collin Dufresne, Goldstein (2008), Chen (2008), Bhamra, Kuhn, and Strebuaev (2008).

that contrasts strongly with extant macro literature where the role of risk aversion and thus risk premia is largely ignored.⁴ As a result we can generate plausible movements in credit prices that are largely driven by fluctuations in risk premia and not by changes in average default rates alone.

Because credit spreads are an important component of the cost of capital for firms, risk premia in corporate bond markets are propagated into the real economy so that our model generates the strong correlation between credit spreads and macro aggregates observed in the data. Thus, the credit risk premium emerges as a common link between credit, equity markets and macroeconomic aggregates and its movements provide a novel and powerful amplification mechanism for macroeconomic fluctuations.⁵

While qualitatively we are able to replicate all the predicted cyclical patterns in macro and financial variables our parsimonious model also performs well quantitatively. We show that we can match both the level and volatility of key macro aggregates as well as that of equity and bond returns with realistic leverage and default levels. Economic fluctuations become amplified and also asymmetric with sharper recessions associated with pronounced spikes in both default rates and credit spreads.

In our model credit spreads predict output and investment growth because they capture important information about the cross-sectional distribution of firms as well as independent shocks to credit supply and not just because they are correlated with aggregate productivity. This suggests that credit spreads may also be of independent use for policy makers seeking to stabilize the economy.⁶

Recently other authors have tried to embed more realistic credit markets into aggregate equilibrium models. Jermann and Quadrini (2010) show that credit shocks are necessary to generate the observed behavior of US economy. Their model is significantly different in that a firm can

⁴An elegant example is Tallarini's (2000) separation result where risk aversion governs asset prices alone while the intertemporal elasticity of substitution determines economic aggregates.

⁵Examples of the ability of credit spreads to forecast economic activity include studies by Stock and Watson (1991), Lettau and Ludvigson (2004), Gilchrist and Zakrajsek (2008), and Mueller (2008). Keim and Stambaugh (1986), Schwert (1989) and Fama and French (1992) study the link between credit spreads and equity markets. There is also a complementary literature documenting that asset prices more broadly have significant forecasting power for macroeconomic variables - see Ang, Piazzesi, Wei (2006) for a recent study and Stock and Watson (1999) for a classic contribution.

⁶Gomes and Schmid (2010) study the optimal response of monetary policy to variations in credit risk in a similar setting.

make intra period loans that are subject to stochastic collateral constraints and face adjustment costs to distributions. Brunnermeier and Sannikov (2010) build a continuous time macroeconomic model with a more detailed financial sector. As in other classic papers by Kyotaki and Moore (1997), Bernanke et al (1999) and Cooley et al (2004) the authors show how the equilibrium could be fragile in that large negative shocks can be amplified by endogenous variations in the price of capital goods. Compared to our paper here, both of these models focus on a different set of financial frictions and offer a more detailed institutional setting. On the other hand they both consider representative firm frameworks and largely abstract from variation in risk premia.⁷

Methodologically, our model is also related to recent quantitative analysis of various aspects of the interactions between frictions at the corporate level, asset prices and macroeconomic fluctuations and to that on equilibrium asset pricing with heterogeneous firms.⁸

The rest of the paper is organized as follows. Section 2 describes our general equilibrium model and some of its properties, while Section 3 discusses some of the issues associated with solving it numerically. A detailed discussion of our findings is provided in Section 4, before we conclude.

2 The Model

In this section we describe a general equilibrium with heterogeneous firms that are financed with both debt and equity. Debt is used because of its tax benefits. Although our economy is often stylized the model presented here preserves tractability and economic intuition, while remaining suitable for a detailed quantitative analysis.

2.1 Production Sector

The production sector of the economy is made of a continuum of firms that differ in their productivity, size and leverage among other characteristics. In characterizing the problem of firms we

⁷Still more recently Miao and Wang (2010) extend our framework to allow for endogenous labor supply, while Gourio (2010) introduces disaster risk in a simplified setting where firms are only alive for two periods. This ensures that firm heterogeneity plays no role in equilibrium and dramatically speeds up computation on the model.

⁸Recent examples of the former include Bolton, Chen, Wang (2010), Hugonnier, Malamud, Morellec (2010), Eberly, Rebelo and Vincent (2010), Eberly and Wang (2010), and Ai, Croce, Li (2010). For the latter see Berk, Green and Naik (1999) and Gomes, Kogan, and Zhang (2003).

take, for now, the stochastic discount factor for the economy as given. Later we show how this is determined in general equilibrium by the optimal consumption and savings decisions of households.

2.1.1 Technology and Investment

All firms produce the same homogeneous final good that can be used for consumption or investment. The production function denoting the instantaneous flow of output is described by the expression⁹:

$$y(x, z, k) = xzk \tag{1}$$

where x and z denote the values of aggregate and firm specific productivity, respectively. The behavior of these follows a first order autoregressive process with normal innovations:

$$\begin{aligned} \log(x_t) &= \rho_x \log(x_{t-1}) + \sigma_x v_{xt}, \\ \log(z_t) &= \rho_z \log(z_{t-1}) + \sigma_z v_{zt}, \end{aligned}$$

where both v_{xt} and v_{zt} are independently and identically distributed shocks drawn from a standard normal distribution.¹⁰

The variable k denotes the firm's productive capacity. This capacity is installed when the firm begins to operate and remains fixed throughout the life of the firm. However existing firms are required to incur an amount δk per year in maintenance costs.

Together the assumptions about the production technology imply that the aggregate economy experiences stochastic and persistent variation in its growth rate over time through fluctuations in aggregate productivity, x_t .

⁹For simplicity of exposition we ignore the role of labor here. It is conceptually straightforward to introduce an endogenous labor supply but at the cost of some clarity of exposition and, we believe, without offering significant additional insights. Miao and Wang (2010) extend our model to include a variable labor input.

¹⁰A small but growing literature has begun to investigate the importance of non-normal or disaster shocks or even time variation in volatility (recent examples include Drechsler and Yaron (2010) and Gourio (2010)). Although these features are absent from our shock processes, our model endogenously generates them in the aggregate quantities.

2.1.2 Firm Entry and Financing

New firms enter the market and start production if market conditions are sufficiently attractive. Entering firms draw the initial realization of the idiosyncratic shock z from its long-run invariant distribution, denoted $G(z)$. For simplicity we assume that this value is only observed *after* entry. We further assume that entering firms are not immediately productive.

Entering firms must invest to build their productive capacity, k . This initial investment can be financed with either debt or equity finance. Debt takes the form of a consol bond that pays a fixed coupon bk as long as the firm is in existence and does not default on its obligations. Defining the coupon as bk allows us to interpret b as a measure of book leverage for the firm.¹¹

2.1.3 Equity Value and Exit

Given production and leverage the firm's per-period operating profits are given by the expression:

$$\pi(x, z, b, k) = (xz - b - \delta)k \quad (2)$$

To economize on notation we henceforth use $s = (x, \mu)$ to denote the aggregate state of the economy. As we will see below this includes the current state of aggregate productivity, x and the cross-sectional distribution of firms, denoted μ .

Taking the households pricing kernel, M , as given, the firm's equity value, $V(s, z, k, b)$ *after* entering the economy is determined through the Bellman equation:

$$V(s, z, b, k) = \max\{0, (1 - \tau)(1 - \lambda)(xz - b)k + E[M(s, s')V(s', z', b, k)]\} \quad (3)$$

where as usual we have used the notation s' to denote the future value of s .

The variable τ is the marginal tax on corporate profits, adjusted for taxes on distributions and personal interest income, and λ is an indicator function that takes the value of 0 when equity distributions are positive and captures the costs of issuing any new equity such as underwriting

¹¹Although we associate new (net) investment to firm creation it is relatively easy to reinterpret the model and think of these new arrivals as simply new (self-financed) projects within existing firms.

fees.

Our assumptions about the nature of cash flows imply that the equity value is linear in k . Thus we can work instead with the market to book ratio:

$$Q(s, z, b) = V(s, z, b, k)/k. \quad (4)$$

The Bellman equation (3) implies that equity holders will default on their debt obligations when equity value falls to zero. This yields a default cutoff value for the idiosyncratic shock, $\bar{z}_d(s, b)$, such that the firm will default whenever $z < \bar{z}_d(s, b)$. Formally, we define this default threshold with the condition:

$$z_d(s, b) = \min\{z : Q(s, z, b) = 0\} \quad (5)$$

All individual firm decisions depend on the aggregate state of the economy, s , which needs to be determined in general equilibrium. Nevertheless since profits are linear in x and z , it is immediate that:

- the market to book ratio, $Q(s, z, b)$, is increasing in x and z , and declining in leverage b , and
- the default cutoff, $z = z_d(s, b)$, is decreasing in x and increasing in the coupon b .

Limited liability endows equity with an exit option that increases the value of uncertainty. Formally this means that $Q(\cdot)$ will be convex in both x and z .

More importantly however, note that when $M(\cdot)$ is constant the default cut-off $z_d(\cdot)$ is (log) linear in x as both shocks enter symmetrically in the value function. With risk averse investors however, $M(\cdot)$ will be tied endogenously to x , and the default threshold will respond differently to positive and negative shocks in aggregate productivity. This will be one of the keys to generate asymmetric behavior of aggregate default rates over the business cycle.

2.1.4 Value of Debt and Credit Spreads

Bondholders receive the coupon payment bk when the firm does not default. In default, the firm's assets (capital plus current cash flows) are liquidated and the proceeds are used to pay the creditors.

A fraction ϕ however is lost in liquidation so that creditors recover an amount equal to $(1 - \phi)(1 - \delta + xz)k$.¹²

Given our assumptions it follows that the market value of debt, $B(s, z, b)$, normalized per unit of capital k , can be defined by the recursion:

$$B(s, z, b) = [b + E[M(s, s')B(s', z', b)]]\chi_{\{z > \bar{z}_d\}} + (1 - \phi)(1 - \delta + xz)(1 - \chi_{\{z > \bar{z}_d\}}) \quad (6)$$

Here χ is an indicator function that takes the value of 1 when the firm survives so that $z > \bar{z}_d(s, b)$.

Costly bankruptcy procedures are empirically plausible but also useful for equilibrium asset pricing models since they imply that investment is, effectively, only partially reversible. In general equilibrium this feature adds to the variation in consumption growth and thus to the market price of risk during recessions which exacerbates the underlying variation in equilibrium asset prices.

It is now straightforward to define the yield $y(s, z, b)$ on corporate debt as:

$$y(s, z, b) = b/B(s, z, b) \quad (7)$$

To construct measures of the credit spread for this economy we compare this yield with that on a riskless bond of identical maturity. Formally then, our measure of credit spreads is:

$$cs(s, z, b) = y(s, z, b) - y_f(s, z, b) \quad (8)$$

where $y_f(s, z, b)$ is the yield on a bond of identical characteristics but assuming no default occurs. This risk free bond is then similar to a AAA rated bond instead of a pure treasury.¹³

As before since profits are linear in x and z , and given the properties of $z = z_d(s, b)$ it is immediate to show that:

¹²Many models of leverage where firms hold no assets assume that bondholders keep a fraction of the continuation value of the firm. This makes less sense in a model with real physical assets and it also implies that default and exit are separate concepts. For credit pricing the specific assets recovered is largely irrelevant. All that really matters is the actual magnitude of the losses upon default relative to face value and these can be easily fitted by specifying the appropriate value for ϕ . We discuss this procedure in more detail in Section 3.2.

¹³Although we can readily construct other measures of credit spreads in the model we prefer this measure since empirically part of the spread between treasuries and corporate yields seems to be driven by liquidity and other microstructure issues that we ignore in this model.

- the market value of debt, $B(s, z, b)$, is increasing in x and z , and declining in leverage b , and
- the credit spread, $cs(s, z, b)$, is decreasing in x and z , but increasing in the coupon b .

2.1.5 Entry and Investment

Each period a mass of potential new entrants arrives in the economy. Each of these firms is endowed with an investment opportunity that expires at the end of the current period.

Given the expression for equity and debt value, the expected value of entry for any of these firms, is given by the expression:

$$A_0(s, b) = \int Q(s, z, b) + B(s, z, b)dG(z) \quad (9)$$

New entrants build up required productive capacity, k , by incurring a unit cost of installation e .

2.1.6 Optimal Capital Structure

Each individual firm finances their initial purchases of capital using a mix of debt and equity. This initial capital structure is chosen to maximize the expected total value of the firm (i.e. debt plus equity). Formally this optimal ex-ante value of the firm $A_0(s)$ is given by the expression:

$$A_0(s) = \max_{b \geq 0} \left\{ \int Q(s, z, b) + B(s, z, b)dG(z) \right\} \quad (10)$$

It follows from this that each potential entrant will enter the economy if and only if the setup cost e is less or equal the ex ante firm value $A_0(s)$. Formally then entry occurs whenever

$$e \leq \bar{e}(s) = A_0(s) \quad (11)$$

The properties of $Q(\cdot)$ and $B(\cdot)$ imply that both entry cutoff, $\bar{e}(s)$, and the optimal leverage choice, $b = \bar{b}(s)$ are increasing in x . As we will see below this implies that in equilibrium relatively costly projects are only adopted in good times while during recessions only the efficient, low e , projects are implemented. Similarly optimal (book) leverage will also be higher for firms entering

during booms.

2.1.7 Summary of Firm Decisions

To summarize the optimal behavior of each individual firm is characterized by:

- The optimal entry cutoff, $\bar{e}(s)$, implied by condition (11),
- An optimal leverage choice, $b = \bar{b}(s)$ implied by (10), and
- An optimal default cutoff, $z = z_d(s, b)$, implied by (5).

2.2 Aggregation

To characterize the general equilibrium of the model we start by aggregating the optimal policies of each individual firm to construct aggregate quantities for our economy.

2.2.1 Cross-Sectional Distribution of Firm

We begin by defining $\mu_t = \mu(z, b)$ as the cross-sectional distribution of firms over leverage and idiosyncratic shocks at the beginning of period t . Our timing means that $\mu(z, b)$ is constructed before any entry and exit decisions and as such is just an accounting measure. Since capital k is installed upon entry and does not vary afterwards, firm heterogeneity is captured by their different choices of b and draws of z so that $\mu(\cdot)$ is a two-dimensional object.¹⁴ It should be apparent that the distribution $\mu(\cdot)$ will also move over time according to the aggregate state of the economy. Henceforth we drop time subscripts and use instead the notation μ' to denote next-period's measure of firms.

We can now define the total mass of firms alive at the beginning of period t , as:

$$F(s) = \int d\mu(z, b) \tag{12}$$

¹⁴In practice this means that if we discretize the state space all cross-sectional heterogeneity (in period t) can be captured with a matrix of size $nz \times nb$ where each entry represents the mass of firms accumulated over a point in the state space.

Our notation $F(s)$ can be slightly misleading. Like $\mu(z, b)$ itself, $F(s)$ is constructed before any entry and exit decisions. Thus, although it is constructed after the current aggregate state is observed (hence $F(s)$), it really does not depend on this state in any way.

As we have seen above, during period t a fraction of firms will optimally choose to default based on the realization of shocks and their own level of leverage. As we have seen above, mathematically default will be concentrated over the subset of the state space where $z < z_d(s, b)$. This means that we can define the default rate, or the fraction of defaulting firms in period t as:

$$D(s) = 1 - \frac{\int \chi d\mu}{F(s)} \quad (13)$$

where as before $\chi = 1$ in the region where $z \geq z_d(s, b)$. Since the default threshold, $z_d(s, b)$, is decreasing in x this default rate will be countercyclical and as discussed above will generally respond asymmetrically to positive and negative shocks in x .

2.2.2 Aggregate Investment and the Price of Capital

At any point in time, there is a mass of potential entrants that may purchase k units of capital at cost e . We assume that individual investment costs are randomly drawn from a uniform measure H . To ensure balanced growth this potential mass of entrants must remain proportional to the number of firms in the economy, i.e. $H = hF(s)$. Of course, at any point in time however, only the firms that draw a realization of e below \bar{e} will find it optimal to start producing.

Net aggregate investment in the economy is given by the sum of the initial setup costs for the entering firms net of the disinvestment by exiting firms. For gross investment we also need to add the maintenance expenditures of all firms. Given the optimal behavior of individual firms, aggregate investment will be equal to:

$$I(s) = \int_0^{\bar{e}(s)} e k dH + \int \delta k d\mu - \int (1 - \chi) k d\mu \quad (14)$$

$$= (h\bar{e}^2(s) + \delta - D(s)) k F(s), \quad (15)$$

Since the entry threshold, $\bar{e}(s)$, is increasing and the default rate, $D(s)$, is decreasing in x , aggregate

investment will naturally be procyclical. The quadratic term implies that the (aggregate) marginal price of capital goods rises in good times as more costly projects are adopted. This feature produces similar results to those obtained in simple aggregate macro models with ad-hoc adjustment costs. The costly transformation between consumption and capital goods is also crucial to generate sufficient variation in the price of financial assets in production economies.¹⁵

We can introduce additional variation in the average quality of available projects and affects the volatility of investment expenditures over the business cycle by allowing for the entry mass H to be more or less responsive to the underlying shocks. For example, making the arrival rate $h = h(x)$ procyclical by setting $h'(x) > 0$, implies that there are more good projects/ideas in good times and makes aggregate investment more responsive to the underlying shocks.¹⁶

2.2.3 Other Quantities

Other aggregate quantities are relatively straightforward to define. Aggregate output can be defined as:

$$Y(s) = \int xzk d\mu = xzkF(s) \quad (16)$$

and the losses associated with bankruptcy are given by:

$$\Phi(s) = \int (1 - \chi)\phi(1 + xz)k d\mu \quad (17)$$

Finally we can also construct the aggregate market value of corporate equity and debt respectively with the expressions:

$$V(s) = k \int Q(s, z, b) d\mu \quad (18)$$

and

$$B(s) = k \int B(s, z, b) d\mu \quad (19)$$

These definitions for the aggregate quantities allow us to identify the aggregate state of our

¹⁵An early discussion is offered in Jermann (1998). For recent applications in similar settings see Kaltenbrunner and Lochstoer (2010) and Croce (2010). Our formulation here is closer to Gomes, Kogan and Zhang (2004).

¹⁶Although we do not explore it here, we can also allow for $h(\cdot)$ to depend on an a separate, investment-sector shock, as in Greenwood, Hercowitz and Huffman (1988) or Fisher (2006) for example.

economy s more specifically with the pair (x, μ) . Intuitively this means that all aggregate quantities and prices will depend not only on the average state of productivity (or profits) but also on the cross-sectional variation in firm productivities and balance sheet positions.

2.3 Households

We now close our general equilibrium model by describing in detail the behavior and constraints faced by the households/investors. We assume that our economy is populated by a competitive representative agent household, that derives utility from the consumption flow of the single consumption good, C_t . This representative household maximizes the discounted value of future utility flows, defined through the Epstein-Zin (1991) and Weil (1990) recursive function:

$$U_t = \{(1 - \beta)u(C_t)^{1-1/\sigma} + \beta \mathbf{E}_t[U_{t+1}^{1-\gamma}]^{1/\kappa}\}^{1/(1-1/\sigma)}. \quad (20)$$

The parameter $\beta \in (0, 1)$ is the household's subjective discount factor and $\gamma > 0$ is the coefficient of relative risk aversion. The parameter $\sigma \geq 0$ denotes the elasticity of intertemporal substitution and $\kappa = (1 - \gamma)/(1 - 1/\sigma)$.

Our household invests in shares of each existing firm as well as riskless bond which in zero net supply and earns a period rate of interest r_t . We also assume that there are no constraints on short sales or borrowing and that households receive the proceeds of corporate income taxes as a lump-sum rebate equal to:

$$T(s) = \int \tau x z k d\mu = \tau x z k F(s) \quad (21)$$

Given these assumptions the equilibrium stochastic discount factor for our economy between two adjacent periods is defined by the expression:

$$M_{t,t+1} = \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-1/\sigma} R_{W,t+1}^{1-1/\kappa} \right]^\kappa. \quad (22)$$

where

$$R_{W,t+1} = \frac{W_{t+1} + C_{t+1}}{W_t}.$$

is the return on total household wealth, including bonds and tax proceeds.

As is well known, the absence of arbitrage implies that all gross asset returns in this economy will satisfy:

$$E_t[M_{t+1}R_{i,t+1}] = 1, \tag{23}$$

for all assets i .

2.4 Equilibrium

We have shown how investor behavior determines the equilibrium stochastic discount factor, $M_{t,t+s}$, given household wealth. Earlier we described optimal firm behavior given the stochastic discount factor and shown how it determines aggregate investment and output as well as household wealth. Ensuring consistency between these two pieces of the economy requires that aggregate consumption by households is equal to aggregate production, net of investment and deadweight losses.

Formally our competitive equilibrium can then be constructed by imposing the additional consistency condition:

$$C_t = C(x, \mu) = Y(x, \mu) - I(x, \mu) - \Phi(x, \mu) \tag{24}$$

This ensures that the stochastic discount factor used by each firm corresponds to that implied by optimal household behavior.¹⁷

Finally we also need to specify a law of motion for the cross-sectional measure of firms over time. Given optimal firm policies this measure satisfies the following relation:

$$\mu(z', b') = Prob(z_{t+1} < z', b_{t+1} < b') = \int \chi N(z'|z) d\mu(z, b') + G(z') \kappa_{\bar{b}(s)=b'} \tag{25}$$

where $N(z'|z)$ is the cumulative normal distribution of z' conditional on z , and $\kappa(\cdot)$ is an indicator function that takes the value of 1 only if $\bar{b}(s) = b'$ and 0 otherwise. The first term in equation (25)

¹⁷We follow the convention of considering that bankruptcy costs are deadweight losses but in a general equilibrium model this is a somewhat debatable choice. Some of these costs might be in the form of legal and accounting fees that accrue to other types of firms in the economy.

aggregates the surviving firms with current leverage b' that move to states z' next period, while the second term adds the entering firms, only if they choose optimal leverage equal to b' . Note that this equation recognizes that firm leverage does not move over time.

Figure ?? illustrates the pattern of this cross-sectional distribution in both good and bad times. Although much of the underlying variation is (log) normal the equilibrium distribution reflects both the effects of truncation by exit and lumpy additions from new entrants. Typically there is less mass over high coupon, b , states in bad times, both because these firms are prone to default and also because potential entrants will typically choose lower leverage in recessions.

3 Computation

We now offer a brief description of our approach to solve the model in section 2 and the choice of parameter values. Although the model is relatively parsimonious, the computation of the competitive equilibrium is difficult because the cross-section measure of firms $\mu(\cdot)$ changes over time.

3.1 Computation

Computing the competitive equilibrium requires the following three basic steps:

- Given an initial stochastic discount factor $M_{t,t+s}$ solve the problem of each individual firms and determine the optimal level of entry and default
- Aggregate individual firm decisions and use the consistency condition (24) to compute aggregate consumption and wealth
- Ensure that the implied aggregate quantities are consistent with the initial process for $M_{t,t+s}$.

Convergence of this procedure delivers the equilibrium values for all individual and aggregate quantities in the model. The appendix described this procedure in more detail.

3.2 Parameter Choices

3.2.1 Basic Parameters

Our benchmark model requires us to specify only eleven parameters: three for preferences, five for technology, and three related to institutional or policy. Table 1 reports our choices for.

We start by setting the preference parameters β , γ and σ to ensure that the model matches the key properties of the risk free rate and the aggregate equity premium in the economy. Several studies have now shown how to combine time non-separable preferences and persistent shocks to aggregate growth to produce these results. Here we show how these dynamics can arise endogenously in general equilibrium and so our parameter values are quite similar to several papers in this literature.¹⁸

For the technology parameters we start by setting the depreciation rate of capital to 5% per year. The volatility and persistence of the aggregate productivity process are set to $\rho_x = 0.96$ and $\sigma_x = 0.015$. These values are largely in line with other macro studies and ensure that we match the volatility and persistence of output growth in the data. The parameters for idiosyncratic shocks determine the amount of cross-sectional variation in firm heterogeneity. Since we are especially concerned with the role of leverage and credit spreads in our economy we set these parameters to match the unconditional means of both of these variables. This implies that $\rho_z = 0.9$ and $\sigma_z = 0.18$

Finally we need the three institutional parameters. The marginal corporate tax rate, τ is set to 20% to reflect the effect of individual taxes on distributions and interest on the effective marginal tax rate. We choose the bankruptcy cost parameter, ϕ to generate average recoveries on defaulted bonds around 75% of face value. This is close to the numbers reported in Warner (1977) and captures both direct and indirect costs of the bankruptcy process. Formally then we set the value of ϕ so that in default:

$$(1 - \phi)(1 - \delta + xz) = 0.75B(s_0, b, z) \tag{26}$$

where $B(s_0, b, z)$ is the value of debt (relative to k) initially raised by the firm on average. Finally

¹⁸For an early example of this approach and findings see Bansal and Yaron (2004). Lochstoer and Kaltenbrunner (2009) and Croce (2010) also show how production economies can generate these results.

we use 2% to denote the marginal cost of issue equity, λ , an estimate that is similar to that used in Gomes (2001) and estimated in Hennessy and Whited (2006) and to popular measures of underwriting discounts in the US.

3.2.2 Additional Parameters

Our model is flexible enough to accommodate several variations and in this paper we also explore three of them. First, as we have discussed earlier, in our baseline model we modify the arrival rate of new firms to generate additional variability in aggregate investment, relative to consumption. Although we do not introduce investment specific shocks, as in Greenwood et al (1987), we do allow for a stochastic arrival rate that depends on the value of aggregate productivity. Specifically we set $h = h(x)$ with $h'(x) > 0$ and an elasticity equal to 0.25. This implies that there are more potentially good projects/ideas in good times which, as we will see, makes aggregate investment more responsive to the underlying shocks.

Separately, we also consider is the introduction of credit market specific shocks, along the lines of Jermann and Quadrini (2010). To do this we assume that recovery rates in bankruptcy fluctuate over time, perhaps as a result of shocks to liquidation values or “liquidity”.¹⁹ Equation (6) shows how fluctuations in the recovery rate, ϕ , directly affect the relative price of credit to the firm so that these liquidity shocks act as effective shocks to credit supply.

Formally we assume that ϕ can take two values: a benchmark value of 0.25 reflecting average bankruptcy costs and an extreme (but rare) value of 0.75 that occurs during liquidity crisis. We also assume that ϕ evolves over time according to a two-state Markov chain with the following transition probabilities:

$$P[\phi_{t+1} = 0.25 | \phi_t = 0.25] = 0.98, \tag{27}$$

$$P[\phi_{t+1} = 0.75 | \phi_t = 0.75] = 0.5, \tag{28}$$

In practice this means that liquidity crisis are both rare and temporary.

¹⁹Einsfelt and Rampini (2007) show these types of shocks can be important to explain measured variation in individual firm investment over time.

Finally, and for comparison purposes, we also study a simplified version of the model where we impose that the firm's initial investment must be entirely financed with equity.

4 Findings

We are now ready to describe our quantitative findings. We begin by summarizing the basic properties of the model by reporting the unconditional means and volatilities of the major aggregate quantities and asset prices. We then examine the model's implications for the behavior of financial variables over the business cycle and compare those with the available empirical evidence. Finally we investigate the role of credit spreads in predicting future movements in both macro quantities.

To construct the statistics reported below we solve the model by numerical dynamic programming as detailed in Section 3. We then simulate the implied equilibrium policies at quarterly frequency to construct 1000 independent panels of 59 years each and report averages across all simulations. Unless otherwise noted we always report the relevant empirical moments for the sample period between 1951 and 2009.

4.1 Basic Properties

The first panel in Table 2 reports the volatility of the key macroeconomic variables as well as the share of investment in GDP for our benchmark model as well as several alternative parameterizations. The baseline parameter choices yield a very close match between the model and the data along these dimensions. Not only is the share of investment (and hence consumption) plausible but both variables also exhibit as much variability as in the actual data.

The lower panel in Table 2 documents the implied properties of the model for the unconditional means and volatilities of the risk free rate and the equity premium. Our benchmark model also does a good job in replicating these facts. Both the level of the risk free rate and the equity premium are very close to those observed in the data, and this match does not require the very large movements in the risk free rate often associated with habit preferences. This is because the persistent stochastic variation in growth rates generated by our model increases the household's precautionary savings thereby lowering equilibrium interest rates.

While Bansal and Yaron (2004) have shown that accounting for long run movements in consumption and dividends, combined with preferences for a early resolution of uncertainty, delivers realistic risk premia in an endowment economy setting, this has proved harder to implement in general equilibrium production economies (Kaltenbrunner and Lochstoer (2010), Campanale, Castro and Clementi (2009), Croce(2010)). This is because in a production economy, general equilibrium restrictions often tie dividends very closely to consumption, while empirically, dividends are much more volatile than consumption.

4.1.1 The Role of Leverage and Persistence

Here however financial leverage (endogenously) breaks the tight link between dividends and consumption and renders dividends an order of magnitude more volatile. This allows us to generate a more realistic amount of stock market volatility and is crucial in matching the aggregate equity premia.²⁰

As Table 2 shows an all equity version of our economy does not generate enough volatility in equity returns and is not capable of matching the observed equity premium. The same occurs if the shocks to the stochastic growth rate are not sufficiently persistent. As we already discussed in this case the model also produces an unrealistically high equilibrium risk free rate. Persistence has profound effects on the asset pricing implications of our model but does not alter significantly the level or volatility of the main macro quantities, at least not at these relatively short horizons.

Predictably, financial leverage increases the volatilities of both financial prices and macro quantities. Compared with an all equity economy, unconditional volatility increases by about 25% to 30% in the baseline, levered, economy, which is roughly in line with the financial accelerator model in Bernanke et al (1999). However as we discuss in more detail below our underlying transmission mechanism is different and relies on movements in credit risk premia.

²⁰Although we do not report these numbers here the model also generates a slow moving pattern in leverage (Lemmon, Roberts and Zender (2008)) and the long run movements in aggregate dividends observed in the data (Bansal and Yaron (2004)).

4.1.2 Investment Shocks

As Table 2 shows modifying the arrival rate of new projects affects the relative volatility of consumption and investment because it changes the marginal cost of allocating resources to the production of capital goods, i.e. creating new firms in the economy.

When $h'(x) = 0$ the arrival rate of new projects does not vary with business cycle conditions and the investment volatility is too low, while consumption volatility rises. This is remediated in our baseline case, because more good projects arrive in good times making it easier to transform consumption into capital. The result is that more firms are created and aggregate investment volatility raises. To see this formally, recall that investment arising from firm creation equals $I = h\bar{e}^2kF(s)$, where $F(s)$ is the mass of firms at the beginning of period t . If the behavior of the entry cutoff \bar{e} remains unaffected, any increase in h increases average investment expenditures. In equilibrium however \bar{e} actually becomes more cyclical when $h'(x) > 0$ reflecting the fact that investment becomes more attractive in good times.

General equilibrium implies that equity prices are also affected. One way to see this has to do with the implied movements in the endogenous stochastic discount factor since, in equilibrium, the cyclical nature of investment prices, \bar{e} , is reflected both on expected consumption growth and on the returns to wealth. In turn this means that $h'(x) > 0$ makes equity -and corporate debt too- riskier.

Simplified versions of our baseline model without issuance costs for equity or that introduce credit shocks have only a negligible impact on these core macro and finance statistics. Their effect is instead felt on the credit market statistics and the conditional responses of economy to various shocks.

4.2 Credit Market Statistics

Table 3 shows the basic properties of the key credit market statistics as well as its empirical counterparts. Note that these statistics are all based on the average properties of the cross-sectional distribution of firms. As before we report the numbers both for our baseline parameter choices and for a few alternative specifications to illustrate some of mechanisms behind our findings.

The benchmark model almost exactly matches the cross-sectional average market leverage (the

ratio of book leverage to the value of market equity plus book leverage). Moreover it also produces a realistic level for credit defaults and the average credit spread.²¹

4.2.1 The Role of Credit Risk

The combination of low default rates and substantial tax benefits to debt is often interpreted as evidence that firms chose sub-optimal levels of leverage in the data. Here we can easily match the observed leverage ratios for two reasons. The first is that our model produces a large credit risk premium. As in recent work by Bhamra et al (2008) and Chen (2008), we exploit the fact that default occurs in periods of very high marginal utility, thereby significantly increasing the effective cost of default and the required compensation to bondholders. Here, however, this link is an explicit product of our general equilibrium structure and not an exogenously assumed covariance structure between default and the market price of risk.

Tying macroeconomic fluctuations to variation in default rates is the key component of the large credit spreads and the reason we can match the data along this dimension. Our baseline model generates a credit spread of 104 basis points with a default rate of only 1.42% while a risk neutral valuation would imply a credit spread of, at most 36 basis points.²² Unlike other macro models with credit markets, it is then the credit risk premium, induced by the (endogenous) covariance between default rates and the market price of risk, and not default rates that account for the large credit spreads in our model.²³

4.2.2 Other Effects

A second, less important, reason for us to be able match credit market data is the fact that our firms anticipate having to make costly equity issues in times of low profits to meet their financial obligations. As a result they will optimally choose somewhat lower leverage ratios ex ante. Since λ is only 2% in the baseline model this effect is relatively small here as we can see from Table 3.

²¹By construction we only focus on spreads for long maturity debt. The existence of large spreads at short maturities is arguably a larger puzzle in the literature. Addressing this requires allowing for multiple forms of debt and probably more complex stochastic processes.

²²Assuming a realistic recovery rate of about 75% which our model is calibrated to match.

²³Our decomposition is also consistent with Elton, Gruber, Agrawal and Mann (2001) who estimate that about two thirds of the credit spreads are due to the credit risk premium.

As before we find that the persistence of the shocks retains a very important effect on prices. A low value for ρ_x variation in the market price of risk and thus credit spreads. This acts as an effective expansion in the supply of credit and which raises equilibrium leverage. Equilibrium default nevertheless falls because, with less persistent shocks, the option value of remaining an active firm rises. Our calibration of (rare) credit supply shocks is specifically designed to have only a negligible impact of these numbers, but it is easy to see how they can be used to generate more pronounced effects. Unsurprisingly larger variability in credit supply conditions will reduce average leverage but the overall effect on default rates and credit spreads depends on specific assumptions about the persistence of these shocks.

4.3 Investment and Finance over the Business Cycle

Table 4 documents the cyclical behavior of several investment and financing variables by reporting their cross-correlations with GDP. The table shows that all variables have the correct cyclical behavior in our baseline model although the implied correlations are sometimes higher than in the data. Intuitively this is because our benchmark calibration probably relies too much on a single source of aggregate uncertainty so that the innovations in output growth are too closely tied to those in aggregate productivity. Allowing for credit shocks in the last column generally produces more realistic numbers, with the possible exception of equity issues which become nearly uncorrelated with GDP growth. Intuitively movements in credit supply affect both leverage and spreads directly but GDP growth only through change in consumption and investment over time.

Persistence in aggregate shocks implies a more strongly pro-cyclical behavior in aggregate investment as new firms enter the market and build up productive capacity in anticipation of higher future profits. As a result the market value of firms (and especially of equity) is also more strongly pro-cyclical implying a countercyclical pattern in market leverage.

Also intuitive is the behavior of both default rates and credit spreads which are strongly countercyclical since default becomes less attractive when profits are temporarily high.

As in the data, our firms are more likely to issue equity during good times in the model, although the correlation with economic activity is relatively low. This is because firms must issue equity both

at entry to finance investment in productive capacity and also when they need additional funds in times of low profits in order to cover coupon expenses. The former is usually stronger and so equity issues remain procyclical in our model. As the last two columns of Table 4 show however, lowering the issuance cost, λ , particularly in conjunction with large credit shocks can lead to countercyclical pattern in equity issuance.

4.4 Amplification and Asymmetry of Business Cycles

Figures 5 and 5 look at the impact of fluctuations in credit markets on key macroeconomic quantities. Figure 5 directly compares the response to exogenous technology shocks in our benchmark economy with levered firms, to the response in an alternative environment where all firms must be financed with equity alone. Unsurprisingly, we find that economic fluctuations are more pronounced in the levered economy, with both output, consumption and investment growth all responding between 35% to 50% more to an increase in the level of aggregate productivity. Again, we see that leverage introduces a powerful amplification mechanism since now positive productivity shocks reduce the risk of default and lower the cost of debt. This raises ex-ante firm value considerably and encourages firm creation and investment spending.²⁴ These amplifications results are quantitatively similar to those in Bernanke et al (1999). This is because both models are calibrated to deliver similar shares of investment in output and average credit spreads. However as we have seen, our transmission mechanism is different, and relies on movements in credit risk premium to produce the required variation in credit spreads and the cost of capital.

An important aspect of the model is the implied asymmetric in the response to shocks that is induced by the non-linear nature of entry and exit rules. Figure 5 illustrates these asymmetries in business cycle fluctuations with negative shocks producing sharper declines in output and investment. While a positive shock to productivity raises GDP growth about 1.2% above its mean, a negative shock of the same magnitude will reduce GDP growth by about 1.7%. Although often documented in the empirical literature this pronounced asymmetry in economic fluctuations is rarely obtained in general equilibrium macroeconomic models even when they do not rely on linear

²⁴Because we abstract from variations in labor supply these results are probably a lower bound on the amount of endogenous propagation that this mechanism can generate.

approximation to the stationary equilibria.²⁵

Figure 5 helps explain this phenomenon. Intuitively a productivity shock has now two effects. First, a negative shock directly lowers output by lowering productivity. Second, it increases default rates and leads to a widening credit spreads. By making debt more expensive this further reduces the value of potential entrants and lowers investment spending. Both aspects are common to the financial accelerator literature but in our model the second effect is not symmetric because of role of risk premia in our model.

Figure 5 shows that risk premia acts in two ways. First as discussed earlier it makes the default rate respond asymmetrically to positive and negative shocks. Second, because bond losses are concentrated in recessions credit spreads widen more sharply here than in models with risk neutrality.²⁶ In turn this then exacerbates the response of aggregate investment to the negative productivity shock.

4.5 Credit Market Shocks

Although we have focused on the economy's response to technology driven fluctuations our framework can easily accommodate several other types of shocks. Figure 5 shows the predicted response of our economy to an unexpected tightening of credit conditions, i.e. an unexpected transition to the low ϕ state. This shock both devalues outstanding bonds, directly reducing household wealth, and makes new debt issues more costly, lowering investment and future output. The top panel confirms these predictions, showing that both output and investment fall, with output response actually peaking few quarters after the initial shock. The bottom panel also shows that default rates also rise, as the negative wealth effect feeds back to lower equity values of incumbent firms in general equilibrium.

Compared with the response to a standard technology shock a credit shock has a significantly larger impact on investment relative to output. Sensibly, this implies that in our model fluctuations

²⁵Kyotaki and Moore (1997), Bernanke et al (1999), Cooley et al (2004).

²⁶Intuitively:

$$\text{Spread} = \text{Default Rate} \times \text{Loss in Default} \times \text{State Price of Default} \quad (29)$$

Because default losses are fixed by assumption, if risk premia does not move, movements in default rates and credit spreads are essentially one-to-one. Here however credit spreads move nearly twice as much as default rates.

in credit conditions impact young (and small) firms much more than the older established ones.

4.6 Credit Spreads and Business Cycle Predictability

We now examine whether our model can match the well documented ability of credit spreads to forecast movements in the aggregate economy.²⁷ Table 5 shows the results of regressing the k period ahead growth in (log) output and investment, respectively, on the value weighted aggregate credit spread at time t for both our baseline model and in the data. Both panels show that credit spreads forecast movements in aggregate output and investment at horizons ranging between 1 quarter and 1 year. In both the data and the model the forecasts are usually statistically and economically meaningful. Moreover the estimated coefficients on the simulated panels are of very similar magnitudes to those found in recent empirical studies.

Table 6 shows that this predictability survives even after we control for the current state of aggregate productivity, x_t . Recall that in our model all aggregate variables such as output and investment depend on the two-dimensional aggregate state $s = (x, \mu(\cdot))$. The results in Table 6 show that, although important, variation in x_t is far from subsuming all of the information about the behavior of aggregate quantities. Unlike general aggregate models there is here an important role for firm heterogeneity and, to some extent, this is here captured by variation in credit spreads. This is not surprising since both $\mu(\cdot)$ and spreads contain much information about firm default probabilities and, as we have seen, these are closely tied to investment and output growth.

Table 7 recalculates the predictability regressions for the augmented version of our model with credit shocks. It shows how credit spreads become potentially more important in this world. Here they will reflect not only variation in cross-sectional default probabilities but also in exogenous credit conditions. Formally, with credit shocks the aggregate state space becomes $s = (x, \phi, \mu)$ and credit spreads now capture at least some of the variation in the last two. If, as Jermann and Quadrini (2010) argue, financial sector shocks are becoming more important, we would then expect the predictability of credit spreads to increase significantly.

²⁷The forecasting ability of credit spreads is documented recently in Gilchrist et al(2008), Lettau and Ludvigson (2004) and Mueller (2008).

4.7 A Recession with Debt Overhang

The independent usefulness of credit spread information is also illustrated in Figures 5 and 5. they compare the typical recession, depicted earlier in Figures 5 and 5, with one that starts from a position of "debt overhang" where firms are exogenously endowed with excessive amounts of leverage.²⁸ Although somewhat arbitrary this experiment captures the possible aftermath of a "over-leverage" crisis like the one in 2008-09. Figure 5 shows how much deeper and more persistent this second recession would be in our model with GDP falling by an extra 0.5% per quarter and investment growth by almost 2%. Although both recessions are triggered by the same movement in aggregate productivity, their extent is only really captured by the sharp differences in credit spreads in Figure 5.²⁹

5 Conclusion

In this paper we propose a tractable general equilibrium asset pricing model with heterogeneous firms that endogenously links movements in stock and bond markets to macroeconomic activity. In our model these movements are associated to endogenous fluctuations in risk premia. As a result movements in financial variables such as credit spreads and expected equity returns will forecast future economic activity.

In our equilibrium setting, endogenous default increases the volatility of consumption during recessions, thereby rendering the market price of risk sharply countercyclical. As a consequence, expected returns on stocks and bonds are higher in recessions, raising the cost of capital and lowering investment and output growth. Endogenous movements in credit markets allow our model to match the observed conditional and unconditional movements in both financial prices and macroeconomic quantities in a parsimonious setting.

While a long theoretical literature in macroeconomics has demonstrated that financial frictions

²⁸Formally we compare average recessions generated by a one standard deviation negative shock to x . In the first case all firms start with the optimal amount of leverage implied by their policy rules. In the second case we endow firms with the optimal amount of leverage implied by setting x one standard deviation above its mean.

²⁹This figure also shows that in theory default rates - and indeed leverage - provide equally useful data that can be used to predict the response of the economy. In practice though default rates and leverage are measured with significant lags and imprecisions and tend not to be used as often as credit spreads by empiricists.

have the potential to deliver a powerful amplification mechanism for macroeconomic shocks, our focus on risk premia is quite distinct and, arguably, more appropriate to understand the recent developments in 2008-09.

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Appendix: Computation Details

Computation of the competitive equilibrium is complicated by the endogeneity of the pricing kernel, which embodies the equilibrium market clearing conditions. The main difficulty here is the dependence of all aggregate quantities on the cross-sectional distribution, μ , a high-dimensional object.

Our solution algorithm exploits the parsimonious characterization of the distribution μ and relies on two basic techniques. First, we re-normalize the value functions for debt and equity to express them in units of marginal utility which is computationally more convenient. Next, following Krusell and Smith (1998), the cross-sectional distribution μ is approximated by a low-dimensional state variable that summarizes the relevant information in μ .

The expression for the pricing kernel (22) guides both our choice of the approximate state space and the re-normalizations. To that end, we define the function:

$$p(C, W) = C^{-\frac{\kappa}{\sigma}} W^{\kappa-1}$$

and rewrite the expression for the market-to-book value of equity as

$$\hat{Q}(s, z, b) = Q(s, z, b)p(C, W)$$

The normalized market value of debt (relative to capital), $\hat{B}(s, z, b)$, is defined in the same way. Our numerical strategy is based on numerically iterating on these two to obtain individual policy functions and then aggregate. Since both values depend on the aggregate state $s = (x, \mu)$ we start by approximating its high-dimensional space by $\hat{s} \equiv (x, W)$. In other words, we assume that aggregate household wealth W captures the relevant information about aggregate quantities contained in the cross-sectional distribution μ . We can then write the equity value of the firm as:

$$\begin{aligned} \hat{Q}(x, W, z, b) = & \max\{0, (1 - \tau)(1 - \lambda)(xz - b)p(\hat{C}(x, W), W) \\ & + E \left[\beta^\kappa \left(\frac{W' + \hat{C}(x', W')}{\hat{C}(x, W)} \right)^{\kappa-1} \hat{Q}(x', W', z', b) \right] \} \end{aligned}$$

where $\hat{C}(x, W) = C(\hat{s})$ in our approximate state space. $\hat{B}(x, W, z, b)$ is defined analogously.

Again following Krusell and Smith (1998) we parameterize both the consumption policy \hat{C} and the law of motion for aggregate wealth W' as log linear functions of the aggregate state, x and W :

$$\begin{aligned}\log C &= \alpha_0 + \alpha_1 \log x + \alpha_2 \log W \\ \log W' &= \eta_0 + \eta_1 \log x + \eta_2 \log W\end{aligned}$$

for some coefficient vectors α and η . With these rules at hand we compute firm value and policies. These are then aggregated and checked for consistency using the general equilibrium condition (24).

More precisely, we use the following iterating procedure:

- Discretize the state space by choosing discrete grids for b and W , and the shocks x and z .^{30, 31}
- Guess initial vectors α^0 and η^0
- Iterate on the functional equations for \hat{Q} and \hat{B} and compute decision rules for investment, default and leverage.
- Simulate decisions rules and compute the implied equilibrium allocations for C and W .
- Use implied time series for x , C and W to revise log linear rules for C and W' and check fit.
- Iterate until convergence.

The simulation uses the law of motion for the cross-section of firms over time defined by equation (25). The key step here is to recognize that at any point in time μ_t can be represented by a matrix of size $nz \times nb$, where each element $\mu_{t,ij}$ represents the mass of firms with coupon b_i which drew idiosyncratic shock z_j at time t . The default decision is efficiently characterized by the selection of μ_t implied by the optimal policy χ , while entry adds a vector corresponding to the ergodic distribution of $G(z)$ at the position implied by the optimal choice of leverage for new entrants at time t and mass equal to $h.F(s)$.

³⁰As b is a function of both x and wealth W , so $nb = nx \times nw$, where n_i is the number of points in the grid for $i = b, z, x, W$.

³¹We use the procedure in Rouwenhorst (1995) since are highly persistent.

Table 1: **Calibration**

Parameter Values	
Preferences	
β	0.993
γ	10
σ	2
Technology	
δ	0.05
ρ_x	0.96
σ_x	0.015
ρ_z	0.9
σ_z	0.18
Institutions	
τ	0.2
λ	0.02
ϕ	0.4

This table reports the basic parameter choices for our model. These choices are discussed in detail in Section 3.2. The model is calibrated at quarterly frequency to match data both at the macro level and in the cross-section.

Table 2: **Aggregate Moments**

Variable	Data	Benchmark	All Equity	$\rho_x = 0.9$	$h'(x) = 0$	$\lambda = 0$	Credit Shocks
Macro Moments							
$\sigma[\Delta C]$	1.68	1.64	1.27	1.53	1.79	1.61	1.74
$\frac{\sigma[\Delta C]}{\sigma[\Delta Y]}$	0.70	0.61	0.63	0.68	0.75	0.60	0.66
$\frac{\sigma[\Delta I]}{\sigma[\Delta Y]}$	4.59	3.73	3.85	3.48	2.91	3.74	3.82
$\frac{I}{\bar{Y}}$	0.19	0.23	0.20	0.20	0.18	0.24	0.23
Asset Pricing Moments							
$E[r^f]$	1.69	1.34	2.11	3.64	1.90	1.38	1.19
$\sigma[r^f]$	2.21	1.59	1.04	0.92	1.14	1.55	1.46
$E[r^e - r^f]$	4.29	4.20	1.43	1.35	2.93	4.24	4.03
$\sigma[r^e]$	17.79	10.86	3.76	3.97	6.62	10.97	11.12

This table reports unconditional sample moments generated from the simulated data of some key variables of our model under different parameter specifications. We report averages across 1000 simulations of 59 years. All data are annualized. The return on equity refers to the value weighted aggregate stock market return. The parameter values used in the benchmark simulation are reported in table 2. Data counterparts come from the BEA and CRSP

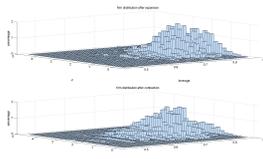


Figure 1: Cross-Sectional Distribution of Firms. This figure depicts the equilibrium cross-sectional distribution of firms, $\mu(s, b, z)$ for our baseline model. The top panel shows the impact of a one standard deviation increase in aggregate productivity, x on $\mu(\dot{c})$ while the bottom panel shows the effects of a one-standard deviation decrease in x .

Table 3: **Credit Market Statistics**

Variable	Data	Benchmark	All Equity	$\rho_x = 0.9$	$h'(x) = 0$	$\lambda = 0$	Credit Shocks
Default Rate	1.48%	1.42%	0.00	0.60%	0.97%	1.29%	1.58%
Credit Spread	0.95%	1.04%	0.00	0.43%	0.80%	0.91%	1.12%
Market Leverage	0.35	0.33	0.00	0.44	0.41	0.38	0.31

This table reports statistics related to credit markets and firms' capital structures. Data from the model comes from averages across 1000 simulations of 59 years. All data are annualized. The default rate is from Jermann and Yue (2007). The average (annualized) credit spread is the AAA-BAA spread. This data and that for the market leverage ratios are for the period between 1951 and 2008 and come from the Board of Governors of the Federal Reserve.

Table 4: **Financing Over Business Cycle**

Variable	Data	Benchmark	All Equity	$\rho_x = 0.9$	$h'(x) = 0$	$\lambda = 0$	Credit Shocks
Investment	0.81	0.72	0.77	0.66	0.69	0.74	0.52
Market leverage	-0.11	-0.60	0.00	-0.51	-0.56	-0.61	-0.33
Equity Issuance	0.10	0.19	0.28	0.14	0.09	0.17	0.03
Default rate	-0.33	-0.81	0.00	-0.84	-0.75	-0.80	-0.59
Credit Spread	-0.36	-0.74	0.00	-0.78	-0.82	-0.72	-0.42

This table reports business cycle properties of key macro and financial variables in the model. Data from the model comes from averages across 1000 simulations of 59 years. For flow variables we use correlations between growth rates. For leverage we report correlation with end of period ratios. Empirical sources are the Bureau of Economic Analysis and the Board of Governors of the Federal Reserve.

Table 5: **Forecasting Output and Investment Growth**

		Data		
Horizon k		1 quarter	2 quarter	1 year
$\Delta Y_{t,t+k}$		-1.20 (-4.26)	-2.24 (-3.58)	-3.89 (-2.82)
$\Delta I_{t,t+k}$		-3.69 (-4.30)	-6.54 (-3.16)	-9.71 (-2.39)
		Model		
Horizon k		1 quarter	2 quarter	1 year
$\Delta \tilde{Y}_{t,t+k}$		-1.69 (-2.41)	-2.19 (-2.28)	-2.85 (-2.15)
$\Delta \tilde{I}_{t,t+k}$		-2.92 (-2.32)	-5.73 (-2.21)	-7.96 (-2.07)

This table reports forecasting regressions for output and investment growth in both the model and the data. We regress k -period ahead log growth in output and investment, respectively: $\Delta Y_{t,t+k} = \log Y_{t+k} - \log Y_t$ and $\Delta I_{t,t+k} = \log I_{t+k} - \log I_t$ on the value weighted aggregate credit spread at time t . T-statistics are reported in parentheses below. Statistics for the model are obtained by averaging the results from simulating the economy 1000 times over 59 years. Standard errors are corrected using Newey-West with 4 lags.

Table 6: **Forecasting with Credit Spreads - Baseline Model**

	CS_t	x_t	R^2
$\Delta Y_{t,t+1}$	-1.69 (-2.41)		0.22
		1.44 (2.92)	0.65
	-0.83 (-2.16)	2.06 (2.54)	0.79
$\Delta I_{t,t+1}$	-2.92 (-2.32)		0.11
		4.58 (3.27)	0.49
	-1.55 (-2.03)	6.19 (2.40)	0.62

This table reports forecasting regressions for output and investment growth in the baseline model. We regress 1 period ahead log growth in output and investment, respectively: $\Delta Y_{t,t+k} = \log Y_{t+k} - \log Y_t$ and $\Delta I_{t,t+1} = \log I_{t+1} - \log I_t$, on the value weighted aggregate credit spread at time t, CS_t , and the aggregate shock at time t, x_t . T-statistics are reported in parentheses below. Statistics for the model are obtained by averaging the results from simulating the economy 1000 times over 59 years. Standard errors are corrected using Newey-West with 4 lags.

Table 7: **Forecasting with Credit Spreads - Credit Shocks**

	CS_t	x_t	R^2
$\Delta Y_{t,t+1}$	-1.25 (-2.38)		0.26
		1.56 (2.29)	0.55
	-1.14 (-2.47)	1.91 (2.43)	0.74
$\Delta I_{t,t+1}$	-3.11 (-2.57)		0.23
		3.82 (2.27)	0.48
	-1.78 (-2.12)	4.46 (2.09)	0.66

This table reports forecasting regressions for output and investment growth in the model with credit shocks. We regress 1 period ahead log growth in output and investment, respectively: $\Delta Y_{t,t+k} = \log Y_{t+k} - \log Y_t$ and $\Delta I_{t,t+1} = \log I_{t+1} - \log I_t$, on the value weighted aggregate credit spread at time t , CS_t , and the aggregate shock at time t , x_t . T-statistics are reported in parentheses below. Statistics for the model are obtained by averaging the results from simulating the economy 1000 times over 59 years. Standard errors are corrected using Newey-West with 4 lags.

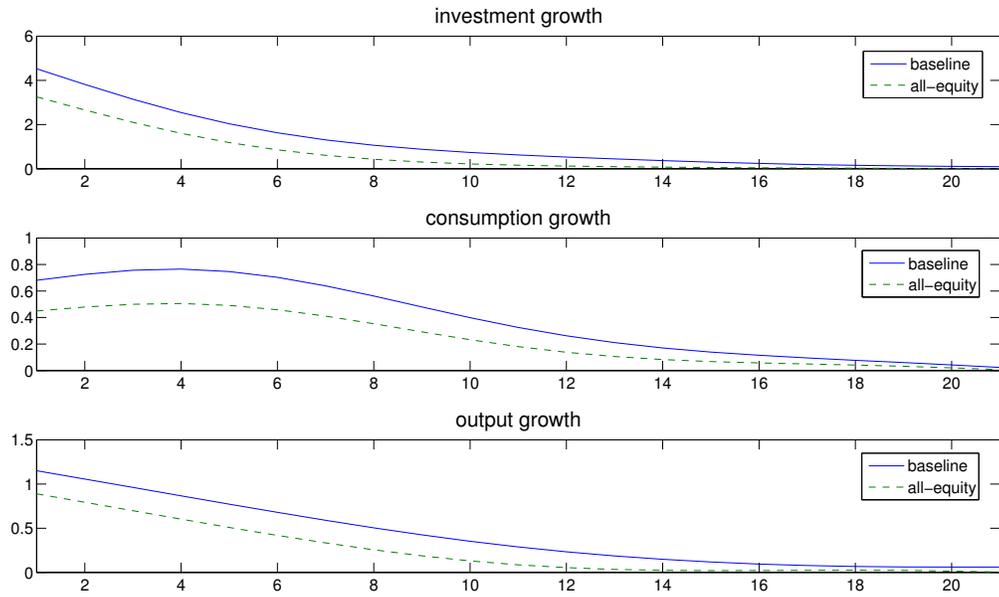


Figure 2: Business Cycle Amplification. This figure shows the response of output, consumption and investment growth to a one standard deviation positive innovation in aggregate technology in both our baseline levered economy and an alternative scenario where all investment is financed with equity alone.

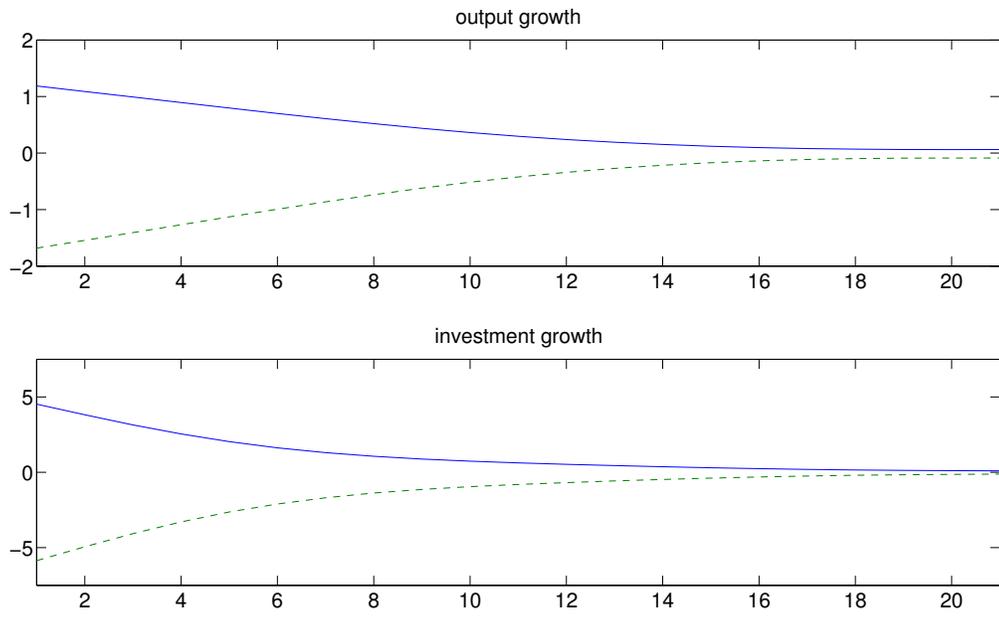


Figure 3: Booms and Busts - Macro Quantities. This figure shows the response of output and investment growth to both a one standard deviation positive and negative innovation in aggregate technology in both our baseline levered economy.

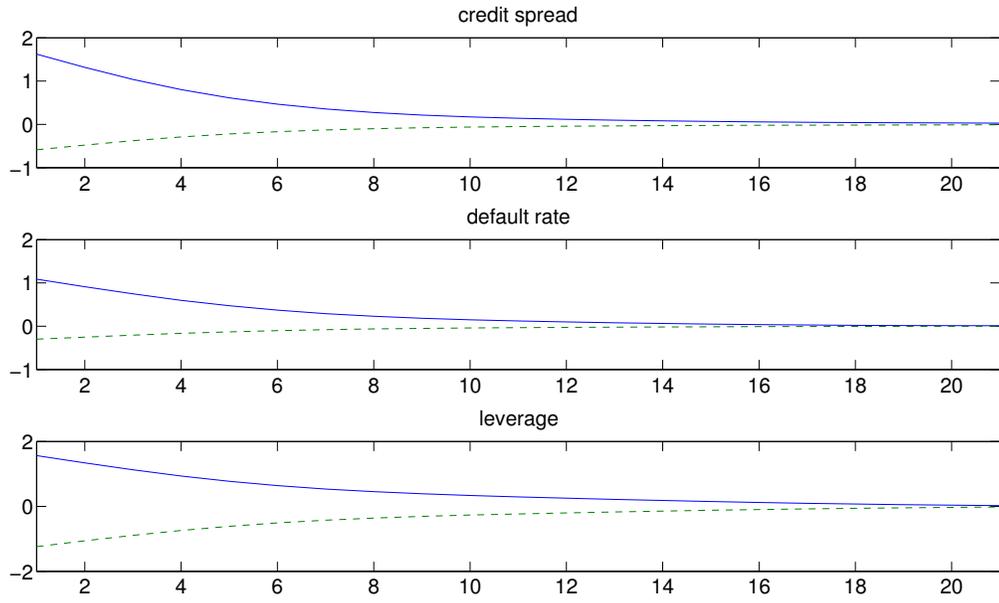


Figure 4: Booms and Busts - Credit Markets. This figure shows the response of default rates and credit spreads to both a one standard deviation positive and negative innovation in aggregate technology in both our baseline levered economy.

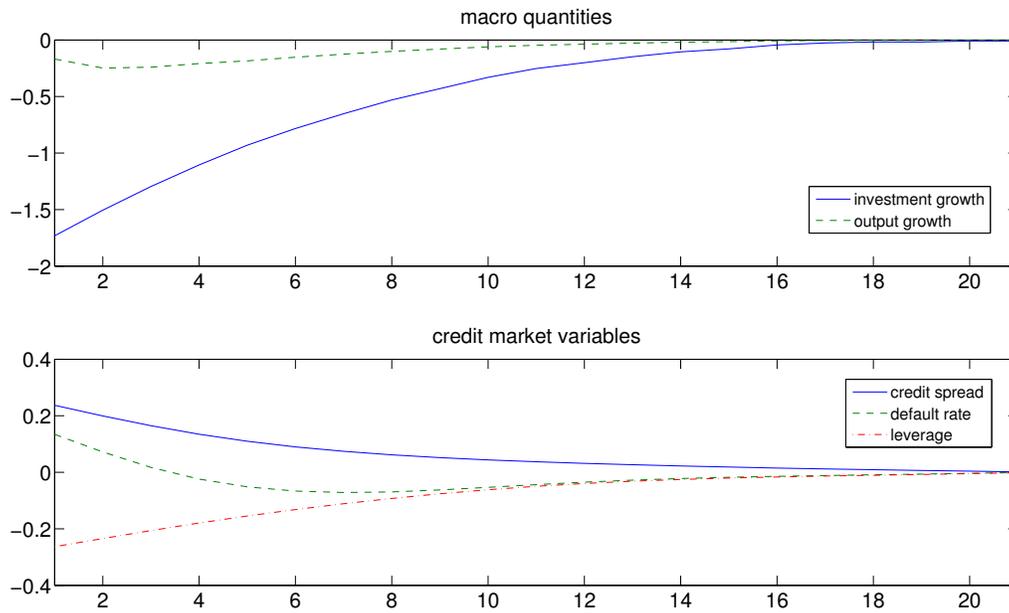


Figure 5: A Credit Supply Shock. This figure shows the response of both macro and credit market variables to a deterioration in credit market conditions induced by a drop in the recovery rate on assets ϕ .

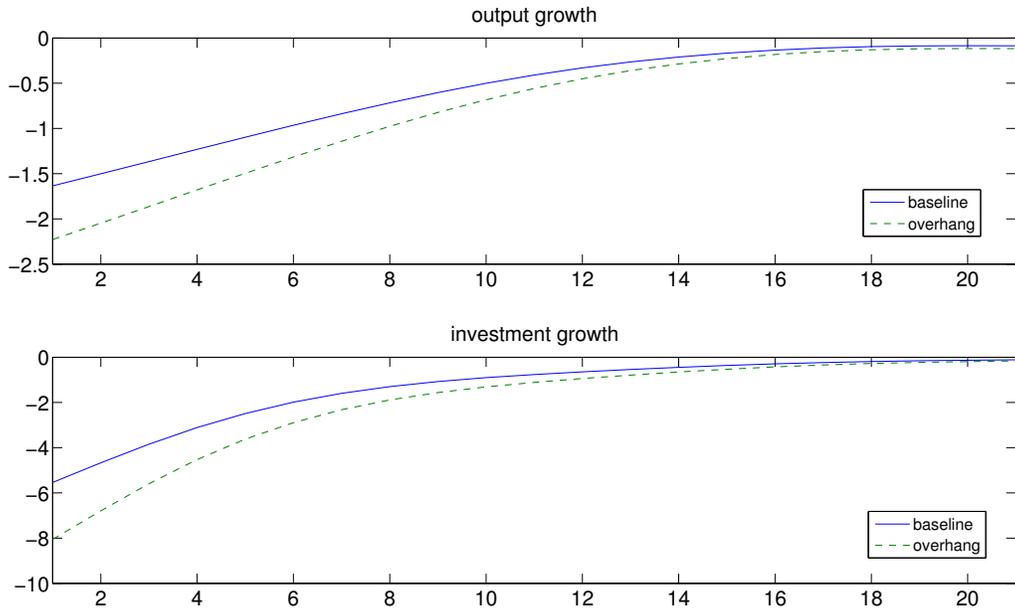


Figure 6: A Bust with Debt Overhang - Macro Quantities. This figure shows the response of output and investment growth to a one standard deviation negative innovation in aggregate technology in both our baseline levered economy and in a world where firms have accumulated excessive debt.

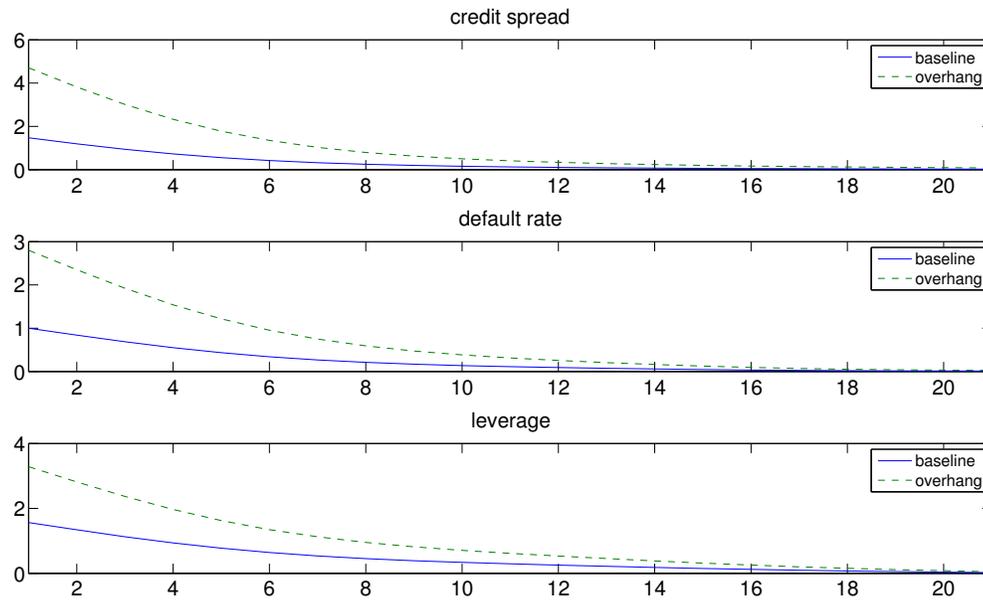


Figure 7: A Bust with Debt Overhang - Credit Markets. This figure shows the response of default rates and credit spreads to a one standard deviation negative innovation in aggregate technology in both our baseline levered economy and in a world where firms have accumulated excessive debt.