

Contingent Capital Structure*

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Abstract

This paper studies the optimal financing contract of a bank with risk-shifting incentives and private information, in an environment with macroeconomic uncertainty. Leverage mitigates adverse selection problems owing to debt information-insensitivity, but leads to excessive risk-taking. I show that the optimal leverage is procyclical in the *laissez-faire* equilibrium, and contingent convertible (CoCo) bonds emerge as part of the implementation of the optimal contingent capital structure. However, the equilibrium entails excessive leverage and risk-taking, due to a bank's private incentives to minimise market mispricing of its securities. It is socially optimal to impose countercyclical capital requirements, implemented by CoCo bonds in addition to straight debt and equity.

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1 Introduction

The recent financial crisis has brought the prudential regulation of financial institutions to the fore as an issue of critical importance. A new type of “contingent capital” – contingent convertible (CoCo) bonds – a form of debt that automatically converts into additional common shareholders’ equity when a bank’s original capital is depleted, has received much attention for its potential to restore the incentives for banks to practice prudent risk management and to prevent the disruptive insolvency of large financial institutions.¹ In this paper, I employ an agency-theoretic approach to show that CoCo bonds emerge as part of the optimal bank capital structure to mitigate risk-shifting problems when banks with private information face economic uncertainty.

CoCo bonds were first proposed as an alternative capital instrument by Flannery (2005), followed by modifications put forward by various scholars in the pursuit of a prudential capital structure of banks.² CoCo bonds have been positively embraced by regulators including the Swiss banking supervisor, Finma. For example, Lloyds Banking Group announced the first issue of £7bn CoCo bonds (Enhanced Capital Notes) through a bond exchange as early as 2009. The conversion will be triggered if the bank’s core capital falls to less than 5% under Basel II rules.³ CoCo bonds have been proven popular among banks and investors ever since, with issuance in 2012 and 2013 exceeding \$20bn, and oversubscription being the norm.⁴

The literature on CoCo bonds, as surveyed below, has expanded in a short period of time, with the focus of discussion on the issues of trigger design and the pricing of the instrument. However, little has been done to address the following fundamental questions. Why does contingent capital improve efficiency as part of the capital structure of financial institutions, if at all? What is the role of financial regulation when contingent capital is available? This paper is the first to formally address these issues in a model of optimal

¹Straight debt can be interpreted as a special case of a CoCo bond. However, unlike straight debt, which “converts” into equity in the event of a default, CoCo bonds allow flexibility in choosing the conversion trigger of the bonds. The proposed CoCo bonds are typically converted into equity well before a bank enters into distress.

²The idea of Flannery (2005) first appeared in a 2002 working paper.

³Source: Bloomberg (2009).

⁴The most recent issue of CoCo bonds is a \$3bn offering of Tier 2 Capital Notes by Credit Suisse. The Credit Suisse CoCo bonds are wiped out if the bank breaches its 5% tier one capital ratio or if the national regulator deems it is near default. Source: Financial Times (2013).

contracting with endogenous risk choice under macroeconomic uncertainty.

The model builds upon two agency problems that are direct consequences of the intermediating functions performed by banks, in an environment with macroeconomic uncertainty. First, banks as informed lenders typically have better information about their investment opportunities than outside investors. Hence there is an inherent asymmetric information problem when banks raise capital. Second, banks as delegated monitors can influence borrowers' behaviour. Without modelling the borrowers explicitly, this paper assumes that banks can affect the riskiness of the businesses they lend to, and charge higher yields on loans to riskier businesses. This creates scope for *ex post* risk-shifting, i.e. the shareholders of levered banks may prefer a portfolio of excessively risky loans at the expense of the debt holders' interests. Moreover, the model takes into account that the general returns on banks' investments fluctuate with macroeconomic conditions, to study the implications of the two agency problems for banks' risk-taking incentives across different economic conditions and the role of a pre-committed contingent capital structure.

The analysis proceeds as follows. I start with showing that, in the *laissez-faire* equilibrium, it is optimal for a bank to raise capital *ex ante* with a contingent capital structure employing procyclical leverage that depends on the subsequent realisation of the macroeconomic conditions. In this baseline model, the asymmetric information and risk-shifting problems uniquely determine the equilibrium leverage because of the trade-off effect of leverage: leverage reduces the signalling cost because debt is an information-insensitive funding instrument, but leads to excessive risk-taking *ex post*. Moreover, the optimal contingent capital structure entails higher leverage in booms when the information asymmetry is relatively more severe. Higher leverage must be employed in booms because it is more difficult to differentiate a good issuer from a bad one when asset values are generally higher and the bank's private information becomes relatively less significant. The model implies procyclical leverage ratios for banks, consistent with the empirical evidence documented by Adrian and Shin (2008a,b). In the resulting equilibrium, the bank's equity value is higher in booms and the default probability is lower in booms.

The optimal procyclicality of the equilibrium contingent leverage can be implemented using CoCo bonds, in addition to straight debt and equity, so that the bank has less leverage entering into an economic downturn. The model yields implications regarding

the practical design of the CoCo bonds. First, two types of conversion features that are seen in existing CoCo bonds can arise in equilibrium. CoCo bonds issued by better capitalised banks should specify a debt write-down when triggered, because the bank is not in need of much outside capital; CoCo bonds issued by poorly capitalised banks should specify a conversion into equity. For example, Credit Suisse has issued CoCo bond with a contingent convertible feature in 2011 and 2012, followed by CoCo bonds with a write-down feature in 2013, as the capital position of the bank improved.⁵ Second, while the model assumes a verifiable macroeconomic state as the trigger of the CoCo bonds, which is analogous to a regulatory declaration proposed by the Squam Lake Working Group (2009), the optimal CoCo bonds can also be implemented with triggers based on the prices of equity (e.g. Flannery 2009; Pennacchi 2011; McDonald 2011) and the CDS spreads of the bank (Hart and Zingales 2011). A concern raised by Sundaresan and Wang (2013) is that the market price of equity may not be unique unless the conversion of the CoCo bonds is designed to not transfer value between the existing shareholders and the bond holders, which precludes penalising the existing shareholders, defeating the purpose of CoCo bonds. This paper endogenises the effect of capital structure on banks' value due to risk-shifting incentives and shows that the existing shareholders can be diluted upon the optimally designed CoCo conversion.

The *laissez-faire* equilibrium warrants regulation as it involves excessive leverage and risk-taking driven by a bank's private incentives to minimise the mispricing in the securities it issues under asymmetric information. In this model, the bank can signal its type, either by increasing the amount of leverage in its capital structure, or by deliberately underpricing the securities it issues. While leverage is preferred by the bank as it minimises mispricing, it incurs a social cost through the risk-shifting incentives of the bank by reducing the value of the businesses the bank lends to. I study the optimal regulation, defined as a set of constraints on banks' leverage designed to maximise bank value, which captures social welfare in this baseline model, when the bank privately optimises its capital structure subject to the regulatory constraints. The extent to which the regulator can cap leverage is thus limited by banks' private incentives. The result confirms the optimality of countercyclical capital requirements as proposed by the Basel Committee on Bank

⁵According to Credit Suisse Regulatory Disclosures (2012), the core tier 1 capital ratio of Credit Suisse under Basel II.5 increased from 10.4% to 14.4% from 2011 to 2012, and the tier 1 capital ratio increased from 15.1% to 18.4%.

Supervision (2010) and advocated by scholars such as Brunnermeier et al. (2009). Faced with binding capital requirements, banks find it (privately) “costly” to issue the excess equity relative to the *laissez-faire* equilibrium because of the mispricing in the market. Banks meet the countercyclical capital requirements by issuing CoCo bonds, in a manner similar to the implementation of the *laissez-faire* equilibrium.

The baseline model thus far considers the role of leverage in a bank’s capital structure in signalling its private information at the cost of inducing risk-shifting, which negatively impacts the value of the businesses the bank lends to. However, risk-taking by banks has systemic effects on the broader economy, as highlighted by recurrent financial crises. Large and correlated bank failures tend to pose large negative externalities.⁶ Consequently, policy-makers are unwilling or unable to allow major financial institutions to fail.⁷ I consider two types of state guarantees, implicit bailouts and explicit deposit insurance, in two extensions of the baseline model, to examine moral hazard problem induced by state guarantees and the implications for the optimal capital regulation.

The first extension assumes the systemic importance of the bank so that the regulator has the incentive to bail out a failed bank *ex post* by repaying the creditors on the bank’s behalf. The second extension includes risk averse, unsophisticated depositors as part of the bank’s funding base, who are protected by deposit insurance. Both forms of state guarantees create moral hazard problems because they provide an implicit subsidy when the bank issues debt. I show that the optimal capital regulation remains countercyclical and can be implemented using CoCo bonds with the same face value as in the baseline model, since the face value of the CoCo bonds is determined by the relative severity of the asymmetric information problems across different economic states. However, in the presence of state guarantees, the optimal capital regulation permits higher leverage in the form of straight debt or demand deposit. This follows the previous intuition that the extent to which the regulator can cap leverage is limited by the bank’s private incentives. Since the moral hazard problem effectively reduces the private cost of leverage to the bank,

⁶For example, Ivashina and Scharfstein (2010) document a run by short-term bank creditors following the failure of Lehman Brothers, contributing to a 46% reduction in the extension of new loans to large borrowers in the fourth quarter of 2008 relative to the previous quarter.

⁷For example, the run on Northern Rock, Britain’s fifth-largest mortgage lender, did not stop until a taxpayer-backed guarantee of all existing deposits was announced in September 2007. The US Treasury and the Federal Reserve System bailed out 282 publicly traded banks and insurance companies under the Troubled Asset Relief Program (TARP) in 2008–9.

higher leverage must be permitted to alleviate the asymmetric information problem. The *ex post* bailout of a systemically important bank and explicit deposit insurance therefore hinder the *ex ante* efficient capital regulation of the bank. Bail-in capital, a form of contingent debt that is wiped out in case the bank fails, has been considered by regulators such as the Basel Committee and the Bank of England as part of the resolution regimes for banks, in order to shield taxpayers from the need to bail out a bank that is “too big to fail”. In light of the results in this extension, if a regulator can credibly commit to bailing in the debt *ex post*, the capital market would correctly price in the risk of default when the bank raises financing *ex ante*, removing the moral hazard problem of bailouts.

Literature review

This paper relates to a growing literature on contingent capital. In the first strand of the literature, Albul et al. (2010) and Barucci and Viva (2011) endogenise contingent capital in banks’ capital structures. These papers extend the Leland (1994) model of tax benefit and bankruptcy cost to consider a firm’s choice of capital structure among equity, straight debt and contingent convertible bonds. Unlike this paper, their approach does not take into consideration adverse selection or moral hazard. Since risk-shifting problems are perhaps the most important motivation for capital regulation, their settings do not provide implications for regulating the risk of the banking system. The second strand of the literature studies the implications of exogenously imposed CoCo bonds in banks’ capital structures. Martynova and Perotti (2012) consider the effect of CoCo bonds on banks’ risk-taking incentives. Others focus on the practical issues associated with CoCo bonds using different trigger mechanisms. For example, see Flannery (2005, 2009), Raviv (2004), Squam Lake Working Group (2009), McDonald (2011), Hart and Zingales (2011), Pennacchi et al. (2010), Pennacchi (2011), Bolton and Samama (2012), Calomiris and Herring (2013) and Sundaresan and Wang (2013). This paper is the first to provide a unified analysis of the optimality of CoCo bonds and the subsequent risk-taking behaviour of a bank. The framework also provides economic intuition for the design of CoCo bonds.

This paper is also related to the discussion of countercyclical capital regulation. The point that optimal bank capital regulations should depend on the state of the business cycle is made by Kashyap and Stein (2004) in a model of (exogenously) expensive equity capital and systemic cost of default. Later works by Hanson et al. (2011), Repullo and Suarez (2013) and Shleifer and Vishny (2010) also discuss time-varying capital

requirements. A recent paper by Gersbach and Rochet (2013) studies a model of financial frictions with complete markets in which inefficient credit fluctuations arise and can be corrected by countercyclical capital regulation. Similar to this paper, Dewatripont and Tirole (2012) show the optimality of countercyclical capital and self-insurance mechanisms such as CoCos, considering the risk-shifting incentives induced by leverage. The trade-off in their model is generated by the creditor control right in default as disciplining device, which implies that agents should be rewarded only for the part of performance that is under managerial control. In contrast, this paper considers the benefit of mitigating adverse selection using debt financing; generating CoCo bonds that are contingent on exogenous macroeconomic conditions.

More generally, this paper relates to the literature on optimal corporate financing structures. While a significant portion of the theory of corporate finance under frictions can be categorised into two distinct paradigms, agency models (e.g. Jensen and Meckling 1976; Myers 1977; Grossman and Hart 1984; Green 1984) and financial signalling models (e.g. Ross 1977; Leland and Pyle 1977), efforts have been made to explore the implications when the two problems are both present. In the presence of private information, John and Kalay (1982) study the agency costs of underinvestment, while Darrough and Stoughton (1986) study the agency problems of effort provision. Similar to this paper, John (1987) considers the problems of risk-shifting and asymmetric information in determining the capital structure and the investment policies of a widely held firm. John (1987) emphasises that the risk-shifting problem increases the signalling cost in the equilibrium, relative to the case in which the firm can commit to an investment policy. This paper differs from John (1987) in two aspects. First, I consider the impact of macroeconomic conditions on the trade-off between risk-shifting and asymmetric information problems, generating a role for contingent capital structure. Second, I recognise that the equilibrium is constrained inefficient, and characterise the optimal regulation, which restores constrained efficiency.

Although this paper shows the optimality of CoCo bonds, which are “reverse” convertible bonds, a number of papers have shown that conventional convertible bonds help to mitigate the asymmetric information problem by allowing the security to be contracted on additional signals. Stein (1992) show that callable convertible bonds are used by firms with medium quality as “back-door equity” financing to prevent bad firms from mimicking, in a setting in which the initial asymmetry of information is completely resolved by the time the security is called. Chakraborty and Yilmaz (2011) recognise

that the resolution of information asymmetry is likely to be imperfect, and conversion only occurs if good news arrives. The “back-door equity” value of the securities is correlated with the manager’s private information, thereby allowing an optimally designed callable convertible bond to resolve the asymmetric information problem without dissipation. In contrast, this paper considers a contingent security contracted upon macroeconomic states, which are uninformative of the private information of the issuer. The optimal conversion of security is therefore determined by the relative severity of the asymmetric information and risk-shifting problems considered in this model across different macroeconomic states, which gives rise to optimal procyclical leverage, implemented by CoCo bonds.

The remainder of the paper is structured as follows. Section 2 outlines the baseline model. Section 3 analyses the *laissez-faire* equilibrium to show the optimality of a contingent capital structure with procyclical leverage. Section 4 illustrates how CoCo bonds are part of the implementation of the optimal contingent capital structure and discusses the optimal design of the CoCo bonds. Noticing that the *laissez-faire* equilibrium entails excessive leverage, Section 5 characterises the optimal capital requirement, which is countercyclical. Section 6 studies two extensions of the model to incorporate the moral hazard of state guarantees. Section 7 concludes.

2 Model

There are four dates: 0, $\frac{1}{2}$, 1 and 2. The model’s participants consist of a bank and a set of outside investors.⁸ All agents are risk neutral and there is no discounting.

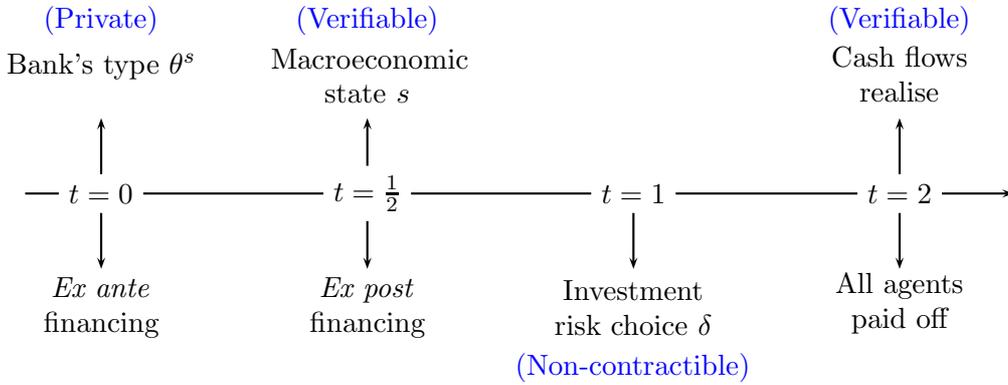
At $t = 0$, the bank has an opportunity to extend a total of 1 unit credit to form a loan portfolio that pays off at $t = 2$. The bank is endowed with private information regarding the payoffs of the portfolio at $t = 0$. However, at $t = 1$ after the loans are made, the bank can influence the riskiness of the borrower, and charge a higher yield on loans to riskier businesses. In order to focus on the capital structure of the bank, I abstract from modelling the borrowers explicitly. Instead, in Section 2.1 I make assumptions on the cash flows of the bank’s loan portfolio directly to capture this intuition.

⁸In Section 6 I consider as an extension the case in which some outside investors are risk averse and the bank raises funds partially through deposits issued to the risk averse investors.

At $t = \frac{1}{2}$, a verifiable macroeconomic state realises, which affects the payoffs of the loan portfolio.⁹ In order to fund the lending, the bank chooses its financing arrangement and raises capital either at $t = 0$ (*ex ante* financing), or at $t = \frac{1}{2}$ after the realisation of the macroeconomic state (*ex post* financing). I detail the financing arrangements in both the *ex ante* and the *ex post* cases in Section 2.1, and analyse both cases in Section 3 for comparison.

The timing of the events is summarised in Fig. 1, where the nature of the private information θ^i , the macroeconomic state s and the risk choice δ of the investment are detailed in the following section.

Figure 1: Timeline



2.1 Assumptions and discussion

This section presents the assumptions on the distribution of the bank's portfolio cash flows to incorporate both the asymmetric information and the risk-shifting problems, and discusses how the bank can structure itself in order to finance its lending.

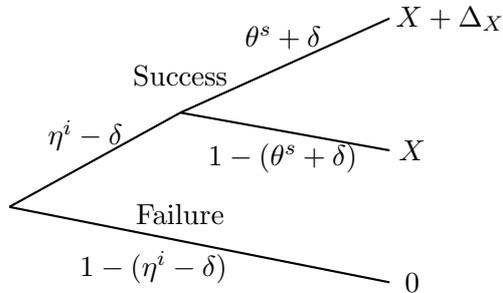
The bank's loan portfolio

At $t = 0$, the bank has the opportunity to extend 1 unit credit and form a portfolio of risky loans that pays off at $t = 2$. The final payoff of the portfolio can be 0, X or $X + \Delta_X$. I will refer to the case where the portfolio returns 0 a failure, and a positive cash flow X

⁹As there is only one bank in this simple framework, the state s is interpreted as a macroeconomic state for intuitive purposes. Whether the shock to the state s is macroeconomic or idiosyncratic depends on factors outside of this model, such as the correlation of the shock across banks. I discuss other interpretations of the shock in Section 4.2.2 in relation to the design of the CoCo bonds that implement the optimal contingent capital structure.

or $X + \Delta_X$ a success.¹⁰ The distribution of the portfolio cash flow over $\{0, X, X + \Delta_X\}$ depends on the type of the bank i , the state of the economy s and the bank's risk choice δ as illustrated in Figure 2 and as specified below.

Figure 2: Distribution of the portfolio cash flow at $t = 2$



The type of the bank $i \in \{G, B\}$ is characterised by its success likelihood η^i . A *Bad* bank (B) has a loan portfolio with a higher failure probability than a *Good* bank (G): $\eta^G > \eta^B$. The bank privately observes its type at $t = 0$. Outside investors do not observe the type of the bank, but they have a prior belief that the bank is good with probability γ .

The state of the economy $s \in \mathcal{S} = \{1, \dots, S\}$ also affects investment opportunities, and is realised at $t = \frac{1}{2}$. Specifically, the state of the economy affects the expected return of the portfolio upon success. Conditional on success, the likelihood of realising a high cash flow is θ^s . I shall interpret the states with relatively higher θ^s as booms, and those with lower θ^s as recessions.

The bank can then privately choose the risk profile δ of the loan portfolio at $t = 1$. The bank can increase the riskiness of the businesses it lends to, but charge a higher yield on the loans. Specifically, the bank can decrease the success probability by δ , but increase the expected payoff of the portfolio upon success by $\delta\Delta_X$, through an increase in the conditional probability of receiving a high cash flow by δ . This setup loosely captures the trade-off between risk and return in financial investments.¹¹

¹⁰The assumption that the loan portfolio returns 0 in case of a “failure” is not without loss of generality. If the bank's loan portfolio produces a positive minimum cash flow, the portfolio contains a portion of cash flow which is safe and therefore does not impose any financing problems on the bank. Backed by this safe part of the cash flow, a bank can issue deposits or safe debt. However, in practice banks typically take on additional risky debt. The model therefore sheds light on understanding the additional leverage taken by banks in the form of risky debt.

¹¹The specific assumption that a decrease in the success probability brings an increase of the same

To summarise, for a given bank of type i in a given economic state s , given the risk choice δ of the bank, the investment succeeds with probability $\eta^i - \delta$. If successful, the loan portfolio returns a high cash flow $X + \Delta_X$ with probability $\theta^s + \delta$, or a low cash flow X with probability $1 - (\theta^s + \delta)$.

Bank value

The value of the bank is determined by the risk choice made at $t = 1$ and the type of the bank, in any given state s . For a type i bank in a given state s , the first best risk choice $\delta_{FB}^{i,s}$ maximises the value of the loan portfolio.¹²

$$\delta_{FB}^{i,s} \equiv \arg \max_{\delta} (\eta^i - \delta)[X + (\theta^s + \delta)\Delta_X] \quad (1)$$

As a result, the probability of success when the risk choice is first best is given by

$$q_{FB}^{i,s} \equiv \eta^i - \delta_{FB}^{i,s} = \frac{1}{2}(\eta^i + \theta^s + \frac{X}{\Delta_X}) \quad (2)$$

Therefore the model suggests that, when operated at the first best risk level, the bank has a higher probability of success in a higher state than in a lower state, and if it is a Good bank than if it is a Bad bank, other things equal.

Denote the portfolio value when operated at the first best risk level by $V_{FB}^{i,s}$. Assume that $V_{FB}^{G,s} > 1 > V_{FB}^{B,s}$ and $\gamma V_{FB}^{G,s} + (1 - \gamma)V_{FB}^{B,s} > 1 \forall s$. That is, a Good bank has a positive NPV investment if operated at an appropriate risk level, whereas a Bad bank always has a negative NPV investment. However, at the first best risk level, an average bank has a positive NPV investment. This implies that pooling equilibria are feasible in this model.

As only a Good bank managed without risk-shifting produces positive NPV, the first best outcome in this economy can be produced if the bank (i) raises financing and invests if and only if it is a Good type at $t = 0$ or $t = \frac{1}{2}$, and (ii) chooses the first best level of risk at $t = 1$. However, because the lending can be value-enhancing on average, a pooling equilibrium with financing is feasible in which the bank invests regardless of its type.

magnitude in the probability of receiving a high cash flow, conditional on success, is made to simplify the expressions. The results do not change qualitatively, as long as there is a trade-off between the success probability and the conditional probability of a high cash flow.

¹²Assume that $\frac{X}{\Delta_X} < \eta^i + \theta^s < 2$, so that the probability of success $q^{i,s}$ and the conditional probability of the loan portfolio paying off a high cash flow $(\theta^s + \delta^{i,s})$ can both lie within the range of $(0, 1)$ in the first best case and in all equilibria derived in this paper. The derivation of this assumption is given in Appendix A.

Ex post and ex ante financing

The main results of the model are derived from the bank's choice of capital structure to finance the lending in equilibrium. I detail the financing game in this section.

After observing its type i , the bank can raise financing either at $t = \frac{1}{2}$ after the resolution of the economic uncertainty (*ex post* financing), or at $t = 0$ prior to the resolution of the economic uncertainty (*ex ante* financing).

I assume that the bank is endowed with internal capital $\bar{e} < 1$. It is therefore unable to self-finance the loans. At $t = 1$ the bank can finance the loan portfolio partly with its internal capital $e \leq \bar{e}$ and partly from outside investors. This endowment can be interpreted as the sum of the internal capital available within the bank and the maximum amount of funds that can be provided by incumbent shareholders.¹³

This paper takes a security design approach and solves for the equilibrium financing contract. Without loss of generality, I express the overall contract given to the outside investors as a combination of debt with state-contingent face value F^s maturing at $t = 2$ and a state-contingent fraction α^s of the residual equity.¹⁴ The model therefore allows financing via debt and equity, which are the forms of financing used in practice. This framework thus also allows the study of hybrid instruments, as most of the commonly adopted hybrid instruments can be thought of as a combination of debt and equity. As will be shown in Section 3, the optimal state-contingent capital structure can be implemented via CoCo bonds, a kind of such hybrid instruments.

In the case of *ex post* financing at $t = \frac{1}{2}$, the state of the economy s is common knowledge. In state s , the bank raises capital by promising the outside investors a combination of debt with face value F^s and a fraction α^s of the residual equity.¹⁵

¹³The model assumes that the bank's asset is solely comprised of the loan portfolio. One can also interpret the portfolio as a marginal investment whose payoff can be contracted upon, which would be the case for securitisation. Alternatively, the portfolio can be a part of the on-going operations of a bank, as long as that at $t = 0$ the bank does not have outstanding risky debts. If the bank has existing risk debt, the bank's incentives for financing and investment are distorted by the debt overhang problem. For an analysis on the efficient recapitalisation of banks under debt overhang, see Philippon and Schnabl (2013).

¹⁴Because of the three point cash flow space $\{0, X, X + \Delta_X\}$, in a given state s , debt with face value $F^s \leq X$ and residual equity replicate any contract that satisfies the usual assumption of monotonicity.

¹⁵In order to allow the difference between debt and equity as funding instruments, I restrict parameter values such that the cost of risk-shifting is sufficiently high relative to the NPV of the bank's investment, so that the bank cannot be purely debt financed.

In the case of *ex ante* financing at $t = 0$, the bank raises capital prior to the resolution of the economic uncertainty. I consider a general contingent capital structure specification that is given by a set of face values of the debt in each state of the economy $\mathbf{F}_C \equiv \{F_C^s\}_{s=1}^S$ and a set of fractions of the equity issued to outside investors $\alpha_C \equiv \{\alpha_C^s\}_{s=1}^S$, so that the *ex post* capital structure of the bank depends on the realisation of the economic state s .¹⁶

For each case of the model, the financing game is played as follows. Firstly the bank decides whether to raise financing and invest given its type and its knowledge regarding the state of the economy. Assume that the bank chooses not to invest if it is indifferent between participating or not. Practically, this is the case if there is a small but non-zero cost to participate in the capital market.

If the bank decides to raise financing and invest, it announces in the capital market debt issue with face value F and equity issue of fraction α , where (F, α) are either (\mathbf{F}_C, α_C) or (F^s, α^s) as specified above in each case of the model. The bank also puts up $e \leq \bar{e}$ of its own capital and retains the remaining fraction $(1 - \alpha)$ of the equity. After observing the financing plan (e, F, α) , capital market investors form a belief regarding the type of the bank, and decide whether or not to accept the terms and provide capital $1 - e$.¹⁷

2.2 Definition of equilibrium

A PBE with financing is a set of financing parameters (e, F, α) representing the amount of internal capital invested by the incumbent bank shareholders, the face value of the debt issued to outside investors and the fraction of the equity issued to outside investors; and a consistent belief assigned by the capital market regarding the type of bank, such that (i) the market valuation is fair given the belief and that the investors at least break even

¹⁶To highlight the economic intuition for the properties of the conversion, I assume that the macroeconomic state is verifiable and therefore contractable. In Section 4 I discuss potential implementation of the optimal capital structure contracted upon alternative variables including equity prices and CDS spreads.

¹⁷I only consider the case in which the bank raises just enough to finance its lending. This is without loss of generality. A bank can technically raise more than $1 - e$ in terms of outside capital, in which case the excess can only be stored as cash. Since this part of the asset yields zero and poses no information problems to investors, it does not affect the residual payoff structure of the model. In other words, every equilibrium in which a bank raises more than necessary corresponds to an equilibrium in which it raises exactly one unit of capital.

at the issuing price, (ii) the bank optimally makes the financing decision at $t = 0$, and (iii) the bank optimally makes the risk decision at $t = 1$.

Consistent with the existing literature on signalling games (e.g. Spence, 1973) there exists a continuum of PBE. I invoke the Intuitive Criterion of Cho and Kreps (1987) in order to focus on equilibria with reasonable out-of-equilibrium beliefs. Intuitively, given the resulting equilibrium, there cannot exist an off-equilibrium-action such that (i) one type (Bad) is strictly worse off deviating to it, and (ii) if the market indeed believes that the deviation can only come from the other type (Good), this type strictly prefers to defect.

In some cases of the analysis in Section 5, the Intuitive Criterion still leaves equilibria with substantially different characteristics. In these cases, I invoke the concept of undefeated equilibrium proposed by Mailath et al. (1993). Consider a proposed equilibrium and an action that is not taken in the equilibrium. Suppose there is an alternative equilibrium in which some types of the player prefer the alternative equilibrium. The criterion then requires that the beliefs at that action in the original equilibrium to be consistent with this set of types. Otherwise, the second equilibrium defeats the proposed equilibrium. In this model, if the Intuitive Criterion leaves both a pooling and a separating equilibrium, the pooling equilibrium defeats the separating equilibrium if both types are better off in the pooling equilibrium.

3 *Laissez-faire* Equilibria

I derive the *laissez-faire* equilibria in this section and discuss the procyclicality of the equilibrium leverage. The cases of *ex post* and *ex ante* financing are analysed separately. Whereas the case of *ex ante* financing is of primary interest in this paper, the case of *ex post* financing is useful for highlighting the trade-off effects of the asymmetric information and the risk-shifting problem in determining the equilibrium bank capital structure.

3.1 Equilibrium with *ex post* financing

In this section I consider the case of *ex post* financing. In the absence of any macroeconomic uncertainty at the time of financing, this version highlights the interaction between the two main frictions considered in this model.

In this case, at $t = \frac{1}{2}$ after the macroeconomic state s becomes common knowledge, the

bank announces its financing plan (e, F^s, α^s) . Following a backward induction process, I firstly inspect the risk choice of the bank at $t = 1$, and then solve for the optimal financing plan at $t = \frac{1}{2}$.

At $t = 1$, for a bank of type i in state s , given a financing plan (e, F^s, α^s) , the risk level δ is chosen to maximise the expected value of the retained cash flow by the bank $(1 - \alpha^s)(\eta^i - \delta)[X + (\theta^s + \delta)\Delta_X - F^s]$. Alternatively, the optimal risk choice $\delta^{i,s}(F^s)$ maximises the equity value of the bank

$$\delta^{i,s}(F^s) = \arg \max_{\delta} (\eta^i - \delta)[X + (\theta^s + \delta)\Delta_X - F^s] \quad (3)$$

The face value of the outstanding debt F fully determines the bank's risk choice, because for a given capital structure, equity value is independent from the ownership structure α^s . In turn F^s also completely determines the success probability, equity value and the total bank value in equilibrium. Denote the equity value and the total value of a type i bank in state s given the optimal risk choice as $E^{i,s}(F^s)$ and $V^{i,s}(F^s)$ respectively. In particular, denote the equilibrium success probability given leverage F^s by $q^{i,s}(F^s)$, given by

$$q^{i,s}(F^s) \equiv \eta^i - \delta^{i,s}(F^s) = \frac{1}{2}(\eta^i + \theta^s + \frac{X - F^s}{\Delta_X}) \quad (4)$$

This highlights the risk-shifting incentives induced by leverage, which decreases the success probability. Therefore the first best risk choice can only be implemented if and only if the bank has an unlevered capital structure, i.e. $F^s = 0$.

I now turn to the security design problem at $t = 0$. Applying the Intuitive Criterion allows the Good firm to select the “least-cost separating equilibria”, as stated in Proposition 1. A separating equilibrium (e, F^s, α^s) is characterised by the following constraints

$$(PC^B) : (1 - \alpha^s)E^{B,s}(F^s) \leq e \quad (5)$$

$$(PC^G) : (1 - \alpha^s)E^{G,s}(F^s) \geq e \quad (6)$$

$$(IR) : V^{G,s}(F^s) - (1 - \alpha^s)E^{G,s}(F^s) \geq 1 - e \quad (7)$$

The participation constraints for the Bad bank and the Good bank, (PC^B) and (PC^G) respectively, dictates that only the Good bank raises financing and invests. The investors' rationality constraint (IR) takes into account that the outside investors, regardless of the kind of securities they hold, claim the total value of the bank less the equity retained by

the insiders. I will henceforth refer to an equilibrium in which the (IR) holds in equality as a fair-price equilibrium.

Proposition 1. *Any equilibrium under ex post financing that satisfies the Intuitive Criterion is a fair-price separating equilibrium in which only a Good bank raises financing and invests. The equilibrium capital structure is given by*

$$\arg \max_{e, F^s, \alpha^s} (1 - \alpha^s) E^{G,s}(F^s) \quad \text{s.t. } e \leq \bar{e}, (PC^B) \text{ and } (IR) \quad (8)$$

Proof. This and all other proofs are provided in Appendix B. □

The above optimisation programme allows us to characterise the equilibrium capital structure that satisfies the Intuitive Criterion in further detail. Firstly, notice that in equilibrium, the bank should always prefer internal financing to outside financing. That is, the equilibrium financing plan involves putting in all the internal capital \bar{e} , whenever outside leverage is used. This is because internal financing is free from either the risk-shifting problem or the asymmetric information problem in this model.

I proceed to consider the optimal mix of debt and equity when outside debt financing is required, and the unique equilibrium leverage is summarised in the following proposition.

Corollary 1 (to Proposition 1). *There exists a threshold \check{e}^s such that, in any equilibrium under ex post financing that satisfies the Intuitive Criterion,*

- *If $\bar{e} \geq \check{e}^s$, the bank issues only equity and no debt.*
- *If $\bar{e} < \check{e}^s$, the unique equilibrium capital structure is $(\bar{e}, \hat{F}^s(\bar{e}), \hat{\alpha}^s(\bar{e}))$, where $\hat{F}^s(\bar{e}) > 0$ and $\hat{\alpha}^s(\bar{e})$ are characterised by the binding participation constraint of the Bad bank (PC^B) and the investors' rationality constraint (IR).*

The intuition for this result is illustrated in Fig. 3, for a Good bank with a given level of internal capital \bar{e} . The figure plots the value of the Good bank $NPV^G + \bar{e}$ (dashed line), and the maximum payoff to the inside shareholders of the Good bank $(1 - \alpha^s) E^{G,s}(F^s)$ (solid line), in any equilibrium with a given leverage F^s . It can be shown that there exist thresholds $\check{F}^s(\bar{e})$ and \hat{F}^s , such that the maximum payoff to the inside shareholders of the Good bank is obtained in a pooling equilibrium amongst equilibria with face value $F^s \leq \check{F}^s(\bar{e})$, it is obtained in a separating equilibrium with underpricing amongst equilibria with face value $F^s \in (\check{F}^s(\bar{e}), \hat{F}^s(\bar{e})]$, and it is obtained in a fair-price separating equilibrium amongst equilibria with face value $F^s \geq \hat{F}^s(\bar{e})$. The leverage in

an equilibrium that satisfies the Intuitive Criterion \hat{F}^s (if $\hat{F}^s \geq 0$) maximises the retained equity payoff in equilibrium to the Good bank, as given by Proposition 1.

Figure 3: Equilibrium selection using the Intuitive Criterion

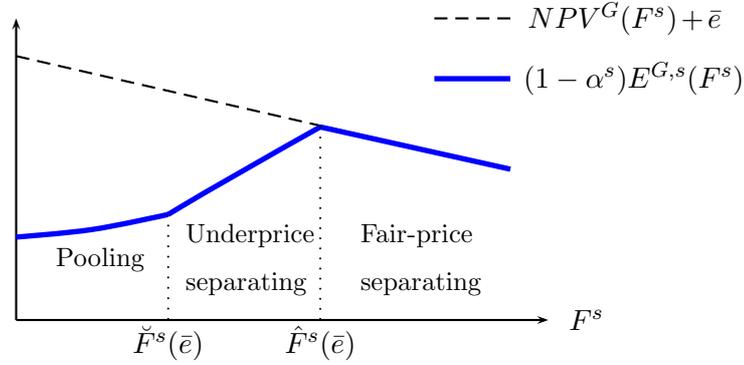


Fig. 3 indicates that the unique leverage level in any equilibrium that satisfies the Intuitive Criterion is the lowest level of leverage that achieves separation at fair-price. This is because, firstly, in any fair-price separating equilibrium, the bank retains the entire NPV created by the bank's lending (the dashed line in Fig. 3, which coincides with the solid line for $F^s \geq \hat{F}^s(\bar{e})$), which is decreasing in the amount of leverage due to risk-shifting problems. It therefore does not have the incentive to increase leverage any further than $\hat{F}^s(\bar{e})$. Secondly, if the bank chooses a leverage level $F^s < \hat{F}^s(\bar{e})$, it has to either underprice the securities issued to signal its type, or pool with a Bad bank. In either case, the Good bank receives less than the full NPV from the lending. By increasing leverage, the Good bank enjoys a private benefit greater than the cost of risk-shifting because of reduced mispricing. The Good bank therefore prefers to separate by using leverage $\hat{F}^s(\bar{e})$.

Corollary 1 shows that in the unique Intuitive equilibrium, only a Good bank raises funds in the capital market, by issuing fairly priced securities. As in the literature on asymmetric information, the constrained efficient outcome can only be achieved when the bank has sufficient internal capital. For $\bar{e} < \check{e}^s$, the bank employs additional leverage and subsequently chooses a higher risk profile.

The framework also demonstrates the intuition of Myers and Majluf (1984) that in the presence of asymmetric information, there is a tendency to rely on internal sources of funds, and to prefer debt over equity if external financing is required. If the bank has sufficient amount of internal capital, the first best result can be achieved.

The pecking order theory, based on asymmetric information alone, is silent about the

determinants of the debt capacity.¹⁸ The interaction between the risk-shifting incentive and the adverse selection problem in this model, similar to that studied by John (1987), endogenously determines the unique equilibrium level of leverage and hence the capital structure. Specifically, leverage mitigates the adverse selection problem, but incurs a cost due to excessive risk-shifting incentives. The debt capacity in this model is thus provided by the extent of the risk-shifting problem.¹⁹

3.2 Equilibrium with *ex ante* financing – Contingent capital structure

I now turn to consider the equilibrium capital structures if the bank raises financing at $t = 0$ prior to knowing the realisation of the underlying economic state. I solve for the equilibrium capital structure within a general class of contingent capital structure given by a set of face values of the debt F_C and a set of fractions of equity issued to outsiders α_C specified for each state s .

A separating equilibrium (e, F_C, α_C) is characterised by the following constraints,

$$(PC_C^B) : \mathbb{E} [(1 - \alpha_C^s)E^{B,s}(F_C^s)] \leq e \quad (9)$$

$$(PC_C^G) : \mathbb{E} [(1 - \alpha_C^s)E^{G,s}(F_C^s)] \geq e \quad (10)$$

$$(IR_C) : \mathbb{E} [V^{G,s}(F_C^s) - (1 - \alpha_C^s)E^{G,s}(F_C^s)] \geq 1 - e \quad (11)$$

This set of conditions is similar to the set of conditions 5–7 for the case of *ex post* financing which was analysed in the previous section. The difference is that in this section, financing is obtained prior to the realisation of the underlying economic state. Since the economic uncertainty only resolves at $t = \frac{1}{2}$, the risk choice at $t = 1$ takes into account the macro state s , while the financing terms at $t = 0$ only relies on the prior distribution of the economic states. The equilibrium conditions are therefore given in expectation, and are weaker than those in the case of *ex post* financing.

Following similar intuition as in the case with *ex post* financing, a bank prefers internal financing to outside financing, and chooses leverage levels to maximise its retained equity payoff, trading off between the asymmetric information and the risk-shifting problems. The resulting equilibria are given as follows.

¹⁸The model of Myers and Majluf (1984) shows that the a firm with private information always issues debt and never issues equity.

¹⁹Other models of capital structure with frictions that predict an interior solution for leverage can also be interpreted as providing a debt capacity, such as Darrough and Stoughton (1986), Leland (1994) among others.

Proposition 2. *Any contingent capital equilibrium under ex ante financing that satisfies the Intuitive Criterion is a fair-price separating equilibrium in which only a Good bank raises financing and invests. The set of equilibrium contingent capital structures is given by*

$$\arg \max_{e, \mathbf{F}_C, \alpha_C} \mathbb{E} [(1 - \alpha_C^s) E^{G,s}(F_C^s)] \quad s.t. \quad e \leq \bar{e}, (PC_C^B) \text{ and } (IR_C) \quad (12)$$

There exists a threshold \check{e}^C such that the bank issues debt if and only if $\bar{e} < \check{e}^C$.

Similar to the case with *ex post* finance, in any equilibrium with *ex ante* financing that satisfies the Intuitive Criterion, the Good bank chooses the leverage level that allows it to separate from the Bad at the least cost of risk-shifting.

3.3 Properties of the *laissez-faire* equilibria

3.3.1 Procyclical equilibrium leverage

This section highlights the procyclicality of the equilibrium leverage in the cases of *ex post* and *ex ante* financing, which creates scope for contingent convertible bonds as discussed in Section 4. In this section I examine the implications of procyclical leverage on the bank's risk-taking incentives and the resulting default probabilities.

Proposition 3. *The face values of debt in an equilibrium that satisfies the Intuitive Criterion are procyclical in both the case with ex post financing and the case with ex ante financing. That is,*

$$\hat{F}^s(\cdot) \geq \hat{F}^z(\cdot), \quad \forall \theta^s > \theta^z \quad (13)$$

where the inequality is strict if and only if $\bar{e} < \check{e}^s$; and

$$\hat{F}_C^s(\cdot) \geq \hat{F}_C^z(\cdot) \quad (14)$$

for any $s, z \in \{s \in \mathbf{S} : \hat{\alpha}_s^C < 1\}$ s.t. $\theta^s > \theta^z$, where the inequality is strict if and only if $\bar{e} < \check{e}^C$.

The procyclicality result is due to the fact that the information asymmetry problem is relatively more severe in a good state when the returns on the loan portfolio are high in general. This is because in this model, the marginal impact on the value of the bank's

claims brought by an increase in the economic fundamentals is greater for a Bad bank than for a Good bank.

$$1 < \frac{E^{G,s}(F)}{E^{B,s}(F)} < \frac{E^{G,z}(F)}{E^{B,z}(F)} \quad \forall \theta^s > \theta^z \quad (15)$$

This is consistent with the view of diminishing marginal return, or when the complementarity between the economic fundamentals and the bank's type is not too high. As a result, the difference between the Good and the Bad banks' (retained) equity value becomes relatively insignificant in booms. Therefore in equilibrium, relatively higher leverage is required during economic booms in order to resolve the information asymmetry.

The result that the equilibrium leverage of banks is procyclical is supported by Adrian and Shin (2008a), who document that changes in total assets are positively correlated with changes in leverage of financial institutions. This model suggests that banks employ procyclical leverage to minimise the cost of asymmetric information and the cost of risk-shifting incentives.²⁰

Although either case produces procyclical leverage in equilibrium, the contingent capital structure in the *ex ante* financing case is less procyclical than the equilibrium in the case with *ex post* financing. This is driven by the fact that the cost of risk-shifting is convex in the amount of leverage in this model, i.e. $\frac{\partial V^{i,s}(F^s)}{\partial F^s} = \frac{F}{2\Delta_X} > 0$.²¹ Financing *ex ante* with a contingent capital structure therefore allows the bank to reduce the cyclicity in its leverage to minimise the expected cost of risk-shifting. This is reflected in the cyclicity of the resulting equilibrium default probabilities (Proposition 4). Intuitively, on the one hand, the intrinsic success probabilities are higher in booms, while on the other hand, procyclical equilibrium leverage implies higher risk-taking in booms which tends to increase the default probabilities of the bank.

Proposition 4. *The equilibrium default probabilities are procyclical in the case of ex post financing. For $\bar{e} \leq \check{e}^z$,*

$$1 - q^{G,s}(\hat{F}^s) > 1 - q^{G,z}(\hat{F}^z) \quad \forall \theta^s > \theta^z \quad (16)$$

²⁰It is widely acknowledged that the leverage ratios of financial institutions are procyclical, contrary to non-financial firms, who tend to exhibit negative correlation between asset returns and leverage ratio, known as the "leverage effect". For example, Adrian and Shin (2008b,a) attribute such observation to banks targeting a leverage ratio given by the value-at-risk, while Gromb and Vayanos (2002) and Geanakoplos (2010) show that collateral constraints generate leverage that tends to be procyclical.

²¹The convexity of the cost of risk-shifting in leverage is also what ensures an interior solution for the choice of leverage.

The equilibrium default probabilities are countercyclical in the case of *ex ante* financing. For $\bar{e} \geq \check{e}^C$ or for any $s, z \in \{s \in \mathbf{S} : \hat{\alpha}_C^s < 1\}$,

$$1 - q^{G,s}(\hat{F}_C^s) \leq 1 - q^{G,z}(\hat{F}_C^z) \quad \forall \theta^s > \theta^z \quad (17)$$

With *ex post* financing, the resulting default probabilities are procyclical. In equilibrium, the bank's choice of leverage overcompensates for the better economic prospects, because they do not fully internalise the cost of leverage in the presence of information asymmetry. To see this, notice that for any leverage level less than \hat{F}^s , a Good bank must issue its securities at a discount because of information asymmetry. The existing shareholders of the bank therefore do not retain the full value created by its loan portfolio. In particular, they share with the outside investors the value destruction brought by an increase in leverage. This conflict of interests leads to excessive risk-taking in booms, sowing the seeds of a bust. Taking the view that securitisation is an important source of funding for banks, this implication is consistent with the findings of Griffin and Tang (2012) that AAA-rated CDO tranches issued between 2003 and 2007, when asset values were high, were of increasingly deteriorating quality leading up to the crisis.

With *ex ante* financing, however, the equilibrium default probabilities given a contingent capital structure are countercyclical. An *ex ante* financing decision allows the bank to take advantage of the relative severity of the information asymmetry problem in different macroeconomic states, which helps the bank to internalise the cost of leverage, alleviating the risk-shifting problem. In particular, to maximise the benefit of leverage in mitigating the information asymmetry problem at the *ex ante* stage, a bank prefers to employ more leverage in the state in which the information asymmetry problem is more severe. This is the state that is less risky as measured by a lower default probability, because the information content in the equity is less pronounced when it is less risky, i.e.

$$1 < \frac{E^{G,s}(F^s)}{E^{B,s}(F^s)} < \frac{E^{G,z}(F^z)}{E^{B,z}(F^z)} \quad \text{iff} \quad q^{i,s}(F^s) > q^{i,z}(F^z) \quad (18)$$

Therefore a bank would never take on so much more leverage in a high state such that it results in a default probability higher than that in a low state, because the excessive leverage employed in the high state is inefficient in resolving either of the two frictions. Specifically, a bank in such a situation would benefit from reducing the excessive leverage employed in the high state and increasing leverage in the low state. In doing so, the bank incurs less cost of risk-shifting in expectation due to the convexity of the risk-shifting

problem in leverage, as well as improves the efficiency in mitigating the information asymmetry problem, as leverage is shifted to the state in which the information asymmetry problem is more severe.

3.3.2 Efficiency of the contingent capital structure

In this section I compare the *ex ante* efficiency of the equilibria in the case of *ex post* financing and the case of *ex ante* financing. *Ex ante* efficiency is measured by the sum of all agents' expected payoffs, or the expected value of the bank, in equilibrium.

Proposition 5. *Raising capital ex ante using a contingent capital structure is (weakly) preferred to raising capital ex post.*

$$\mathbb{E} \left[V^{G,s}(\hat{F}_C^s(\bar{e})) \right] \geq \mathbb{E} \left[V^{G,s}(\hat{F}^s(\bar{e})) \right] \quad \forall \bar{e} \quad (19)$$

where the above inequality is strict for $\bar{e} < \check{e}^C$.

Since the *ex ante* financing problem nests the *ex post* financing problem, it is clear that it is at least as efficient as the *ex post* problem, because the set of constraints for the *ex ante* financing problem is weaker. Moreover, the *ex ante* financing equilibrium is strictly preferred to the *ex post* financing equilibrium whenever the two are not identical. This is for $\bar{e} < \check{e}^C$, as evident from the earlier discussion regarding the cyclicity of the equilibrium default probabilities.

This section asserts that the optimal capital structure is contingent on the realisation of the economic state in the *laissez-faire* equilibrium. The contingent capital structure equilibrium minimises the expected cost of risk-shifting necessary to signal the bank's private information, by employing procyclical leverage to balance the information-sensitivity of the residual equity in different states. The bank would therefore voluntarily issue contingent capital securities to implement the procyclical equilibrium leverage, without restriction.²²

4 Implementation using contingent convertible bonds

The key result of the baseline model is that the optimal contingent capital structures feature procyclical leverage that is higher in booms and lower in busts. Contingent

²²In practice, if the banks are subject to a leverage constraint that binds in all states, they would not employ contingent capital structure, because their capital structure is determined by the leverage constraints.

convertible bonds are a natural addition to debt and equity in order to implement the optimal capital structures of the bank, as it contractually specifies a reduction of the face value of the debt in an economic downturn. This section presents an example of an economy with the possibility of a tail event to illustrate the key features of the CoCo bonds that implement the optimal contingent capital structure. I then discuss the issues surrounding the practical design of CoCo bonds.

In this section, I consider the special case of an economy with two possible states $\mathcal{S} = \{L, H\}$. The High state θ^H occurs with probability β , and the Low state θ^L occurs with probability $1 - \beta$, where β is large. We can interpret the High state as the normal state, and the Low state as an unlikely adverse state – the “tail event”.

4.1 CoCo bonds and procyclical leverage

I characterise the two types of the equilibrium that satisfy the Intuitive Criteria in the tail event economy, and then present an implementation of the equilibrium capital structure using contingent convertible bonds contracted upon the verifiable macroeconomic states in this model.

By Proposition 2–4, the equilibrium of contingent capital structure that satisfies the Intuitive Criterion is a fair-price separating equilibrium in which only a Good bank raises financing and invests. The equilibrium capital structure $(\mathbf{F}_C, \boldsymbol{\alpha}_C)$ are such that the equilibrium leverage and equity values are procyclical, i.e. $\hat{F}_C^H \geq \hat{F}_C^L$ and $E^{G,H}(\hat{F}_C^H) \geq E^{G,L}(\hat{F}_C^L)$.²³

The optimality of procyclical leverage naturally points towards contingent convertible bonds as instruments for the implementation of the optimal capital structure. The reverse convertible feature of CoCo bonds reduces the face value of the debt in the bank’s capital structure in the Low state. By contrast, conventional convertible bonds, which have been shown to play a role in mitigating adverse selection problems (e.g. Brennan and Kraus, 1987; Constantinides and Grundy, 1989) and moral hazard problems (e.g. Green, 1984; Mayers, 1998), do not implement the required contingency in this model.

The following proposition summarises the equilibrium contingent capital structure and proposes an implementation of the equilibrium contingent capital structure that involves CoCo bonds. Two scenarios of contingent capital structures can arise in this tail event

²³That the equity value is procyclical follows the fact that the default probability is countercyclical, because $E^{G,s}(F) = [q^{G,s}(F)]^2 \Delta_X$.

economy depending on the value of \bar{e} .

Proposition 6. *There exist thresholds \check{e}^C and \check{e}^T such that any contingent capital equilibrium that satisfies the Intuitive Criterion is a fair-price separating equilibrium in which only a Good bank raises financing and invests.*

- If $\bar{e} \geq \check{e}^C$, the bank issues only equity and no debt.
- If $\bar{e} \in [\check{e}^T, \check{e}^C)$, the equilibrium contingent capital structure is $(\bar{e}, \{\hat{F}_C^H(\bar{e}), \hat{F}_C^L(\bar{e})\}, \{\hat{\alpha}_C^H(\bar{e}), \hat{\alpha}_C^L(\bar{e})\})$, where $\hat{F}_C^L(\cdot) = \hat{\alpha}_C^L(\cdot) = 0$, and $\hat{F}_C^H(\cdot) > 0$, $\hat{\alpha}_C^H(\cdot) \geq 0$ are given by the binding (PC_C^B) and (IR_C) .

The bank implements the equilibrium contingent capital structure by issuing

- CoCo bonds with face value $\hat{F}_C^H(\bar{e})$ that is written down to zero contingent on the Low state; and,
- Warrants of fraction $\hat{\alpha}_C^H$ designed such that they are only exercised in the High state.

- If $\bar{e} < \check{e}^T$, the equilibrium contingent capital structure is $(\bar{e}, \{\hat{F}_C^H(\bar{e}), \hat{F}_C^L(\bar{e})\}, \{\hat{\alpha}_C^H(\bar{e}), \hat{\alpha}_C^L(\bar{e})\})$, where $\hat{F}_C^H(\cdot) = \hat{F}_C^L(\cdot) + \frac{\theta_H - \theta_L}{\Delta x}$ so that $q^{G,H}(\hat{F}_C^H(\cdot)) = q^{G,L}(\hat{F}_C^L(\cdot))$, and $\hat{F}_C^L(\cdot)$, $\hat{\alpha}_C^H(\cdot)$, $\hat{\alpha}_C^L(\cdot)$ are given by the binding (PC_C^B) and (IR_C) .

The bank implements the equilibrium contingent capital structure by issuing

- Straight bonds with face value $\hat{F}_C^L(\bar{e})$;
- Equity of fraction $\hat{\alpha}_C^H(\bar{e})$; and,
- CoCo bonds with face value $\frac{\theta_H - \theta_L}{\Delta x}$ that convert into equity in the Low state so that the fraction of outside equity becomes $\hat{\alpha}_C^L(\bar{e}) \geq \hat{\alpha}_C^H(\bar{e})$.

The first scenario (corner solution) is for a bank with an intermediate level of internal capital $\bar{e} \in [\check{e}^T, \check{e}^C)$, that issues CoCo bonds with a write-down feature. As the bank is relatively well capitalised, the amount of leverage required to achieve separation is small. Since the debt issued in the High state is relatively less information-sensitive, leverage is only used in the High state, i.e. $\hat{F}_C^H(\bar{e}) > \hat{F}_C^L(\bar{e}) = 0$. This is implemented with CoCo bonds that write down to zero in the Low states. Moreover, given the equilibrium leverage, the residual equity is still less information-sensitive in the High state when the default probability is lower. The equilibrium equity issuance is therefore only in the High state but not in the Low state, implemented with warrant, in order to minimise the “cost

of capital".²⁴ The insiders thus retain full ownership in the Low state to align their incentives, i.e. $\hat{\alpha}_C^H(\bar{e}) \geq \hat{\alpha}_C^L(\bar{e}) = 0$.

The second scenario (interior solution) is for a bank with a low level of internal capital $\bar{e} < \check{e}^T$, that issues CoCo bonds with a contingent convertible feature, in addition to straight debt and equity. In this case, a high amount of leverage is required to achieve separation, and debt must be issued in the low state as well as in the high state. In equilibrium the default probability $1 - q^{G,s}(\hat{F}_C^s(\cdot))$ is equalised between both states in order to minimise the costs of risk-shifting associated with leverage. Moreover, since in this case the equity value is equal in both states, there is one degree of freedom in determining the equity allocation between the High and the Low states. The optimal contingent capital structure in this scenario is therefore implemented using CoCo bonds with face value $\frac{\theta^H - \theta^L}{\Delta_X}$ that converts into equity in the Low state, in addition to straight bonds and equity.²⁵ This scenario highlights the efficiency gain provided by using CoCo bonds to implement the optimal contingent capital structures, which take advantage of the relative severity of the asymmetric information and risk-shifting problems across different states. This is evident from the fact that the face value of the CoCo bonds required $\frac{\theta^H - \theta^L}{\Delta_X}$ is determined by information regarding the verifiable state θ^s but not the private information of the bank η^i .²⁶

This simple model of agency frictions endogenously gives rise to two types of CoCo bonds that are seen in the market. The model predicts that well capitalised banks issue CoCo bonds with a write-down feature, whereas banks in need of much capital issue

²⁴This is in line with the results of Chemmanur and Fulghieri (1997) that warrants can be part of the equilibrium signalling device employed a good issuer with private information. In their model, firms issue warrants because the risk-averse inside shareholders of the good firms, which are also riskier, find it less costly to issue warrants than those of the safer firms that also have lower expected values. The warrants in this model, by contrast, are driven by the fact that in equilibrium, equity is less information-sensitive in the High state than in the Low state.

²⁵Because there is one degree of freedom in determining the fractions of equity held by outsiders across states, implementation using CoCo bonds is feasible for $\hat{\alpha}_C^H(\bar{e}) \geq \hat{\alpha}_C^L(\bar{e})$. The write-down feature is therefore also a special case of the conversion of the CoCo bonds in this scenario, if the fractions of the equity issued to outsiders are equal in both states.

²⁶In this model, the state s is verifiable and thus serves as the trigger for the conversion of the CoCo bonds. Although for the ease of interpretation I have referred to it as a macroeconomic state, the model is also consistent with the interpretation that the state s is driven by idiosyncratic events, provided that they are contractible. I discuss CoCo bonds with market triggers in the following Section

CoCo bonds that convert into equity. For example, the first issues of CoCo bonds were by Lloyds Banking Group with a contingent convertible feature in November 2009 and by Rabobank with a write-down feature in March 2010. The tier 1 capital ratios of the two banks were 8.6% and 13.8% respectively prior to the issuances.²⁷ Credit Suisse issued CoCo bond with a contingent convertible feature in 2011 and 2012, followed by CoCo bonds with a write-down feature in 2013, as the capital position of the bank improved.²⁸

4.2 Interpreting the contingent convertible bonds

4.2.1 The role of commitment and CoCo bonds

It is worth noting that the optimal contingent capital structure can only be implemented with *ex ante* contingent contract. As shown in Section 3.1, a bank raising capital *ex post* after the realisation of the macroeconomic state employs leverage that is excessively procyclical.

After financing is arranged at $t = 0$, it is crucial for the implementation of the optimal contingent capital structure that the bank commits to its pre-specified contingent capital structure until the payoff of the loan portfolio is realised. The model derives the optimal contingent capital structure assuming that the bank commits to the capital structure it chooses at $t = 0$. In fact, this commitment is necessary, as the equilibrium outcome may not be supported in a sequential PBE if the bank is able to alter its capital structure subsequent to the initial offering.

Specifically, leverage in this model sends a credible signal regarding the type of the issuer because the substitution of debt for equity financing is more costly for the Bad bank than for the Good. After the initial offering, however, a levered bank would be better off buying back the debt by issuing equity in order to remove the risk-shifting incentives whenever the market believes that it is of a Good type. Anticipating that the eventual capital structure would be unlevered, at $t = 0$ the investors would not be able to form the separating belief.

In practice, such commitment is likely to be enforced for a number of reasons. Firstly, it is likely to be costly to issue equity subsequently because of new asymmetric information

²⁷Sources: Lloyds Banking Group (2009) and Rabobank Group (2009).

²⁸According to Credit Suisse Regulatory Disclosures (2012), the core tier 1 capital ratio of Credit Suisse under Basel II.5 increased from 10.4% to 14.4% from 2011 to 2012, and its tier 1 capital ratio increased from 15.1% to 18.4%.

problems that can arise. Under the current tax regime, there is also a tax disadvantage to issuing equity. Furthermore, a levered bank would be unwilling to issue additional equity or buy back existing debt because of the “debt-overhang” problem.

Current outstanding CoCo bonds are mostly long term. This helps to create a “debt-overhang” problem that reinforces the commitment by the bank to its optimal contingent capital structure. For example, the contingent convertible bonds issued by Credit Suisse have a 30-year maturity, and the Enhanced Capital Note issued by Lloyds Banking Group has fixed maturities ranging between 10–22 years.²⁹

4.2.2 Trigger design

While so far in this model I refer to the verifiable state s as a macroeconomic state, the model is consistent with a trigger based on idiosyncratic events, provided that they are contractible. Examples of proposals for CoCo bond design based on idiosyncratic events include Flannery (2009), Hart and Zingales (2010, 2011), Pennacchi et al. (2010) and Pennacchi (2011). Others have proposed a dual trigger structure based on both an idiosyncratic event and a measure of macroeconomic downturn, such as Squam Lake Working Group (2009), McDonald (2011) and Kyle (2013).

Alternatives for a trigger that is placed on the idiosyncratic events of the issuing bank include regulatory capital ratios, or market prices of claims on the bank. Although a market price is a “criteri[on] that is informative, objective, timely, difficult to manipulate, and independent of regulators’ intervention, avoiding the problems associated with other types of triggers”, according to Sundaresan and Wang (2013), all the CoCo bonds that are issued thus far have triggers based on regulatory capital ratios. This is because a conversion trigger based on the market price of equity can suffer from the multiplicity or the absence of equilibria in the price of the equity of the issuing bank. Sundaresan and Wang (2013) show that a unique competitive equilibrium exists only if the conversion is designed to leave the equity price equal before and after the conversion. Moreover, the authors claim that this condition precludes penalising the existing shareholders, defeating the purpose of the CoCo bonds.

²⁹This model provides a supply-side rationale for the long-term nature of CoCo bonds. Bolton and Samama (2012) and Hart and Zingales (2010) present a demand-side argument that CoCo bonds are likely to be purchased by long-term investors seeking to enhance yield in good times by risking losses in bad times.

This model can be modified to allow a trigger based on the equity price of the bank. In this model, there exists a conversion ratio of the CoCo bonds that gives rise to a unique pricing equilibrium, while diluting the existing shareholders. This contrasts with the result of Sundaresan and Wang (2013), because unlike their model, this model endogenises the risk taking incentives of the bank and hence the bank value. The conversion of the CoCo bonds creates value since it lowers the leverage of the bank and reduces the bank's incentive to take excessive risk. The condition that the equity price remains constant therefore implies that the fraction of equity held by existing shareholders is diluted upon conversion.

Formally, assume that the state s is observable but not contractible, and that there is no frictions in the secondary market so that the price of the equity reflects perfectly the value of the equity in state s , given the capital structure of the bank and the market belief.³⁰ Suppose the bank has outstanding straight debt, CoCo bonds and common equity according the optimal contingent capital structure, and the number of common shares outstanding is N . The equity price in the high state is thus $p_{w/o}^H \equiv \frac{E^{G,H}(\hat{F}_C^H(\bar{e}))}{N}$ without conversion. Suppose that the trigger price is set at $p_{w/o}^H$. That is, conversion occurs if the price falls below $p_{w/o}^H$, and the CoCo bonds convert into n shares of common equity, which correspond to a fraction $\frac{n}{N+n} \equiv \hat{\alpha}_C^L(\bar{e}) - \hat{\alpha}_C^H(\bar{e})$ of the equity. In the Low state, the equity prices with and without conversion are given by $p_w^L \equiv \frac{E^{G,L}(\hat{F}_C^L(\bar{e}))}{N+n} < p_{w/o}^H$ and $p_{w/o}^L \equiv \frac{E^{G,L}(\hat{F}_C^H(\bar{e}))}{N}$ respectively.³¹ Consider a conversion ratio characterised by n such that the equity prices with and without conversion in the Low state are equal, i.e. $p_w^L = p_{w/o}^L$. It then follows that such a conversion ratio is indeed dilutive, i.e. $\hat{\alpha}_C^L(\bar{e}) - \hat{\alpha}_C^H(\bar{e}) = 1 - \frac{E^{G,L}(\hat{F}_C^H(\bar{e}))}{E^{G,L}(\hat{F}_C^L(\bar{e}))} > 0$.³²

³⁰I abstract from the discussion regarding the informativeness of the equity price. Martynova and Perotti (2012) discuss the efficiency of an exogenously given CoCo bond triggered by equity prices that are noisy signals of the equity value. The authors find that a mandatory conversion based on such equity prices leads to more frequent conversions, and a regulatory trigger produces fewer conversions.

³¹That $p_w^L = \frac{E^{G,L}(\hat{F}_C^L(\bar{e}))}{N+n} < p_{w/o}^H = \frac{E^{G,H}(\hat{F}_C^H(\bar{e}))}{N}$ is due to the fact that the equity values are equal in both states given the optimal contingent capital structure, i.e. $E^{G,H}(\hat{F}_C^H(\bar{e})) = E^{G,L}(\hat{F}_C^L(\bar{e}))$.

³²It is straightforward to check that given this conversion ratio, the unique equilibrium is indeed for the CoCo bonds to convert only in the Low state. Firstly, since the equity prices in the Low state with and without conversion are both below the trigger price, $p_w^L = p_{w/o}^L < p_{w/o}^H$, the unique equilibrium in the Low state is for the CoCo bonds to convert. Secondly, the unique equilibrium in the High state is for the CoCo bonds not to convert, and for the equity price of be equal to $p_{w/o}^H$. In the High state, the equity price, if the CoCo bonds convert, is given by $p_w^H \equiv \frac{E^{G,H}(\hat{F}_C^L(\bar{e}))}{N+n}$. However, since the conversion ratio

Hart and Zingales (2010) propose an alternative trigger mechanism based on the price of the CDS spreads, which reflects the default risk of the issuing bank. In this model, for a bank with a low level of internal capital $\bar{e} < \check{e}^T$, the optimal contingent capital structure specifies a conversion of the CoCo bonds in the Low state to keep the default probability constant. This property suggests that the CDS spread can also act as a conversion trigger that is free from the equilibrium problem, following similar arguments as those given above for a trigger based on equity prices.

5 Optimal countercyclical capital regulation

Although the equilibrium capital structure trades off the cost and benefit of leverage, the bank takes on excessive leverage when its level of internal capital is low, due to its private incentive to minimise the mispricing in the securities it issues. The *laissez-faire* equilibrium thus warrants regulation, since excessive risk-shifting by the bank decreases the value of the businesses the bank lends to, incurring a social cost. In this section I characterise the optimal capital requirements, which are countercyclical, and discuss how CoCo bonds can be used to implement the regulated equilibrium.

In any given state s , a minimum capital requirement is defined as a cap on the face value of the debt, \bar{F}^s . Given a capital requirement, there exist Intuitive equilibria in which the bank issues debt with face value no higher than \bar{F}^s in state s . The optimal *ex ante* capital requirement is a set of state caps $\bar{\mathbf{F}} \equiv \{\bar{F}^s\}_{s=1}^S$ such that it maximises the expected value of the bank in the resulting equilibria that satisfy the Intuitive Criterion.

Imposing a cap on the face value of the debt \bar{F}^s is equivalent to requiring a minimum capital ratio $c^{i,s}(\bar{F}^s) \equiv \frac{E^{G,s}(\bar{F}^s(\cdot))}{V^{G,s}(\bar{F}^s(\cdot))}$ in a given state s , because the capital ratio of the bank is monotonically decreasing in \bar{F}^s . In this section I refer to a minimum capital requirement as a cap on the face value of the debt, as opposed to a capital ratio, to simplify exposition and to allow direct comparison with the previous section. The optimal capital regulation $\bar{\mathbf{F}}$ derived below can be implemented with a state-contingent minimum capital ratio $\mathbf{c} \equiv \{c^s\}_{s \in \mathcal{S}}$.

results in less dilution in the High state than in the Low state when the equity value is higher for any given amount of leverage, a conversion would result in a higher equity price $p_w^H > p_{w/o}^H$, which contradicts with the conversion trigger of the CoCo bonds.

5.1 Optimal capital regulation

The following proposition summarises the optimal capital regulation.

Proposition 7. *The optimal minimum capital requirement can improve the efficiency of the laissez-faire (contingent) capital structure equilibrium and produce the constrained efficient outcome. There exists \check{e}_P^C and \check{e}_{LP}^C such that*

- For $\bar{e} \geq \check{e}^C$, the capital requirement never binds. The bank issues only equity in the capital market and invests if and only if it is of the Good type.
- For $\bar{e} \in [\check{e}_P^C, \check{e}^C)$, the capital requirement of maximum leverage $\bar{\mathbf{F}}(\bar{e}) = \mathbf{0}$ binds. The bank issues only equity in the capital market and invests if and only if it is of the Good type.
- For $\bar{e} \in [\check{e}_{LP}^C, \check{e}_P^C]$, the capital requirement of maximum leverage $\bar{\mathbf{F}}(\bar{e}) = \check{\mathbf{F}}(\bar{e})$ binds. The bank issues both debt and equity in the capital market and invests if and only if it is of the Good type. $\check{\mathbf{F}}(\bar{e})$ is the least-cost leverage cap a regulator can impose that implements a separating equilibrium, which is given by

$$\arg \max_{\mathbf{F}_C} \mathbb{E} [V^{G,s}(F_C^s)] \quad \text{s.t.} \quad v^G(\mathbf{F}_C; \bar{e}) \geq v_P^G(\mathbf{F}_C; \bar{e}) \quad (20)$$

where $v^G(\mathbf{F}_C; \bar{e})$ and $v_P^G(\mathbf{F}_C; \bar{e})$ are the expected retained equity value of the Good bank in the least-cost separating equilibrium that in the least-cost pooling equilibrium respectively, given the capital regulation.

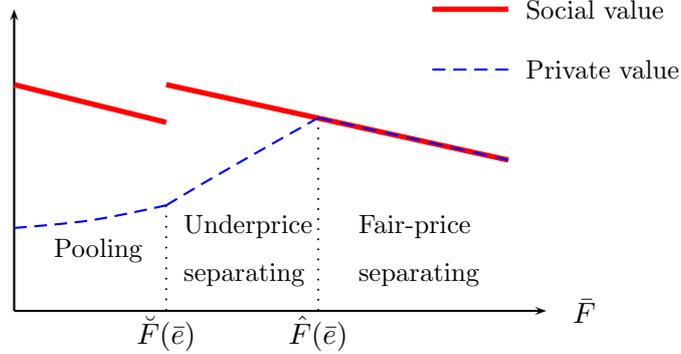
- For $\bar{e} < \check{e}_{LP}^C$, the capital requirement of maximum leverage $\bar{\mathbf{F}} = \mathbf{0}$ binds. The bank issues only equity in the capital market and invests regardless of its type.

I discuss the intuition for Proposition 7 using, as an example, the simplest case where there is only one macroeconomic state, i.e. no macroeconomic uncertainty. In this case the capital regulation is a single cap on leverage \bar{F} . I will comment on the cyclicity of the optimal capital regulation in relation to macroeconomic uncertainty in the next section (Section 5.2).

Fig. 4 highlights the inefficiency in the *laissez-faire* equilibrium and hence the rationale for capital regulation. The figure plots the social value produced by the bank (solid line) and the payoff to the inside shareholders of the Good bank $(1 - \alpha^s)E^{G,s}(F^s)$, in the equilibrium that maximises the payoff to the inside shareholders of the Good bank

for a given leverage F^s . The social value is measured as the value of the Good bank, $NPV^G(F^s) + \bar{e}$, if the equilibrium is separating, and as the expected value of the bank, $\gamma NPV^G(F^s) + (1 - \gamma)NPV^B(F^s) + \bar{e}$, if the equilibrium is pooling.

Figure 4: Inefficiency in the *laissez-faire* equilibrium



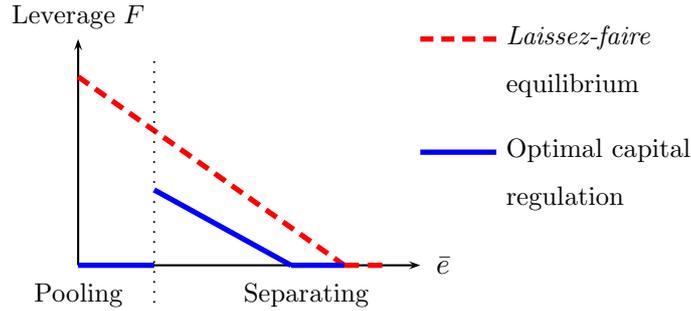
Section 3 has shown that in the *laissez-faire* equilibrium, a bank with $\bar{e} < \check{e}^C$ chooses leverage level $\hat{F}(\bar{e})$ when raising capital, which maximises its retained payoff. Figure 4 illustrates that the *laissez-faire* equilibrium leverage \hat{F} is too high and induces excessive risk-taking by the bank. Although a lower leverage level increases the value of the bank and the social value produced by the bank's lending, the bank is unwilling to reduce its leverage because of the underpricing in the securities it issues due to asymmetric information.

A capital regulation that reduces the equilibrium leverage improves the social value of a Good bank with $\bar{e} < \check{e}^C$, as represented by the solid line in Fig. 4. However, the extent to which the regulator can cap leverage is limited by the bank's private incentives to maximise its retained payoff. As the bank's leverage is capped to be less than $\hat{F}(\bar{e})$, the bank finds it privately costly to signal its type through underpricing. Faced with a stringent capital requirement $\bar{F} < \check{F}(\bar{e})$, the Good bank would prefer to pool with the Bad bank, instead of incurring the heavy cost of signalling (Eq. 20 is violated). This results in another social loss, as it enables the Bad bank to raise capital and make value-destroying investments.

Constrained by the private incentives of a bank with little internal capital $\bar{e} < \check{e}_P^C$, the optimal capital regulation is either $\bar{F}(\bar{e}) = \check{F}(\cdot)$ to implement a separating equilibrium, or $\bar{F}(\bar{e}) = 0$ to implement a pooling equilibrium, depending on whether it is less costly to resolve the asymmetric information problem or to curb the risk-shifting incentives. This result highlights the inherent tension between how the information asymmetry and the

risk-shifting problems can be solved in this model. That is, higher leverage mitigates the information asymmetry problem as it reduces the information-sensitiveness of the securities issued and hence the potential mispricing, but it also incurs social losses due to the excessive risk-taking incentives it creates. Fig. 5 illustrates how this trade-off varies with the bank’s internal capital \bar{e} .

Figure 5: Optimal capital regulation



It is worth noting that the optimal capital requirement $\bar{F}^s(\bar{e})$ depends on the level of the bank’s internal capital \bar{e} . This raises an important distinction between “inside” and “outside” capital. Regardless of the type of the security issued to obtain outside financing, the equilibrium outcome depends on the bank’s “skin in the game” \bar{e} , rather than the total debt to equity ratio.

Depending on the bank’s internal capital \bar{e} , the optimal capital regulation falls into one of the two regions (Fig 5). For a relatively better capitalised bank, the optimal capital regulation limits the amount of leverage the bank can employ, in order to reduce the cost of risk-shifting while allowing the bank to signal through underpricing in addition to leverage (the separating region). For poorly capitalised banks, however, the amount of leverage required to achieve separation is high, and the optimal capital regulation imposes zero leverage and implements a pooling equilibrium (the pooling region).

The optimal capital requirement, while improving social value by curbing excessive risk-taking induced by leverage, nevertheless makes financing costly for the Good bank in terms of mispricing. The amount of mispricing is illustrated by the wedge between the private value received by the bank and the social value (Fig. 4). This is consistent with the observation that equity capital is generally perceived to be expensive. For example, Elliott (2009, p.12) states that “the problem with capital is that it is expensive. If capital were cheap, banks would be extremely safe because they would hold high levels of capital.” In this model capital is costly in the regulated equilibrium because of the two frictions,

in the absence of which the bank would indeed choose no capital market debt and the first best risk level. As argued by Admati et al. (2010), however, it should be noted that capital is not necessarily socially costly, which creates scope for capital regulation.

5.2 Countercyclical capital regulation and CoCo bonds

Having characterised the optimal capital requirements, this section explores the countercyclical property of the optimal capital regulation using again the example of the tail event economy, and discusses the role of CoCo bonds in the regulated equilibrium.

A minimum capital requirement is defined as countercyclical, if in the resulting equilibrium, the bank has a countercyclical capital ratio.

Proposition 8. *In the tail event economy, the optimal minimum capital requirement is countercyclical. That is, given the procyclical leverage caps $\bar{F}^H(\bar{e}) \geq \bar{F}^L(\bar{e})$, the bank has a countercyclical capital ratio $c^H \leq c^L$, where $c^s \equiv \frac{E^{G,s}(\bar{F}^s)}{V^{G,s}(\bar{F}^s)}$. The inequalities are strict for $\bar{e} \in [\check{e}_{LP}^C, \check{e}_P^C]$.*

In particular, there exists a threshold \check{e}_{LP}^T such that for $\bar{e} \in [\check{e}_{LP}^C, \check{e}_{LP}^T]$, the bank implements the equilibrium contingent capital structure subject to capital regulation by issuing CoCo bonds with the same face value as in the laissez-faire equilibrium $\frac{\theta^H - \theta^L}{\Delta_X}$ that convert into equity in the Low state, in addition to straight debt with a lower face value than in the laissez-faire equilibrium $\bar{F}^L(\bar{e}) < \hat{F}_C^L(\bar{e})$ and equity.

That the capital regulation imposes procyclical leverage follows the same reasoning to those of Proposition 3. As the capital regulation in this case intends to achieve separation (following Proposition 7), higher leverage is required in the High state, when asset values are high and the asymmetric information problem is more severe. The amount of CoCo bonds in the bank's capital structure is therefore the same as in the *laissez-faire* equilibrium, capturing the relative severity of the asymmetric information problem across the High and the Low states (for parameters that give rise to an interior solution). Nevertheless, the optimal capital regulation lowers the amount of leverage in the resulting equilibrium, improving the social value of the bank.

The proposition suggests that the optimal capital regulation is countercyclical. The capital ratio in the regulated equilibrium is countercyclical because the optimal procyclicality of the leverage equalises the equity values across the High and the Low states, in order to maximise the efficiency of the leverage in mitigating the asymmetric

information problem, following similar reasoning to those of Proposition 4. The capital ratio is therefore higher in the Low state, as the bank value is lower. This result confirms the optimality of countercyclical capital proposed by the Bank for International Settlement (Basel Committee on Bank Supervision, 2010), in which the regulators call for a capital buffer that takes account of the macro-financial environment in which banks operate. The proposal suggests that the buffer be “deployed ... when excess aggregate credit growth is judged to be associated with a build-up of system-wide risk to ensure the banking system has a buffer of capital to protect it against future potential losses.” This idea, advocated by scholars such as Brunnermeier et al. (2009) and Griffith-Jones and Ocampo (2011), is shown to be optimal in this model of risk-shifting incentives and adverse selection.

CoCo bonds emerge as part of the implementation of the optimal countercyclical capital regulation. Currently Basel III developed by the Basel Committee on Bank Supervision (2011, p. 58) requires that the countercyclical capital buffer is deployed by national jurisdictions with a pre-announcement by up to 12 months before the system-wide risk materialises, to allow time for banks to adjust to a buffer level. In practice, it is not clear that the build-up of the buffer can always be achieved in a timely fashion. In light of the results in this model, the optimal countercyclical capital buffer can be implemented using CoCo bonds, which are voluntarily issued by banks well before the build-up of the system-wide risk to meet the countercyclical capital requirement. The CoCo bonds, subject to a mandatory conversion triggered by a regulatory declaration of a state of systemic risk, implements an immediate recapitalisation of the banks.

6 Extensions

The baseline model thus far considers the role of leverage in a bank’s capital structure in mitigating the asymmetric information problem at the cost of inducing risk-shifting by the bank. The optimal contingent capital structure entails procyclical leverage implemented with CoCo bonds, because the asymmetric information problem is relatively more severe in booms. Due to the bank’s private incentive to reduce any mispricing in the securities it issues, however, the bank employs excessive leverage, and incurs a social cost as its subsequent risk-shifting behaviour negatively impacts the value of the businesses the bank lends to. The *laissez-faire* equilibrium thus warrants countercyclical capital regulation,

which curbs excessive leverage and risk-taking, while preserving the procyclicality of the bank's leverage.

Recurrent financial crises have highlighted the systemic effect of large and correlated bank failures, including a direct impact on the real economy through credit contraction, and a network effect on the financial health of other financial institutions.³³ Consequently, regulators are unwilling or unable to allow major financial institutions to fail.³⁴ While extending state guarantee can be *ex post* optimal for a regulator to alleviate the systemic impact of a bank failure, it creates a moral hazard problem that incentivises *ex ante* risk-taking by the bank.

This section provides two extensions of the model to consider two types of government guarantees: implicit bailouts and explicit deposit insurance. I analyse the moral hazard problem created by the government guarantees and examine the implications for the optimal capital regulation in relation to the frictions considered in the baseline model, using again the example of a tail event economy.

6.1 Bailout and bail-in

This section modifies the baseline model to incorporate the systemic importance of the bank. I assume that the regulator has the incentive to bail out a failed bank *ex post* by repaying the creditors on the bank's behalf, to reduce the systemic externalities posed by the failure of the bank. I analyse the moral hazard problem created by the bailouts and study the optimal *ex ante* capital regulation. Noticing that the *ex post* bailout incentives hinder the efficiency of the *ex ante* capital regulation, I discuss the role of "bail-in" capital in alleviating the moral hazard problem created by bailouts.

Assume that a systematically important bank failing to meet its promised repayment to creditors poses a large cost to the rest of the economy $\bar{\xi}$. Although not modelled explicitly, this large social cost intends to capture the systemic effects of a bank failure and the costs to the real economy. Assume also that an *ex post* bailout by the regulator

³³For example, Ivashina and Scharfstein (2010) document a run by short-term bank creditors following the failure of Lehman Brothers, which contributed to a 47% reduction in the extension of new loans to large borrowers in the fourth quarter of 2008 relative to the previous quarter.

³⁴For example, the run on Northern Rock, Britain's fifth-largest mortgage lender, did not stop until a taxpayer-backed guarantee of all existing deposits was announced in September 2007, and the US Treasury and the Federal Reserve System bailed out 282 publicly traded banks and insurance companies under the Troubled Asset Relief Program (TARP) in 2008–9. Sources: Economist (2007) and Wilson and Wu (2012).

can reduce the cost to $\underline{\xi} < \bar{\xi}$, which accounts for any remaining externalities that cannot be resolved by a bailout and the deadweight loss associated with financing the bailout.

6.1.1 Bailouts and moral hazard

Suppose the regulator maximises social welfare but cannot commit to any time-inconsistent policies. *Ex post* at $t = 2$, the government has the incentive to bail out a failed bank, by repaying the creditors on the bank's behalf to reduce the social cost from $\bar{\xi}$ to $\underline{\xi}$.

The incentives for a bailout at $t = 2$ creates moral hazard problems for the bank. At $t = 2$, the bank receives a bailout whenever its cash flow is less than its promised repayment F^s . Anticipating a bailout, the bank issues effectively risk-free debt in the capital market at $t = 0$ and receives an implicit subsidy of the amount $\mathbb{E}[(1 - q^{i,s}(F^s))F^s]$ for any given debt structure \mathbf{F} . Since the value of the implicit subsidy is increasing in F^s , the bank without restriction takes on unlimited leverage and destroys the value in the loans through subsequent risk-taking. This simple extension of the model captures the moral hazard problem of government bailouts, such as those documented by Duchin and Sosyura (2013).³⁵

Recognising the moral hazard problem as well as the inefficiency discussed in Section 5, the regulator at $t = 0$ designs the optimal *ex ante* capital regulation $\bar{\mathbf{F}}$ as defined in 5, with the expectation of a bailout at $t = 2$. The *ex ante* objective of the regulator is to maximise the expected social value of the bank less the social loss associated with bank failures, i.e. $\mathbb{E}[V^G, s(\bar{\mathbf{F}}^s) - (1 - q^{G,s}(\bar{\mathbf{F}}^s))\underline{\xi}]$. The following proposition characterises the optimal *ex ante* capital regulation of a systemically important bank, given the expectation of a bailout *ex post*.

Proposition 9. *In the tail event economy, with the expectation of a bailout ex post, the optimal minimum capital requirement is countercyclical.*

In particular, there exist thresholds \check{e}_{LP}^{BO} and \check{e}_P^{BO} such that, for $\bar{e} \in [\check{e}_{LP}^{BO}, \check{e}_P^{BO}]$, the optimal capital requirement for a systemically important bank implements a contingent capital structure equilibrium in which the bank issues CoCo bonds with the same face value as in the baseline model $\frac{\theta^H - \theta_L}{\Delta_X}$ that convert into equity in the Low state, in addition

³⁵Duchin and Sosyura (2013) show that banks make riskier loans and shift investment portfolios towards riskier securities after being approved for government assistance.

to straight debt with a higher face value than in the baseline model under optimal capital regulation $\bar{F}_{BO}^L(\bar{e}) > \bar{F}_T^L(\bar{e})$, and equity.

The optimal capital regulation of a systemically important bank remains countercyclical, and can be implemented using CoCo bonds with the same face value as in the baseline model (for parameters that give rise to an interior solution). This is consistent with earlier results that the amount of CoCo bonds in a bank’s capital structure is determined by the relative severity of the asymmetric information problem across the High and the Low states, in order to minimise the expected cost of risk-shifting induced by the leverage required to signal the private information of the bank.

Relative to the optimal capital regulation derived in Section 5, the optimal capital regulation of a systemically important bank permits higher leverage in the form of straight debt. This follows from the earlier intuition that, the extent to which the regulator can impose leverage caps is limited by the bank’s private incentives. Since an *ex post* bailout creates a moral hazard problem which reduces the private cost of risk-shifting internalised by the bank, higher leverage must be allowed to alleviate the asymmetric information problem. The *ex post* bailout of a failed bank to mitigate the *ex post* social loss therefore hinders the efficient capital regulation of the bank *ex ante*.

6.1.2 Bailout and bail-in

Bail-in capital, a form of contingent debt that is automatically wiped out in case the bank fails, has been considered by regulators such as the Basel Committee on Bank Supervision (2011) and the Bank of England, in order to shield taxpayers from the need to bail out a bank that is “too big to fail”.³⁶ In light of the results in this extension, if a regulator can credibly commit to bailing in the debt *ex post*, strictly capital regulation, as characterised in Section 5, can be imposed, resulting in lower leverage and less risk-taking by banks in equilibrium.

With all capital market debts “bail-inable”, the bank writes down its liabilities at $t = 2$ in case it cannot meet the promised repayments. As the bank now fully internalises the cost of risk-taking when it issues risky debt in the capital market *ex ante*, the equilibrium and the optimal capital regulation are identical to those characterised in Section 3–5 and can be implemented using CoCo bonds. That is, the bank issues the optimal amount of

³⁶Source: Federal Deposit Insurance Corporation and Bank of England (2012).

equity, straight debt and CoCo bonds at $t = 0$, where both the straight debt and CoCo bonds have a bail-in feature. At the interim, if a low microeconomic state realises, the CoCo bond conversion is triggered, subsequently reducing the incentives for risk-shifting. Finally at $t = 2$ when the loan portfolio pays off and the securities mature, the debt holders and CoCo bond holders (if they remain unconverted) are paid off if the bank produces sufficient cash flows. Otherwise, all bonds are written off according to the bail-in arrangement, and the bank enters into an orderly resolution.

This extension highlights the differential roles played by the two types of contingent capital instruments. While CoCo bonds provide early conversion well before bankruptcy to implement the optimal procyclical leverage, bail-in capital features a write-down of the debt only in the event of a default. Bail-in capital is effective in avoiding socially costly bank failures and forcing the bank to internalise the cost of risk-taking *ex ante*, thereby resolving the moral hazard problem created by the government's incentive to bail out failed banks *ex post*. Given the *ex post* incentives to bail out a failed bank, not only does bail-in capital improve the efficiency of the bank in managing its loan portfolio, it also leads to fewer incidences of bank failures *ex post*, reducing the cost associated with the failure of a systemically important bank.

6.2 Depositors and deposit insurance

Banks perform the important functions of maturity transformation and risk transformation by offering deposit contracts. The model thus far has abstracted from this aspect by assuming risk neutral investors, to focus on the trade-off effect of leverage across different macroeconomic states. This section modifies the model to extend the funding base of the bank to include a set of risk averse, unsophisticated depositors with liquidity needs.

I provide conditions for when deposit insurance is essential, and show that deposit insurance creates a moral hazard problem similar to that of a bailout, in that it allows the bank access to cheap funding via insured deposits, without internalising the cost of its risk-taking decision. As a result, the optimal *ex ante* capital regulation in this extension remains countercyclical and can be implemented with CoCo bonds, but permits high levels of leverage using a combination of demand deposits and straight debt, because the extent to which the bank can cap leverage is limited by the bank's private incentives.

6.2.1 Depositors

In this extension I assume that the risk neutral capital market investors have limited capital $k < 1 - \bar{e}$, but there is an unlimited supply of funds from a continuum of depositors. The bank thus must raise financing from the depositors in order to finance its lending.

Each depositor has 1 unit of endowment and faces idiosyncratic liquidity shocks following Diamond and Dybvig (1983). I assume the unsophisticated depositors can only observe whether or not their bank runs out of funds. Therefore they would only accept a debt-like contract.

Each depositor demands early consumption at $t = 1$ with probability λ or late consumption at $t = 2$ otherwise. The utility of each depositor is given as

$$U(c_1, c_2) = \begin{cases} u(c_1) & \text{if Early, with prob. } \lambda \\ u(c_2) & \text{if Late, with prob. } 1 - \lambda \end{cases} \quad (21)$$

At $t = 0$, a depositor is willing to deposit with the bank if the bank offers expected utility of at least $u(1)$. I normalise $u(0) = 0$, $u(1) = 1$. The liquidity shock is i.i.d. across depositors and unobservable while the probability λ is common knowledge.

6.2.2 Deposit insurance and moral hazard

As the bank must raise financing at least partially via demand deposits, this section provides conditions for when deposit insurance is essential for the depositors to be willing to provide capital. I then discuss the moral hazard problem associated with deposit insurance and the implications for the optimal capital regulation.

Deposit insurance is a commitment from the government to inject up to \bar{D} to the bank in case the bank runs out of funds to repay the depositors. I assume that the deposit insurance is financed by charging an *ex ante* fair insurance premium so there is no deadweight loss associated with proving the insurance *ex post*.

Suppose the bank takes $\frac{D}{1-\lambda}$ amount of deposits from a mass $\frac{D}{1-\lambda}$ of depositors, promising each depositor a consumption plan (c_1, c_2) upon request, where $c_1 \leq c_2$ so that it is incentive compatible for a late depositor to wait. *Ex post* exactly a mass $\frac{\lambda}{1-\lambda}D$ of the depositors demand early repayment at $t = 1$, and the remaining mass D of depositors wait until $t = 2$.³⁷ In order to prevent illiquidity at $t = 1$, the bank holds $\frac{\lambda}{1-\lambda}D$ in liquid

³⁷Assuming sequential service of deposits and given the government guarantee at $t = 2$, the Late depositors prefer waiting until $t = 2$ to joining the queue at $t = 1$.

reserve (storage) to meet the withdrawal at $t = 1$.³⁸ The bank's $t = 2$ liability to the depositors is therefore Dc_2 . Denote the total liability of the bank in state s as F^s .

At $t = 2$, there is a positive probability that the bank returns 0. Assuming sequential service of deposits, in case the bank runs out of funds, a depositor who joins the queue late receives nothing, whereas one who is at the front of the queue receives full promised repayment. With any given deposit insurance $\bar{D} \leq Dc_2$, in a symmetric equilibrium in which all late depositors wait until $t = 2$, a late depositor receives the full repayment with probability $\frac{\bar{D}}{Dc_2}$, and 0 otherwise. A depositor is willing to deposit with the bank only if his expected utility provided by the deposit contract is at least that if he enjoyed his endowment with certainty, given the capital structure of the bank. Having normalised $u(0) = 0$ and $u(1) = 1$, the break-even condition of a depositor is

$$\lambda u(c_1) + (1 - \lambda) \mathbb{E} \left[q^{i,s}(F^s) + [1 - q^{i,s}(F^s)] \frac{\bar{D}}{Dc_2} \right] u(c_2) \geq 1 \quad (22)$$

The following proposition provides a sufficient condition for full deposit insurance to be necessary to fund the bank.

Proposition 10. *If the depositors are sufficiently risk averse such that $u'(1) \leq \frac{(1-\lambda)(1-\bar{e}-k)}{2\Delta_X}$, the bank cannot raise deposits without a deposit insurance. The cost to the government for providing the insurance is minimised at full coverage up to $\bar{D} = 1 - \bar{e} - k$.*

Proof. Appendix B.9 □

The intuition is as follows. For a given amount of deposit insurance $\bar{D} < Dc_2$, a risk averse depositor would demand a payoff $c_2 > 1$ at $t = 2$, knowing that the bank defaults with positive probability. In turn, a higher liability induces the bank to increase its portfolio risk in an attempt to maximise shareholders' value, resulting in higher default probability. If the depositors are sufficiently risk averse, they require a high yield which creates a risk-shifting problem so severe that the depositors would not deposit with the bank whenever the deposit is risky. Therefore the government must provide full deposit insurance $\bar{D} = D$. Given full coverage, the demand deposit is risk free and the bank

³⁸The bank optimally chooses to hold reserve to prevent premature bankruptcy given its own capital input $e > 0$. If there is a new generation of depositors at $t = 1$, the bank can also raise fresh capital via deposits at $t = 1$ to meet the withdrawal by the first cohort of depositors. This extension of the model refrains from the coordination problem at $t = 1$ to focus on the moral hazard problem associated with possible bank failures at $t = 2$.

chooses to promise $c_1 = c_2 = 1$ to allow the depositor to break even. This part of the result suggests that the risk aversion of the outside investors exacerbates the risk-taking incentives of the bank.

A full coverage deposit insurance creates a moral hazard problem similar to that of a government bailout. Given the total $t = 2$ liability \mathbf{F} and a full coverage guarantee on the deposit \bar{D} , the bank has the incentive to maximise its deposit base up to the coverage of the deposit insurance programme, as it enjoys an implicit subsidy of the value $\mathbb{E}[1 - q^{i,s}(F^s)]$ per unit deposit taken. The total value of the bank, therefore, consists of the value of the loan portfolio $\mathbb{E}[V^{i,s}(\cdot)]$ and the implicit subsidy of value $\mathbb{E}[1 - q^{i,s}(F^s)]\bar{D}$, which is increasing in the deposit coverage \bar{D} . As the bank cannot be financed without deposits, which induce risk-shifting in equilibrium, the first best outcome is not attainable. For constrained efficiency, the government should provide the minimum amount of deposit insurance to enable financing $\bar{D} = 1 - \bar{e} - k$ as long as the value of the loan portfolio $V^{G,s}(\bar{D}) > 1$. I will assume this is true for comparability with the baseline model.

With the optimal deposit insurance \bar{D} in place, the financing plan of a bank is now given by $(e, D, \mathbf{F}, \boldsymbol{\alpha})$, where $F^s \geq D$ is the total book leverage of the bank at $t = 2$, of which $(F^s - D)$ is capital market debt. The bank, when making the capital structure choice, considers the value of the bank as the sum of the value of the loan portfolio and the implicit subsidy provided by the deposit insurance. That is, the bank of type i in state s given the leverage level is $B^{i,s}(F^s) \equiv V^{i,s}(F^s) + [1 - q^{i,s}(F^s)]\bar{D}$. Assuming that this is the case, the following proposition summarises the consequence of the moral hazard problem associated with deposit insurance, which is similar to that of bailouts.

Proposition 11. *Lower availability of capital market funds increases the reliance of banks on fully insured deposits. This induces greater leverage and risk-taking by the banks. The procyclicality of the leverage in the optimal contingent capital structure and the countercyclicality of the optimal capital requirements derived in Section 3–5 go through. In particular, the face value of CoCo bonds remain the same at $\frac{\theta^H - \theta^L}{\Delta_x}$ for banks with little internal capital.*

This result contrasts with the baseline case with only risk neutral investors. Deposit insurance provides the bank with an implicit subsidy, similar to the bailout of capital market debts analysed in Section 6.1. However, while capital market debts can be made “bail-inable” to mitigate the moral hazard problem associated with *ex post* bailouts,

depositors must be protected in all circumstances, leaving an implicit subsidy of the amount $\mathbb{E}[1 - q^{i,s}(F^s)]\bar{D}$, which lowers the effective cost of leverage for the bank. The equilibrium capital structure chosen therefore entails higher leverage, trading off the benefit of mitigating asymmetric information problems and the cost of inducing risk-shifting. However, it remains optimal for a bank to issue CoCo bonds, since CoCo bonds improve the efficiency of the bank by allowing the bank to employ higher leverage in booms when the asymmetric information problem is relatively severe. In particular, the face value of the CoCo bonds remain the same as in the baseline model, and is only determined by information regarding the microeconomic states θ^s . Since the extent to which the regulator can limit leverage is constrained by the bank's private incentives, the optimal countercyclical capital regulation permits higher leverage than in the baseline case.

7 Concluding remarks

Capital regulation of financial institutions has long been at the centre of policy discussions. The recent financial crisis led to the realisation that a single risk-weight capital ratio fails to address macro-prudential concerns. Since then scholars and regulators have called for higher capital requirements and contingent capital to address the procyclical problems of bank leverage. However, a theory is required that incorporates the problems under consideration in order to assess the plausibility of any capital regulation as a solution.

This paper presents a model of the financial structure of a bank that is in need of outside capital for investment. The bank has private information regarding the returns of the investment opportunities, and has a risk-shifting incentive *ex post* to increase the shareholders' value at the expense of the debt holders. These two agency frictions endogenously determine the equilibrium capital structures of the bank, trading off the benefit of leverage as an information-insensitive security and the cost of risk-shifting induced by leverage. Moreover, since the asymmetric information problem is relatively more severe in booms, it is optimal for the bank to raise capital *ex ante* specifying *ex post* procyclical capital structures.

The optimal contingent capital structure can be implemented using contingent convertible (CoCo) bonds in addition to straight debt and equity. The model generates both the write-down feature and the contingent convertible feature seen in the CoCo

bonds issued by banks.

Although the model predicts that banks have the incentives to voluntarily issue CoCo bonds in a *laissez-faire* equilibrium, banks are subject to financial regulation in practice. The model notes that the privately chosen leverage levels are generally excessive due to the bank's incentives to minimise market mispricing of its securities. The optimally designed capital requirements, which are countercyclical, can improve the efficiency of the banks. Subject to the optimal capital regulation, the bank maximises its shareholder value by issuing CoCo bonds to meet the capital requirements.

Banks' excessive risk-taking not only harms shareholders' value, but also brings significant instability and associated costs as highlighted by the recent financial crisis. It introduces state guarantees in the form of government bailouts and deposit insurance. The moral hazard problems associated with state guarantees reduce the cost of risk-shifting internalised by a bank. This limits the extent to which a regulator can restrict bank leverage *ex ante*. Nevertheless, countercyclical capital regulation remains optimal, because the optimality of the procyclical leverage implemented by CoCo bonds is determined by the relative severity of the asymmetric information problem across different macroeconomic states.

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Appendices

A Parameter restriction on $\eta^i + \theta^s$

As given in Eq. 3, the optimal risk choice $\delta^{i,s}(F^s)$ in state s given a financing plan with debt of face value F^s is $\frac{1}{2}(\eta^i - \theta^s - \frac{X-F^s}{\Delta_X})$. This implies that the success probability is $q^{i,s}(F^s) \equiv \eta^i - \delta^{i,s}(F^s) = \frac{1}{2}(\eta^s + \theta^s + \frac{X-F^s}{\Delta_X})$ and the conditional probability of realising a high cash flow is $\theta^s + \delta^{i,s}(F^s) = \frac{1}{2}(\eta^i + \theta^s - \frac{X-F^s}{\Delta_X})$, where the first best case is produced when $F^s = 0$.

That the success probability lies in $(0, 1)$ is equivalent to $-\frac{X-F^s}{\Delta} < \eta^i + \theta^s < 2 - \frac{X-F^s}{\Delta}$. That the conditional probability of realising a high cash flow lies in $(0, 1)$ is equivalent to $\frac{X-F^s}{\Delta} < \eta^i + \theta^s < 2 + \frac{X-F^s}{\Delta}$.

As it will become clear later, the equilibrium face value of the debt $F^s \in [0, X]$. Therefore the necessary condition that guarantees that the equilibrium probabilities lie in $(0, 1)$ is that

$$\frac{X - F^s}{\Delta} < \eta^i + \theta^s < 2 \quad (23)$$

B Proofs

B.1 Proof of Proposition 1 and 2

The problem in the case of *ex post* financing can be interpreted as a special case of the problem in the case of *ex ante* financing, when the set of macroeconomic states \mathbf{S} is a singleton. Proposition 1 is therefore implied by Proposition 2. Here I present the proof for the latter proposition, which then also implies the former.

I first show in Claim 1 that the set The set of equilibria that satisfy the Intuitive Criterion is given by the programme given in Proposition 2. I then show in Claim 2 that all equilibria that satisfy the Intuitive Criterion are fair-price separating. Finally I derive the threshold \check{e}^C such that the bank issues debt if and only if $\bar{e} < \check{e}^C$.

Claim 1. *The set of equilibria that satisfy the Intuitive Criterion is given by the programme given in Proposition 2.*

Proof of Claim 1. Denote $(\hat{e}, \mathbf{F}_C, \boldsymbol{\alpha}_C)$ a maximiser of the programme given in Proposition 2. The proof is consisted of two parts. Firstly I apply the Intuitive Criterion to discard

any equilibrium in which the capital structure is not a maximiser of the said programme. Intuitively, the Good bank always has the incentive to deviate to a capital structure that gives it a higher payoff which still allows it to separate from the bad. I then show that a maximiser of the programme satisfies the Intuitive Criterion.

The first part of the proof is to show that any equilibrium capital structure that satisfies the Intuitive Criterion a maximiser of the programme given in Proposition 2. I show this in the following two steps: (i) any separating equilibrium that is not a maximiser of the said programme does not satisfy the Intuitive Criterion, and (ii) any pooling equilibrium does not satisfy the Intuitive Criterion.

(i) Consider a separating equilibrium with a contingent capital structure $(e, \mathbf{F}, \boldsymbol{\alpha})$, which is not a maximiser of the said programme. A deviation to the capital structure $(\hat{e}, \mathbf{F}_C, \boldsymbol{\alpha}_C)$ provides the Good bank with a strictly higher payoff whereas the Bad bank receives a lower payoff than in the equilibrium, while the outside investors at least break even, if the outside investors hold a belief that the deviation can only come from the Good type. The separating equilibrium $(\mathbf{F}, \boldsymbol{\alpha})$ thus does not satisfy the Intuitive Criterion. Therefore any contingent capital structure equilibrium that survives the Intuitive Criterion must be a maximiser of the said programme.

(ii) Consider a pooling equilibrium with a contingent capital structure $(e, \mathbf{F}, \boldsymbol{\alpha})$. The equilibrium capital structure satisfies the following constraints,

$$(PC_{Pool}^B) : \mathbb{E}[(1 - \alpha^s)E^{B,s}(F^s)] \geq e \quad (24)$$

$$(PC_{Pool}^G) : \mathbb{E}[(1 - \alpha^s)E^{G,s}(F^s)] \geq e \quad (25)$$

$$(IR_{Pool}) : \mathbb{E}[\gamma V^{G,s}(F^s) + (1 - \gamma)V^{B,s}(F^s) - (1 - \alpha^s)(\gamma E^{G,s}(F^s) + (1 - \gamma)E^{B,s}(F^s))] \geq 1 - e \quad (26)$$

In equilibrium, the payoff to the existing shareholders of the bank, if the bank issues securities according to the equilibrium financing plan, is given by

$$\mathbb{E}[(1 - \alpha^s)E^{i,s}(F^s)] + (\bar{e} - e) = u \quad (27)$$

for some constant u . Implicitly different F^s in Eq. 27 with regard to α^s to find the derivative

$$f_{\alpha^s, F^s}^{i,s}(e, \mathbf{F}, \boldsymbol{\alpha}) \equiv \frac{\partial \alpha^s}{\partial F^s} = -\frac{1 - \alpha^s}{q^{i,s}(F^s)\Delta_X} \quad (28)$$

The derivative is negative, and more so for the Bad type than for the Good type. That is, the Bad bank requires the equity issuance to be decreased more, in order to compensate

for an increase in the leverage. Therefore there exists a deviation $(e, \mathbf{F}', \boldsymbol{\alpha}')$, such that $F^{s'} > F^s$, $\alpha^{s'} < \alpha^s$, and all other financing parameters are the same as in the equilibrium. This deviation, compared to the original pooling equilibrium, provides the Good bank with a strictly higher payoff whereas the Bad bank receives a lower payoff than in equilibrium, if the outside investors are willing to provide capital.

If the investors believe that such a deviation can only come from a Good bank, they should accept as long as they at least break even. In a separating equilibrium, the investors' rationality constraint is given by Eq. 7. Binding the (IR) and implicitly differentiating it yields the derivative

$$f_{\alpha^s, F^s}^{IR, s}(e, \mathbf{F}, \boldsymbol{\alpha}) \equiv \frac{\partial \alpha^s}{\partial F^s} = \frac{F^s}{2\Delta_X} - \frac{1 - \alpha^s}{q^{G, s}(F^s)\Delta_X} \quad (29)$$

Notice that $|f_{\alpha^s, F^s}^{G, s}(\cdot)| > -f_{\alpha^s, F^s}^{IR, s}(\cdot) \forall F > 0$. That is, an increase in F and a decrease in α that leaves the marginally lower payoff to a Bad bank would strictly benefit the investors. Therefore there indeed exists such a deviation that the investors would be willing to accept and allow the Good bank to be strictly better off.

(i) and (ii) thus collectively suggest that any contingent capital structure equilibrium that survives the Intuitive Criterion must be a maximiser of the said programme.

The second part establishes that a maximiser of the programme satisfies the Intuitive Criterion. Suppose that there exists a capital structure $(\mathbf{F}, \boldsymbol{\alpha})$ which is a maximiser of the programme but it does not satisfy the Intuitive Criterion. That is, there exists a capital structure $(\mathbf{F}', \boldsymbol{\alpha}')$ such that the good bank receives a strictly higher payoff and the bad bank receives a lower payoff than at $(\mathbf{F}, \boldsymbol{\alpha})$, and that the outside investors at least break even. This suggests that $(\mathbf{F}', \boldsymbol{\alpha}')$ satisfies the constraints (PC_C^B) and (IR_C) and gives the good bank a strictly higher payoff than $(\mathbf{F}, \boldsymbol{\alpha})$. This contradicts with the assumption that $(\mathbf{F}, \boldsymbol{\alpha})$ is a maximiser of the programme.

Therefore, the set of equilibrium contingent capital structures that satisfy the Intuitive Criterion is given by the set of maximisers of the programme given in Proposition 2. This result implies Proposition 1. \square

Claim 2. *Any equilibrium that survives the Intuitive Criterion is fair-price separating.*

Proof of Claim 2. I prove this claim by showing that otherwise there exists a deviation to eliminate the equilibrium according to the Intuitive Criterion. This part of the proof builds upon the previous claim and only considers separating equilibria.

Intuitively, if the (IR_C) is not binding given a separating belief, a Good bank can increase its leverage and decrease its equity issue to maintain the same payoff, while still allowing the investors to at least break even, despite the value destruction due to increase leverage and risk-taking. Moreover, substituting debt for equity also hurts the Bad bank more than the Good because it reduces the mispricing. A change that leaves the Good bank indifferent should then make the Bad strictly worse off.

This argument is formally established below. Consider an underpricing separating equilibrium with financing plan $(e, \mathbf{F}, \boldsymbol{\alpha})$ such that

$$\mathbb{E}[V^{G,s}(F^s) - (1 - \alpha^s)E^{G,s}(F^s)] > 1 - e \quad (30)$$

There thus exists a deviation to financing plan $(e, \mathbf{F}', \boldsymbol{\alpha}')$, where where $F^{s'} > F^s$, $\alpha^{s'} < \alpha^s$ and all other financing parameters are the same, such that, if accepted by the investors, this financing plan makes a Good bank strictly better off, whereas it makes a Bad bank strictly worse off in comparison to the equilibrium outcome, following similar argument as in Part 1(ii) of the proof of Claim 1.

If the investors believe that such an deviation can only come from a Good bank, they are willing to accept the financing terms if the increase in leverage is small. This is because the investors make a positive payoff in equilibrium. A sufficiently small increase in leverage reduces the payoff left for investors but they can still at least break even.

Therefore in any Intuitive equilibrium, (IR_C) binds. The equilibrium thus must be a fair-price separating equilibrium. \square

I now derive the threshold \check{e}^C in Proposition 2. First consider the fair-price separating equilibria with no capital market debt that survives the Intuitive Criterion. Combining (IR_C) with (PC_C^B) yields that, at $\mathbf{F} = \mathbf{0}$, the conditions can be satisfied if

$$e \geq \check{e}^C \equiv \frac{\mathbb{E} \left[(1 - \alpha^s) E_{FB}^{G,s} \right]}{\mathbb{E} \left[(1 - \alpha^s) (E_{FB}^{G,s} - E_{FB}^{G,s}) \right]} \left(\mathbb{E} \left[V_{FB}^{G,s} \right] - 1 \right) \quad (31)$$

That is, a Good bank must put in at least \check{e}^C of its internal capital in order to separate from the Bad. This implies that no separating equilibrium exists if $\bar{e} < \check{e}^C$. Moreover, for $\bar{e} \geq \check{e}^C$, the bank is able to achieve separation without leverage. A separating equilibrium without leverage therefore cannot satisfy the Intuitive Criterion. In summary, the bank issues debt in equilibrium if and only if $\bar{e} < \check{e}^C$.

B.2 Proof of Corollary 1

Because of the complex nature of the problem, I impose parameter restrictions to guarantee an interior solution, that $\max_{F^s} V^{B,s}(F^s) + \frac{\eta^G - \eta^B}{2} F < 1$. Intuitively, this is the case if the Good and the Bad types are not too different, i.e. $\eta^G - \eta^B$ small.

To characterise the unique equilibrium leverage that satisfies the Intuitive Criterion, I start by establishing the following claims. The Intuitive Criterion is applied to discard equilibria in which there exists an out-of-equilibrium action such that (i) a Bad bank is strictly worse off deviating to it, and (ii) if the investors believe that such a deviation can only come from a Good bank, the Good bank is strictly better off deviating to it. This process is equivalent to establishing that there is a unique solution \hat{F}^s to the maximisation programme given by Proposition 1.

Claim 3. *Any equilibrium with capital market debt $F^s > 0$ that survives the Intuitive Criterion has a binding (PC^B) .*

Proof of Claim 3. I show this claim by constructing a deviation according to the Intuitive Criterion. Intuitively, if the (PC^B) is slack, a Good bank can reduce its leverage to enjoy the value created by the reduction in risk-shifting incentive, while still achieving separation from the Bad.

Formally, consider a fair-price separating equilibrium with financing plan (\bar{e}, F^s, α^s) such that $(1 - \alpha^s)E^{B,s}(F^s) < \bar{e}$ where $F^s > \bar{D}$. There exists a deviation to financing plan (\bar{e}, F', α') , where $F' < F^s$ and $\alpha' > \alpha^s$, such that a Good bank strictly benefits if the financing plan is accepted by the investors, and that $(1 - \alpha')E^{B,s}(F') < \bar{e}$ so that a Bad bank has no incentive to deviate to this financing plan.

Consider any financing plans (e, F, α) such that the payoff to the insiders of a bank of type i is

$$(1 - \alpha)E^{i,s}(F) + (\bar{e} - e) = u \quad (32)$$

where u is any constant. The existence of such a deviation thus follows from the properties of the functions $f_{\alpha^s, F^s}^{i,s}(\cdot)$ and $f_{\alpha^s, F^s}^{IR,s}(\cdot)$ given by Eq. 28–29 and discussed in Part 1(ii) of the proof of Claim 1. \square

Claims 2–3 thus suggest that both (PC^B) and (IR) bind in an equilibrium that satisfy the Intuitive Criterion. Combining both binding constraints implies that, for a given

F^s , the equilibrium input of internal capital e is given by

$$e = \check{e}^s(F^s) \equiv \frac{E^{B,s}(F^s)}{E^{G,s}(F^s) - E^{B,s}(F^s)} [V^{G,s}(F^s) - 1] \quad (33)$$

This implies an equilibrium that satisfies the Intuitive Criterion with F^s exists only for $\bar{e} \geq e$. Particularly, for $\bar{e} \geq \check{e}^s$, fair-price separation can be achieved without leverage. The financing plan in the unique Intuitive equilibrium is thus $(\hat{e}, 0, \hat{\alpha}^s(\bar{e}))$ where $\hat{e} \geq \check{e}^s$ and $\alpha^s(\cdot)$ is given by a binding (IR). That is, $\hat{\alpha}^s(\cdot) = (1 - \bar{e})/V_{FB}^{G,s}$.

For $\bar{e} < \check{e}^s$, however, the bank must resort to taking leverage in order to achieve fair-price separation.

Claim 4. *In any equilibrium with capital market debt $F^s > 0$ that survives the Intuitive Criterion, the bank puts up all of its internal capital \bar{e} .*

Proof of Claim 4. The proof builds upon that of Claims 2–3 and only considers fair-price separating equilibria with a binding (PC^B).

Intuitively, because internal capital is even less information sensitive than debt, substituting internal capital for debt hurts the Bad bank and benefits the Good by reducing the mispricing from debt issuance. Therefore a Good bank can deviate to a lower leverage level and puts up more internal capital. This deviation still allows the outside investors to break even since a reduction in leverage increases the total value of the bank.

Formally, I implicitly differentiate Eq. 32 to find the derivative

$$f_{\alpha,e}^i(e, F, \alpha) \equiv \frac{\partial \alpha}{\partial e} = -\frac{1}{E^{i,s}(F)} \quad (34)$$

The derivative is negative, suggesting that increasing internal capital input by a marginal unit while decreasing the fraction of equity issued by $1/E^{i,s}(F)$ leaves a type i bank with the same amount of payoff, holding the face value of the debt outstanding unchanged.

Notice that $|f_{\alpha,e}^B(\cdot)| > |f_{\alpha,e}^G(\cdot)|$. That is, a Bad bank requires to retain more equity in order to compensate for the additional internal capital input than a Good bank.

Consider a fair-price separating equilibrium (e, F^s, α) , where $F^s > 0$. There exists a financing plan (e', F', α) , where $e' > e$ and $0 < F' < F$ such that if raises financing successfully, a Good bank is strictly better off, whereas a Bad bank is strictly worse off than in equilibrium.

If the investors believe that such an deviation can only come from a Good bank, they are willing to accept the financing terms. Because the reduction in leverage increases the

value of the bank, a Good bank can be better off and still allow the investors to at least break even.

Therefore in any Intuitive equilibrium in which the bank issues capital market debt, it puts up all of its internal capital. \square

Therefore for $\bar{e} < \check{e}^s$, any fair-price separating equilibrium in which the (PC^B) holds with strict inequality does not satisfy the Intuitive Criterion because it entails some leverage $F^s > 0$. Intuitively, because reducing leverage increases the bank's value, the bank can share some of it with the investors and still benefit from the deviation. The unique Intuitive equilibrium is thus characterised by a binding (PC^B) and a binding (IR) .

Specifically, the equilibrium face value of debt $\hat{F}^s(\bar{e})$ is given by $\bar{e} = \check{e}^s(\hat{F}^s(\cdot))$. The equilibrium fraction of outside equity issued is then given by $\hat{\alpha}^s(\bar{e}) = 1 - \bar{e}/E^{B,s}(\hat{F}^s(\cdot))$.

B.3 Proof of Proposition 3 and 4

B.3.1 *Ex post* financing

In the equilibrium with *ex post* financing, the equilibrium leverage is given by binding (PC^B) and (IR) , which yields

$$\check{e}(\hat{F}^s) \equiv \frac{E^{G,s}(\hat{F}^s)}{E^{G,s}(\hat{F}^s) - E^{B,s}(\hat{F}^s)} [V^{G,s}(\hat{F}^s) - 1] = \bar{e} \quad (35)$$

The procyclicality of book leverage $\hat{F}^s(\cdot)$ is derived by implicitly differentiating Eq. 35 with regard to θ^s . The equity value of a bank given the face value of the debt and the optimal risk choice can be expressed as $E^{i,s}(F^s) \equiv [q^{i,s}(F^s)]^2 \Delta_X$. I then express Eq. 35 as

$$\left[q^{G,s}(\hat{F}^s(\cdot)) \right]^2 \Delta_X + q^{G,s}(\hat{F}^s(\cdot)) \hat{F}^s(\cdot) - \bar{e} \frac{\left[q^{G,s}(\hat{F}^s(\cdot)) \right]^2}{\left[q^{B,s}(\hat{F}^s(\cdot)) \right]^2} = 1 - \bar{e} \quad (36)$$

Denote by A the total derivative of the left hand side of Eq. 36 with respect to $q^{G,s}(\cdot)$, when expressing $q^{B,s}(\cdot) = q^{G,s}(\cdot) - \frac{\eta^G - \eta^B}{2}$. Implicitly differentiating Eq. 36 with regard to θ^s yields

$$\frac{\partial \hat{F}^s(\cdot)}{\partial \theta^s} = \Delta_X \frac{A}{A - 2q^{G,s}(\cdot)\Delta_X} > \Delta_X > 0 \quad (37)$$

$$\text{where } A \equiv 2q^{G,s}(\cdot)\Delta_X + \hat{F}^s(\cdot) + \bar{e} [q^{G,s}(\cdot) - q^{B,s}(\cdot)] \frac{q^{G,s}(\cdot)}{q^{B,s}(\cdot)} \quad (38)$$

$$> 2q^{G,s}(\cdot)\Delta_X \quad (39)$$

The partial derivative of the success probability $q^{G,s}(\hat{F}^s(\bar{e}))$ with respect to θ^s is therefore

$$\frac{dq^{G,s}(\cdot)}{d\theta^s} = \frac{\partial q^{G,s}(\cdot)}{\partial \theta^s} + \frac{\partial q^{G,s}(\cdot)}{\partial F^s} \frac{\partial \hat{F}^s(\cdot)}{\partial \theta^s} < \frac{1}{2} - \frac{1}{2\Delta_X} \Delta_X = 0 \quad (40)$$

B.3.2 *Ex ante* financing

The intuition for cyclical leverage is as follows. If the default probability is higher in one state than in another, effectively the debt in the higher leverage state is more information sensitive than in the other state. Then Good bank should have incentive to equalise the resulting default probabilities across states to reduce the mispricing. Since a better economic fundamental sustains higher leverage to produce the same default probability, leverage tends to be procyclical. However, since high leverage is required in booms to equalise the resulting default probabilities, it may become too costly in terms of the risk shifting incentives. Therefore in equilibrium, the default probabilities may not be equal across the states, but countercyclical.

Formally, I implicitly differentiate Eq. 28 to find the derivative

$$f_{F^s, F^z}^i(e, \mathbf{F}, \boldsymbol{\alpha}) \equiv \frac{\partial F^s}{\partial F^z} = -\frac{1 - \alpha^z}{1 - \alpha^s} \frac{q^{i,z}(F^z) p(z)}{q^{i,s}(F^s) p(s)} \quad (41)$$

where $p(s)$ is the probability of state s .

The derivative is negative if $\alpha^z < 1$, suggesting that increasing the leverage in state z by a marginal unit while decreasing leverage in state s by $\frac{1 - \alpha^z}{1 - \alpha^s} \frac{q^{i,z}(F^z)}{q^{i,s}(F^s)}$ leaves a type i bank with the same amount of payoff, holding all other financing parameters unchanged.

Notice that $|f_{F^s, F^z}^B(\cdot)| > |f_{F^s, F^z}^G(\cdot)|$ iff $q^{i,z}(F^z) > q^{i,s}(F^s)$. That is, a Bad bank requires to reduce leverage in the highly levered state by more in order to compensate for the increase in leverage in the less levered state than a Good bank.

If the investors believe that a financing plan can only come from a Good bank, they should accept the financing plan if they can at least break even. I implicitly differentiate the binding (IR_C) (Eq. 11) to find the derivative

$$f_{F^s, F^z}^{IR}(e, \mathbf{F}, \boldsymbol{\alpha}) \equiv \frac{\partial F^s}{\partial F^z} = -\frac{(1 - \alpha^z)q^{G,z}(F^z) - \frac{F^z}{2\Delta_X} p(z)}{(1 - \alpha^s)q^{G,s}(F^s) - \frac{F^s}{2\Delta_X} p(s)} \quad (42)$$

That is, increasing leverage in state z by a marginal unit while decreasing leverage in state s by $\frac{(1 - \alpha^z)q^{i,z}(F^z) - \frac{F^z}{2\Delta_X} p(z)}{(1 - \alpha^s)q^{i,s}(F^s) - \frac{F^s}{2\Delta_X} p(s)}$ leaves the investors with the same amount of payoff, holding all other financing parameters unchanged.

I now show that there exists a deviation to eliminate any equilibrium in which the contingent leverage that is countercyclical or in which the default probability is procyclical. Consider any states s, z s.t. $\theta^s > \theta^z$.

Consider a fair-price separating equilibrium financing plan $(\bar{e}, \mathbf{F}_C, \boldsymbol{\alpha}_C)$ such that (PC_C^B) binds, where $F^z > F^s$. This implies that $q^{i,z}(F^z) < q^{i,s}(F^s)$. There exists a financing plan $(\bar{e}, \mathbf{F}', \boldsymbol{\alpha}')$, where $F^{s'} > F^s$, $F^{z'} < F^z$ and all other financing parameters are the same, such that if raises financing successfully, a Good bank is strictly better off, whereas a Bad bank is strictly worse off than in equilibrium. Thus only a Good bank has the incentives to deviate to this financing plan.

Notice that $|f_{F^z, F^s}^G(\cdot)| < -f_{F^z, F^s}^{IR}(\cdot)$ if $F^z > F^s$ and $q^{i,z}(F^z) < q^{i,s}(F^s)$. That is, an increase in leverage in state s and a decrease in leverage in state z that leaves the same payoff to the Good bank would strictly benefit the investors. Therefore there indeed exists such a deviation as described above that the investors would be willing to accept.

Consider now a fair-price separating equilibrium financing plan $(\bar{e}, \mathbf{F}, \boldsymbol{\alpha})$ such that (PC_C^B) binds, where $F^z < F^s$ and $q^{i,z}(F^z) > q^{i,s}(F^s)$. By the same reasoning, there again exists a financing plan $(\bar{e}, \mathbf{F}', \boldsymbol{\alpha}')$, where $F^{s'} < F^s$, $F^{z'} > F^z$ and all other financing parameters are the same, such that if raises financing successfully, a Good bank is strictly better off, whereas a Bad bank is strictly worse off than in equilibrium; and the investors are willing to accept the financing plan given a belief that it can only come from the Good bank.

Therefore in any equilibrium that satisfies the Intuitive Criterion, leverage is such that $\hat{F}_C^s > \hat{F}_C^z$ and $E^{i,s}(\hat{F}^s) \geq E^{i,z}(\hat{F}^z) \forall \theta^s > \theta^z, s, z \in \{s \in \mathbf{S} : \hat{\alpha}_C^s < 1\}$. This also implies that $1 - q^{i,s}(\hat{F}^s) \leq 1 - q^{i,z}(\hat{F}^z)$.

B.4 Proof of Proposition 5

First, I notice that the equilibrium capital structures in the case of *ex post* financing also satisfy the equilibrium conditions for the case of *ex ante* financing, since in the former case the conditions are satisfied in each state *ex post*. Proposition 2 then implies that the Intuitive contingent capital structure equilibrium must give the Good bank at least as high an expected payoff as the equilibrium with *ex post* financing. Moreover, given that the outside capital market investors always break even in equilibrium, the bank captures the entire value created by the bank. The contingent capital structure therefore produces an expected bank value that is at least as high as in the other two cases.

Secondly, the contingent capital structure is strictly preferred to the *ex post* capital structure when capital market debt is issued in equilibrium, i.e. $\bar{e} < \check{e}^C$. This is because by Proposition 3, the default probability in a contingent capital structure equilibrium is countercyclical, while the face value of the debt is procyclical. The *ex post* capital structure equilibrium has procyclical default probability, whereas the non-contingent capital structure equilibrium has constant face value of debt. Therefore neither capital structure belongs to the optimal set of contingent capital structures. The optimal contingent capital structure thus must deliver strictly higher bank value than either of the other two cases.

B.5 Proof of Proposition 6

The equilibrium contingent capital structure in the example of a tail event economy is given by Proposition 2. In order to fully characterise the equilibrium, consider the following two scenarios for $\bar{e} < \check{e}^C$.

Firstly, suppose an interior solution such that $E^{i,H}(\hat{F}_C^H) = E^{i,L}(\hat{F}_C^L)$, i.e. $\hat{F}_C^H = \hat{F}_C^L + \frac{\theta^H - \theta^L}{\Delta_X}$. This also implies that $q^{i,H}(\hat{F}_C^H) = q^{i,L}(\hat{F}_C^L)$. Given this restriction, denote the equity value and the success probability with $E^i(\mathbf{F}_C)$ and $q^i(\mathbf{F}_C)$ respectively, which are equal in both states. The maximisation programme can then be rewritten as

$$\max_{e, F_C^L, \alpha_C^H, \alpha_C^L} [\beta(1 - \alpha_C^H) + (1 - \beta)(1 - \alpha_C^L)]E^G(\mathbf{F}_C) \quad (43)$$

$$s.t. \quad e \leq \bar{e} \quad (44)$$

$$[\beta(1 - \alpha_C^H) + (1 - \beta)(1 - \alpha_C^L)]E^B(\mathbf{F}_C) \geq e \quad (45)$$

$$\begin{aligned} & \beta V^{G,H}(F_C^H) + (1 - \beta)V^{G,L}(F_C^L) \\ & - [\beta(1 - \alpha_C^H) + (1 - \beta)(1 - \alpha_C^L)]E^G(\mathbf{F}_C) \geq 1 - e \quad (46) \end{aligned}$$

Applying similar reasoning as in the proof given in Appendix B.2, it is easy to show that objective function is maximised when all three constraints bind, which determines $e = \bar{e}$ and the equilibrium $\hat{F}^H = \hat{F}_C^L + \frac{\theta^H - \theta^L}{\Delta_X}$, where $\hat{\mathbf{F}}_C$ is given by

$$\bar{e} = \frac{E^B(\hat{\mathbf{F}}_C)}{E^G(\hat{\mathbf{F}}_C) - E^B(\hat{\mathbf{F}}_C)} \left[\beta V^{G,H}(\hat{F}_C^H) + (1 - \beta)V^{G,L}(\hat{F}_C^L) - 1 \right] \quad (47)$$

as well as $\hat{\alpha}_C$ up to one degree of freedom, i.e.

$$\beta(1 - \alpha_C^H) + (1 - \beta)(1 - \alpha_C^L) = \frac{\bar{e}}{E^B(\hat{\mathbf{F}}_C)} \quad (48)$$

The restrictions $\hat{F}_C^H = \hat{F}_C^L + \frac{\theta^H - \theta^L}{\Delta_X}$ implies that $\hat{F}_C^H \geq \frac{\theta^H - \theta^L}{\Delta_X}$. Therefore the condition for the first scenario to arise is that in equilibrium, $E^i(\hat{F}_C) \leq V_{FB}^{i,L}$. This is the case if

$$\bar{e} \leq \check{e}^T \equiv \frac{V_{FB}^{B,L}}{V_{FB}^{G,L} - V_{FB}^{B,L}} \left[\beta V^{G,H} \left(\frac{\theta^H - \theta^L}{\Delta_X} \right) + (1 - \beta) V_{FB}^{G,L} - 1 \right] \quad (49)$$

For $\bar{e} \in [\check{e}^T, \check{e}^C)$, the second scenario arises, in which case $\hat{F}_C^H < \frac{\theta^H - \theta^L}{\Delta_X}$ and $\hat{F}_L = 0$. Imposing the restriction that $\hat{F}_L = 0$, the maximisation programme can then be rewritten as

$$\max_{e, F_C^H, \alpha_C^H, \alpha_C^L} \beta(1 - \alpha_C^H) E^{G,H}(F_C^H) + (1 - \beta)(1 - \alpha_C^L) V_{FB}^{G,L} \quad (50)$$

$$s.t. \quad e \leq \bar{e} \quad (51)$$

$$\beta(1 - \alpha_C^H) E^{B,H}(F_C^H) + (1 - \beta)(1 - \alpha_C^L) V_{FB}^{B,L} \geq e \quad (52)$$

$$\begin{aligned} \beta V^{G,H}(F_C^H) + (1 - \beta) V_{FB}^{G,L} - \beta(1 - \alpha_C^H) E^{G,H}(F_C^H) \\ - (1 - \beta)(1 - \alpha_C^L) V_{FB}^{G,L} \geq 1 - e \end{aligned} \quad (53)$$

It is straightforward that the solution entail $e = \bar{e}$. The restriction $F_C^H < \frac{\theta^H - \theta^L}{\Delta_X}$ implies that $E^{G,H}(F_C^H) > V_{FB}^{G,L}$. Therefore the equity in the high state is relatively less information sensitive in the low state. The solution should therefore entail $\hat{\alpha}_C^L = 0$. The equilibrium leverage \hat{F}_C^H and the equity issuance $\hat{\alpha}_C^H$ in the high state is therefore given by binding the (IR) and (PC^B) ,

$$\bar{e} = \frac{E^{B,H}(\hat{F}_C^H)}{E^{G,H}(\hat{F}_C^H) - E^{B,H}(\hat{F}_C^H)} \left[\beta V^{G,H}(\hat{F}_C^H) + (1 - \beta) V_{FB}^{B,L} \frac{E^{G,H}(\hat{F}_C^H)}{E^{B,H}(\hat{F}_C^H)} - 1 \right] \quad (54)$$

$$\hat{\alpha}_C^H = 1 - \frac{\bar{e} + (1 - \beta) V_{FB}^{B,L}}{\beta E^{B,H}(\hat{F}_C^H)} \quad (55)$$

The implementation using CoCo bonds in addition to straight debt and equity provided in the proposition follows intuitively from the characterisation of the equilibrium contingent capital structure.

B.6 Proof of Proposition 7

To derive the optimal capital regulation, I first show in Claim 5 that any regulation that imposes a leverage cap below the equilibrium level will bind in the resulting equilibrium. I then characterise \check{F} , the minimum leverage cap the regulate can impose, while implementing a separating equilibrium. Finally I provide conditions for when this is feasible, and derive the optimal capital regulation for the other cases.

Claim 5. *The capital regulation $\bar{\mathbf{F}}$ binds in any Intuitive equilibrium if the bank has insufficient internal capital $\bar{e} < \check{e}^s(\bar{\mathbf{F}})$.*

Proof. This result follows the intuition of Claim 2. At any leverage level \mathbf{F} such that $\bar{e} < \check{e}^s(\mathbf{F})$, the Good bank's payoff is increasing in the leverage level, in any separating equilibrium. It therefore has the incentive to increase leverage until the capital regulation binds. This is due to the underpricing of the claims issued by the Good bank in order to separate at low leverage. \square

This effectively allows the regulator to set leverage levels in the equilibrium. I now characterise $\check{\mathbf{F}}$, the minimum leverage cap the regulator can impose, while implementing a separating equilibrium.

I invoke the concept of undefeated equilibrium proposed by Mailath et al. (1993). Consider a pooling equilibrium and a separating equilibrium. If the pooling equilibrium provides the Good bank with a strictly higher payoff, the pooling equilibrium defeats the separating equilibrium, by restricting that the out-of-equilibrium belief upon observing the pooling equilibrium action in the separating equilibrium to be consistent with the set of types who strictly benefit from such a deviation. $\check{\mathbf{F}}$ is therefore characterised by the optimisation programme given by Eq. 20, where the expected retained payoffs to the Good bank in the least-cost separating equilibrium $v^G(\mathbf{F}; \bar{e})$ and the least-cost pooling equilibrium $v_P^G(\mathbf{F}; \bar{e})$ are given by, respectively,

$$v^G(\mathbf{F}_C; \bar{e}) \equiv \max_{e, \alpha_C} \mathbb{E} [(1 - \alpha_C^s) E^{G,s}(F_C^s)] \quad s.t. \ e \leq \bar{e}, (PC_C^B), (IR_C) \quad (56)$$

$$v_P^G(\mathbf{F}_C; \bar{e}) \equiv \max_{e, \alpha_C} \mathbb{E} [(1 - \alpha_C^s) E^{G,s}(F_C^s)] \quad s.t. \ e \leq \bar{e}, (PC_{Pool}^B) \text{ and } (IR_{Pool}) \quad (57)$$

I now derive the thresholds \check{e}_P^C and \check{e}_{LP}^C . $\check{e}_P^C < \check{e}^C$ is the threshold for a separating equilibrium that satisfy the Intuitive Equilibrium, given a leverage cap of zero $\bar{\mathbf{F}} = 0$, to be undefeated by a pooling equilibrium. Because the equilibrium payoff to the Good bank is increasing in its internal capital \bar{e} , the threshold \check{e}_P^C is given by when the Good bank is indifferent between a pooling equilibrium and a separating equilibrium at zero leverage. That is,

$$v^G(\mathbf{0}; \check{e}_P^C) = v_P^G(\mathbf{0}; \check{e}_P^C) \quad (58)$$

For $\bar{e} < \check{e}_{LP}^C$, the leverage required to implement a separating equilibrium is too socially costly because of the risk-shifting problem, and the regulator prefers to implement

a pooling equilibrium at zero leverage. Such threshold \check{e}_{LP}^C is therefore given by

$$\mathbb{E}[V^{G,s}(\check{F}^s(\check{e}_{LP}^C))] = \mathbb{E}[\gamma V_{FB}^{G,s} + (1 - \gamma)V_{FB}^{B,s}] \quad (59)$$

B.7 Proof of Proposition 8

For $\bar{e} \in [\check{e}_{LP}^C, \check{e}_P^C]$, the optimal capital regulation $\bar{\mathbf{F}} = \check{\mathbf{F}}$, where $\check{\mathbf{F}}$ is the least-cost leverage cap a regulator can impose that implements a separating equilibrium given by the optimisation programme Eq. 20. Within the example of a tail event economy, I first characterise the functions $v_P^G(\mathbf{F}; \bar{e})$ and $v^G(\mathbf{F}; \bar{e})$. This allows me to then show that $\check{\mathbf{F}}$ is procyclical and derive the results in comparison to the *laissez-faire* equilibrium. Lastly show that this implies that the capital ratio under the optimal capital regulation is countercyclical for all \bar{e} .

Characterisation of $v_P^G(\mathbf{F}; \bar{e})$

In this part of the proof I characterise the function $v_P^G(\mathbf{F}; \bar{e})$ given by Eq. 57. The solution to this maximisation programme must entail the constraints $e \leq \bar{e}$ and (IR_{Pool}) binding. (i) $e = \bar{e}$ following the reasoning for Claim 4. If $e < \bar{e}$, there exists an alternative financing plan that involves more internal capital investment and less equity issuance that produces a higher payoff to the Good bank while satisfying all the other constraints. (ii) Similarly, if the (IR_{Pool}) is slack, there exists an alternative financing plan that involves less equity issuance that produces a higher payoff to the Good bank while satisfying all the constraints.

(iii) Given \mathbf{F} , equity should be issued primarily in the state in which the equity value is higher. This is shown in the following. I differentiate α^H in Eq. 27 with regard to α^L to find the derivative

$$f_{\alpha^H, \alpha^L}^i(e, \mathbf{F}, \boldsymbol{\alpha}) \equiv \frac{\partial \alpha^H}{\partial \alpha^L} = -\frac{E^{i,L}(F^L)}{E^{i,H}(F^H)} \frac{1 - \beta}{\beta} \quad (60)$$

This derivative is negative, suggesting that increasing the equity issuance in the Low state by a marginal unit while decreasing leverage in the High state by $\frac{E^{i,L}(F^L)}{E^{i,H}(F^H)} \frac{1 - \beta}{\beta}$ leaves a type i bank with the same payoff.

Notice that $|f_{\alpha^H, \alpha^L}^B(\cdot)| > |f_{\alpha^H, \alpha^L}^G(\cdot)|$ iff $E^{i,H}(F^H) < E^{i,L}(F^L)$. That is, a Bad bank requires to equity issuance in the highly levered state by more in order to compensate for an increase in equity issuance in the less levered state than a Good bank.

If the investors believe that a financing plan can only come from a Good bank, they should accept the financing plan if they can at least break even. I implicitly differentiate α^H in the binding (IR_{Pool}) with regard to α^L to find the derivative

$$f_{\alpha^H, \alpha^L}^{IR, Pool}(e, \mathbf{F}, \boldsymbol{\alpha}) \equiv \frac{\partial \alpha^H}{\partial \alpha^L} = - \frac{\gamma E^{G,L}(F^L) + (1-\gamma)E^{B,L}(F^L)}{\gamma E^{G,H}(F^H) + (1-\gamma)E^{B,H}(F^H)} \frac{1-\beta}{\beta} \quad (61)$$

That is, increasing equity issuance in the Low state by a marginal unit while decreasing equity issuance in the High state by $\frac{\gamma E^{G,L}(F^L) + (1-\gamma)E^{B,L}(F^L)}{\gamma E^{G,H}(F^H) + (1-\gamma)E^{B,H}(F^H)} \frac{1-\beta}{\beta}$ leaves the investors with the same amount of payoff, holding all other financing parameters unchanged. In particular, $|f_{\alpha^H, \alpha^L}^{IR, Pool}(\cdot)| \leq |f_{\alpha^H, \alpha^L}^B(\cdot)|$, iff $E^{i,H}(F^H) < E^{i,L}(F^L)$. That is, an increase in equity issuance in the less levered state and a decrease in equity issuance in the highly levered state such that the Bad bank enjoys the same payoff, leaves the investors strictly better off.

I can now establish that an maximiser of the programme given by Eq. 57 should entail equity issuance primarily in the state in which the is higher. Otherwise, there exists an alternative financing plan with high equity issuance in the state in which the equity value is higher, and lower equity issuance in the state in which the equity value is lower. Such a financing plan provides the Bad bank with lower payoff while leaving the Good bank and the investors strictly better off.

This discussion allows us to examine the marginal impact of a change in \mathbf{F} on $v_P^G(\mathbf{F})$. Specially, given that (IR_{Pool}) binds and that the equity issuance is a corner solution, consider the marginal effect of an increase in F_C^s on the equilibrium equity issuance in the state z in which the solution is interior, where z can be either H or L . This is obtained by implicitly differentiating (IR_{Pool}).

$$\frac{\partial \alpha^z}{\partial F^s} = \frac{\frac{F}{\Delta_X} + (1-\alpha^s) [\gamma q^{G,s}(F^s) + (1-\gamma)q^{B,s}(F^s)]}{\gamma E^{G,z}(F^s) + (1-\gamma)E^{B,z}(F^z)} \frac{p^s}{p^z} \quad (62)$$

where p^s is the probability of state s realising, $s \in \{H, L\}$. Substituting this into the total differentiation equation of the payoff of the Good bank yields

$$\begin{aligned} \frac{\partial v_P^G(\mathbf{F})}{\partial F^s} &= p^s \frac{F}{\Delta_X} \frac{E^{G,z}(F^z)}{\gamma E^{G,z}(F^z) + (1-\gamma)E^{B,z}(F^z)} \\ &+ p^s (1-\alpha^s) \left(\frac{E^{G,z}(F^z) [\gamma q^{G,s}(F^s) + (1-\gamma)q^{B,s}(F^s)]}{\gamma E^{G,z}(F^s) + (1-\gamma)E^{B,z}(F^z)} - q^{G,s}(F^s) \right) \end{aligned} \quad (63)$$

Characterisation of $v^G(\mathbf{F}; \bar{e})$

In this part I characterise the function $v^G(\mathbf{F}; \bar{e})$ given by Eq. 56 by examining the constraints in the maximisation programme. (i) The constraint $e \leq \bar{e}$ binds given the

solution. This follows similar reasoning for Claim 4. (ii) (IR_C) is slack, implied by Claim 5, for $\bar{e} < \check{e}^s(\bar{F})$. (iii) Given that the (IR_C) is slack, (PC_C^B) binds. This is because otherwise there exists an alternative financing plan with lower equity issuance that still satisfies all constraints but leaves the Good bank a strictly higher payoff. (iv) Given \mathbf{F} , equity is issued primarily in the state in which the equity value is higher. This follows the same reasoning as given in Part (iii) of the previous section when characterising $v_P^G(\mathbf{F}; \bar{e})$.

Characterisation of $\check{\mathbf{F}}$

I now turn to characterise $\check{\mathbf{F}}$, the least-cost capital regulation that implements a separating equilibrium.

(i) The constraint that such a separating equilibrium is undefeated by a pooling equilibrium binds given the $\check{\mathbf{F}}$, i.e. $v^G(\check{\mathbf{F}}; \bar{e}) = v_P^G(\check{\mathbf{F}}; \bar{e})$. I show this by contradiction. Suppose $(e, \mathbf{F}, \boldsymbol{\alpha})$ is a solution to the maximisation programme, such that $v^G(\mathbf{F}; \bar{e}) > v_P^G(\mathbf{F}; \bar{e})$. Consider a financing plan $(e, \mathbf{F}', \boldsymbol{\alpha}')$ such that $F^{s'} < F^s$, $\alpha^{s'} > \alpha^s$ and all other financing parameters are the equal. Because of the property of Eq. 28 discussed in Part 1(ii) of the proof of Claim 1, there exists a financing plan $(e, \mathbf{F}', \boldsymbol{\alpha}')$ such that it provides the Bad bank with the same payoff in the separating equilibrium as the equilibrium financing plan, but reduces the Good bank's payoff relative to the equilibrium financing plan. This also relaxes the (IR_C) since it increases the value of the Good bank in equilibrium. This alternative financing plan produces a higher social value as it entails lower leverage. This contradicts with the supposition that the original financing plan $(e, \mathbf{F}, \boldsymbol{\alpha})$ is a solution to the maximisation programme, as the constraint $v^G(\mathbf{F}; \bar{e}) \geq v_P^G(\mathbf{F}; \bar{e})$ can still be satisfied for a sufficiently small deviation in F^s .

I now examine the procyclical property of $\check{\mathbf{F}}$ in the following. I show in the following that $\check{\mathbf{F}}$ is procyclical and that the resulting default probability is procyclical.

(ii) Consider $F^H < F^L$. This implies that $q^{i,H}(F^H) > q^{i,L}(F^L)$. I eliminate this case by constructing a socially preferred alternative. Consider another set of leverage caps \mathbf{F}' such that $F^{H'} > F^H$ and $F^{L'} < F^L$. Notice that in this case equity is only issued in the high state in both the least-cost pooling and separating equilibria. By the reasoning in Appendix B.3.2, there exists \mathbf{F}' such that the resulting least-cost separating equilibrium produces a higher payoff for the Good bank than \mathbf{F} , which implies that the social value is also higher. Such an alternative is therefore socially preferred, if $v^G(\mathbf{F}'; \bar{e}) \geq v_P^G(\mathbf{F}'; \bar{e})$ is still satisfied.

Indeed the constraint is satisfied. The payoffs to the Good bank in the least-cost pooling and separating equilibria can be expressed as

$$v_P^G(\mathbf{F}; \bar{e}) = \frac{E^{G,H}(F^H)}{\underbrace{\gamma E^{G,H}(F^H) + (1-\gamma)E^{G,H}(F^H)}_B} \left(\mathbb{E} [\gamma V^{G,s}(F^s) + (1-\gamma)V^{B,s}(F^s)] - 1 + \bar{e} \right) - (1-\beta) (E^{G,L}(F^L) - [\gamma E^{G,L}(F^L) + (1-\gamma)E^{B,L}(F^L)] \underbrace{\frac{E^{G,H}(F^H)}{\gamma E^{G,H}(F^H) + (1-\gamma)E^{B,H}(F^H)}}_B) \quad (64)$$

$$v^G(\mathbf{F}; \bar{e}) = \underbrace{\frac{E^{G,H}(F^H)}{E^{B,H}(F^H)}}_C \bar{e} - (1-\beta) \left[E^{G,L}(F^L) - E^{B,L}(F^L) \underbrace{\frac{E^{G,H}(F^H)}{E^{B,H}(F^H)}}_C \right] \quad (65)$$

where $C > B$, and

$$\frac{\partial B}{\partial F^H} = (1-\gamma) \frac{q^{G,H}(F^H)q^{B,H}(F^H) [q^{G,H}(F^H) - q^{B,H}(F^H)]}{[\gamma E^{G,H}(F^H) + (1-\gamma)E^{B,H}(F^H)]^2} \Delta_X > 0 \quad (66)$$

$$\frac{\partial C}{\partial F^H} = (1-\gamma) \frac{q^{G,H}(F^H)q^{B,H}(F^H) [q^{G,H}(F^H) - q^{B,H}(F^H)]}{[E^{B,H}(F^G)]^s} \Delta_X > \frac{\partial B}{\partial F^H} \quad (67)$$

This suggests, for β sufficiently large, an increase in F^H and a decrease in F^L such that it leaves the social value unchanged would increase $v^G(\cdot)$ more than $v_P^G(\cdot)$. Therefore there exists \mathbf{F}' such that the social value is improved while satisfying $v^G(\cdot) \geq v_P^G(\cdot)$.

(iii) The case in which $F^H > F^L$ such that $q^{i,H}(F^H) < q^{i,L}(F^L)$ can also be eliminated as it does not arise in equilibrium. Notice that, in this case, equity is only issued primary in the Low state. Similar reasoning as above rules out this case as the optimal $\check{\mathbf{F}}$.

To conclude, $\check{\mathbf{F}}$ is such that $\check{F}^H \geq \check{F}^L$ and $q^{i,H}(\check{F}^H) \geq q^{i,L}(\check{F}^L)$.

I now explicitly derive the optimal capital regulation that implements a separating equilibrium $\check{\mathbf{F}}$. This part of the derivation is similar to those in of Proposition 6 given in Appendix B.5. There are two scenarios.

Firstly, suppose an interior solution such that $E^{i,H}(\check{F}^H) = E^{i,L}(\check{F}^L)$, i.e. $\check{F}^H = \check{F}^L + \frac{\theta_H - \theta_L}{\Delta_X}$. This also implies that $q^{i,H}(\check{F}^H) = q^{i,L}(\check{F}^L)$. Given this restriction, denote the equity value and the success probability with $E^i(\check{\mathbf{F}})$ and $q^i(\check{\mathbf{F}})$ respectively, which are both equal in both states. The solution is characterised by imposing $v_P^G(\cdot) = v^G(\cdot)$ as given by Eq. 64–65, given the result in Part (i) of this section. The solution is given by

$$\bar{e} = \frac{E^B(\check{\mathbf{F}})}{\gamma E^G(\check{\mathbf{F}}) + (1-\gamma)E^B(\check{\mathbf{F}})} \left(\mathbb{E} [\gamma V^{G,s}(\check{F}^s) + (1-\gamma)V^{B,s}(\check{F}^s)] - 1 \right) \quad (68)$$

The condition for the first scenario to arise is that in equilibrium, $E^i(\check{\mathbf{F}}) \leq V_{FB}^{i,L}$. This is the case if

$$\bar{e} \leq \check{e}_{LP}^T \equiv \frac{E_{FB}^{B,L}}{\gamma E_{FB}^{G,L} + (1-\gamma)E_{FB}^{B,L}} \left(\beta \left[\gamma V^{G,H} \left(\frac{\theta^H - \theta^L}{\Delta_X} \right) + (1-\gamma) V^{B,H} \left(\frac{\theta^H - \theta^L}{\Delta_X} \right) \right] + (1-\beta) \left[\gamma V_{FB}^{G,L} + (1-\gamma) V_{FB}^{B,L} \right] - 1 \right) \quad (69)$$

For $\bar{e} \in [\check{e}_{LP}^T, \check{e}_P^C]$, the second scenario arises, in which case $\check{F}^H < \frac{\theta^H - \theta^L}{\Delta_X}$ and $\check{F}^L = 0$. The solution in this case is

$$\begin{aligned} \bar{e} = & \frac{E^{B,H}(\check{\mathbf{F}})}{\gamma E^{G,H}(\check{F}^H) + (1-\gamma)E^{B,H}(\check{F}^H)} \left(\beta \left[\gamma V^{G,H}(\check{F}^H) + (1-\gamma) V^{B,H}(\check{F}^H) \right] \right. \\ & \left. + (1-\beta) \left[\gamma V_{FB}^{G,L} + (1-\gamma) V_{FB}^{B,L} \right] - 1 \right) \\ & + (1-\beta) \left(\frac{\gamma E_{FB}^{G,L} + (1-\gamma)E_{FB}^{B,L}}{\gamma E^{G,H}(\check{F}^H) + (1-\gamma)E^{B,H}(\check{F}^H)} - \frac{E_{FB}^{B,L}}{E^{B,H}(\check{F}^H)} \right) E^{B,H}(\check{F}^H) \quad (70) \end{aligned}$$

Countercyclical capital ratio

It is now straightforward to show that the capital ratio under the optimal capital regulation is countercyclical. For all cases in which $\bar{F}^L = \bar{F}^L = 0$, the capital ratio is always 100%. For $\bar{e} \in [\check{e}_{LP}^T, \check{e}_P^C]$, the capital ratio is 100% in the Low state and less than 100% in the High state. For $\bar{e} \in [\check{e}_P^C, \check{e}_{LP}^T]$, the capital ratio is $c^H < c^L$, because the bank's equity value is equalised in both state, but the bank has more debt in the high state.

B.8 Proof of Proposition 9

Given the *ex post* bailout guarantee, the bank receives an implicit subsidy when it issues debt to financing its loan portfolio. Therefore the only difference between this extension and the baseline model is that the (IR_C) is replaced with the following

$$(IR_C^{BO}) : \mathbb{E} [V^{G,s}(F_C^s) + (1 - q^{G,s}(F_C^s)) F^s - (1 - \alpha_C^s) E^{G,s}(F_C^s)] \geq 1 - e \quad (71)$$

All results the baseline model therefore hold qualitatively. I show below that the optimal regulation in this case must permit higher leverage in order to implement a separating equilibrium.

Consider imposing $\check{\mathbf{F}}$, the optimal regulation in the baseline model, in this economy with bailouts. Since (PC_C^B) binds in the regulated separating equilibrium, $v^G(\cdot)$ is equal to that in the baseline model. However, $v_P^G(\cdot)$ is higher than before, because the (IR_{Pool}) is

now relaxed, which enables the Good bank to issue less equity in the pooling equilibrium. This violates the constraint $v^G(\cdot) \geq v_P^G(\cdot)$. Therefore leverage must be reduced in order to implement a separating equilibrium.

In the interior solution region, the optimal capital regulation is given by

$$\bar{e} = \frac{E^B(\check{\mathbf{F}}_{BO})}{\gamma E^G(\check{\mathbf{F}}_{BO}) + (1-\gamma)E^B(\check{\mathbf{F}}_{BO})} \left(\mathbb{E} \left[\gamma V^{G,s}(\check{F}_{BO}^s) + (1-\gamma)V^{B,s}(\check{F}_{BO}^s) \right] - 1 \right. \\ \left. + \left[1 - \gamma q^G(\check{\mathbf{F}}_{BO}) - (1-\gamma)q^B(\check{\mathbf{F}}_{BO}) \right] \mathbb{E}[\check{F}_{BO}^s] \right) \quad (72)$$

For a given \bar{e} , the RHS of Eq. 72 is strictly higher than the RHS of Eq. 68 for a given $\check{\mathbf{F}}$ such that $\check{F}^H = \check{F}^L + \frac{\theta^H - \theta^L}{\Delta_X}$, the solution to Eq. 72 is strictly higher than the solution to Eq. 68. That is, higher leverage must be permitted than in the baseline model to implement a separating equilibrium, when there is the expectation of bailouts.

B.9 Proof of Proposition 10

Denote $b \equiv q^{i,s}(F^s) + [1 - q^{i,s}(F^s)] \frac{\bar{D}}{Dc_2} \leq 1$ the probability of receiving the repayment c_2 at $t = 2$ given the deposit insurance coverage \bar{D} . A depositor breaks even at a deposit contract (c_1, c_s) if

$$\lambda u(c_1) + (1-\lambda)bu(c_2) = u(1) \quad (73)$$

This condition is satisfied for a risk-free deposit contract $c_1 = c_2 = 1$ under full deposit insurance $\bar{D} = D$.

I implicitly differentiate Eq. 73 to find that $\frac{\partial b}{\partial c_2} = -\frac{bu'(c_2)}{u(c_2)}$. That is, if increase the $t = 2$ repayment by a marginal unit, the probability of repayment b must decrease by at most $\frac{bu'(c_2)}{u(c_2)} \leq u'(1)$ to allow the depositor to break even.

Holding other liabilities of the bank constant, a marginal increase in the promised repayment c_2 reduces the success probability of the bank by $\frac{\partial q^{i,s}(F)}{\partial c_2} = -\frac{D}{2\Delta_X}$. Holding the deposit insurance level \bar{D} constant, the probability of repayment to a depositor b decrease by

$$\frac{\partial}{\partial c_2} \left(q^{i,s}(F^s) + [1 - q^{i,s}(F^s)] \frac{\bar{D}}{Dc_2} \right) \quad (74)$$

$$= \frac{D}{2\Delta_X} + \frac{\bar{D}}{Dc_2^2} [1 - q^{i,s}(F^s - Dc_2)] \geq \frac{D}{2\Delta_X} \quad (75)$$

If $u'(1) \leq \frac{(1-\bar{e}-k)}{2\Delta_X}$, a depositor will not be willing to deposit with the bank unless he receives a certain repayment $c_1 = c_2 = 1$, given that the bank has to raise at least

$\frac{D}{1-\lambda} = \frac{1-\bar{e}-k}{1-\lambda}$ from the depositors. This is because Eq. 73 is not satisfied for any $c_2 > 1$ for a given level of D and $\bar{D} = D$. Therefore deposits must be raised with a certain promised repayment $c_1 = c_2 = 1$, backed by a full deposit insurance coverage $\bar{D} = D$.