

A Model of Anomaly Discovery*

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April 30, 2014

*We thank our colleagues at Peking University and Yale School of Management for helpful discussions. Please direct all correspondence to Hongjun Yan, Email: hongjun.yan@yale.edu. The latest version of the paper is available at <http://faculty.som.yale.edu/hongjunyan/>.

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Abstract

Our model shows that the discovery of an anomaly reduces its magnitude, regardless of whether the anomaly is due to risk or mispricing. After the discovery, the return from the anomaly becomes more correlated with the returns from other existing anomalies. While risk- and mispricing-based anomalies have similar implications for asset prices, they have sharply different implications on investors' asset holdings, as well as welfare. Our model also produces new testable implications on the effect of anomaly discovery.

Keywords: Anomaly, Arbitrage, Discovery, Arbitrageur-based asset pricing.

JEL Classifications: G11, G23.

1 Introduction

A significant portion of the asset-pricing literature has been devoted to discovering and interpreting “anomalies,” empirical patterns that appear inconsistent with existing asset-pricing models. While it is relatively less challenging to reach conclusions on the existence and robustness of the empirical patterns, their interpretations are often more controversial. One prominent example is the value premium discovered in Basu (1977), which has both risk-based and mispricing-based interpretations since the early work by Fama and French (1993) and Lakonishok, Shleifer, and Vishny (1994). Twenty years later, the literature still has not reached a consensus on its interpretation.

One effort to distinguish the two types of interpretations has been “out-of-sample” examinations of the anomalies after their discovery. The idea is that if an anomaly is mispricing-based, after the anomaly is publicized, its magnitude should decay. If an anomaly is risk-based, however, its magnitude should persist. Earlier studies examine one or a few anomalies (e.g., Johnson and Schwartz (2000), Jegadeesh and Titman (2001), Schwert (2003)), and more recent studies attempt to analyze a large number of anomalies accumulated in the literature (e.g., Chordia, Subrahmanyam, and Tong (2013), Green, Hand, and Zhang (2013), McLean and Pontiff (2013)). For example, McLean and Pontiff (2013) analyze the post-discovery performances of 82 characteristics that have been identified in published academic studies, and find that post-discovery anomaly returns decay by 44% on average.

While this approach can assess the influence of data mining in the original discoveries, our analysis casts doubt on its effectiveness in distinguishing risk-based anomalies from mispricing-based ones. In particular, our model shows that after the discovery of an anomaly, its magnitude decreases *regardless* of whether the anomaly is caused by risk or mispricing. The logic is straightforward once we recognize that, by definition, a discovery

should inform some investors of the anomaly for the first time.

Let us use value premium as an example. Suppose that, before the discovery in Basu (1977), value stocks have higher average returns because they are riskier to some investors—for example, value stocks have larger exposures to economy-wide financial distress. Unless we believe that Basu was the last person to discover this return pattern (i.e., everyone else was aware of this pattern before), his publication should have informed some investors of the high returns from value stocks. Those newly informed investors may not find those value stocks risky, perhaps because they are wealthy and are less concerned about value stocks' low returns during the economy-wide financial distress. Therefore, those investors are happy to exploit this anomaly, and consequently reduce its magnitude. The same logic applies to the case where the value premium is caused by mispricing. If the higher return in value stocks was due to investors' behavioral biases, its discovery would also induce more capital to exploit the anomaly and reduce its post-discovery magnitude.¹

In essence, discovery of an anomaly increases the amount of arbitrage capital that exploits the anomaly and so reduces its magnitude, regardless of whether the anomaly is caused by risk or mispricing in the first place. Therefore, the post-discovery decay of an anomaly does not necessarily imply that it was caused by mispricing.

Another implication from this view is that the discovery of an anomaly makes the anomaly return (e.g., the return from buying value and selling growth stocks in our earlier example) more correlated with the returns from other existing anomalies. The intuition is the following. Suppose the returns of value and growth stocks are independent of arbitrageurs' pre-discovery investment opportunity, which is presumably based on another anomaly. That is, before the discovery of the value anomaly, its return is uncorrelated

¹Behavioral explanations are usually based on the biases that are considered systematic and are derived from robust and deeply formed aspects of human psychology. That is, those biased investors do not adjust their behavior even after the discovery of the anomaly.

with the return from the existing anomaly. After the discovery, however, arbitrageurs start exploiting the value anomaly, as well as the existing anomaly. This creates a correlation between the two anomalies through a wealth effect. Suppose the return from the existing anomaly is unexpectedly high one period, thus increasing the wealth of the arbitrageurs. Everything else being equal, the arbitrageurs will allocate more investment to all his opportunities, including the value anomaly. This higher investment pushes up the price of value stocks and pushes down the price of growth stocks, leading to a high return for the value anomaly this period. Hence, the wealth effect increases the correlation between the value anomaly return and the return from the existing anomaly. Interestingly, McLean and Pontiff (2013) not only find the post-discovery decay in the large number of anomalies they examine; they also find that post-discovery, the return of the newly discovered anomaly becomes more correlated with the returns from other existing anomalies.

Our simple model also produces two new testable implications on asset prices. First, our model shows that the discovery of an anomaly reduces the correlation coefficient between the returns of “overvalued” stocks and “undervalued” ones. In the value premium example, after the discovery, arbitrageurs long value stocks and short growth stocks. The above-mentioned wealth effect reduces the correlation coefficient between the returns from value and growth stocks. Suppose value stocks have a higher return one period. This leads to a larger profit to the arbitrageurs, who then allocate more investment to their long-short portfolio to exploit the value premium. The short position in growth stocks pushes down their prices, leading to lower returns this period. Hence, the discovery of the value premium reduces the correlation coefficient between value and growth stocks.

The second new prediction is that after the discovery of an anomaly, the market liquidity of the “undervalued” stocks improves, while the liquidity of the “overvalued” stocks deteriorates. In the above value premium example, after the discovery, selling value stocks

has a smaller price impact because of arbitrageurs' purchase. That is, the market liquidity, as suggested by the Amihud (2002) measure, improves. Similarly, growth stocks become less liquid, because selling growth stocks has a larger price impact after the discovery, when arbitrageurs start shorting them.

Our analysis casts doubt on the approach of distinguishing risk- and mispricing-based anomalies through examining post-discovery asset prices. The return of an anomaly decreases after its discovery, for both risk- and mispricing-based cases. Even the two new predictions from our model are similar across risk- and mispricing-based cases. So how can we distinguish them?

One possibility is to examine investors' *portfolios* directly, since risk- and mispricing-based anomalies have starkly different implications on investors' portfolios. For instance, to examine a risk-based explanation of value premium, one can check if the investors who underweight value stocks appear to be those who should be more concerned about the risk that is proposed in the explanation. For example, we can examine whether their labor income or other assets are indeed more exposed to that risk. While this direct approach is demanding for the required dataset, in this "big data" era, with more and more micro-level datasets becoming available, this approach might not be a fantasy, either.

Our model shows that the welfare implication of a discovery is different across risk- and mispricing-based anomalies. In the risk-based version of the value premium example, the financial distress risk was narrowly shared among investors before the discovery. The discovery makes it possible for those investors to offload some of their value stocks to the newly arrived arbitrageurs. Hence, the better risk sharing improves the welfare of all investors. In contrast, if the anomaly is mispricing-based, its discovery increases the biased investors' subjective expected utility but decreases their objective expected utility. In the value premium example, if the anomaly was due to investors' incorrect belief that growth

stocks will have higher future profitability than value stocks, then its discovery will make investors worse off, although they mistakenly think they are better off. When arbitrageurs start exploiting the anomaly, biased investors end up investing even more in growth stocks and less in value stocks. This increases the biased investors' subjective expected utility but decreases their expected utility evaluated under the objective belief.

The essence of our analysis is the role of capital-constrained arbitrageurs in determining asset prices, a point increasingly appreciated since Shleifer and Vishny (1997). Our model is closely related to the analysis of arbitrageurs' risk-bearing capacity (e.g., Xiong (2001) and Kyle and Xiong (2001)). More broadly, our paper belongs to the growing literature that explores the role of arbitrageurs (see, e.g., Gromb and Vayanos (2002), Liu and Longstaff (2004), Basak and Croitoru (2006), and Brunnermeier and Pedersen (2009), Kondor (2009), He and Krishnamurthy (2013), Kondor and Vayanos (2013)). These studies focus on the impact of arbitrageurs in contagion, risk sharing, liquidity, portfolio choice, and so on, while our paper focuses on the impact of the discovery of anomalies and highlights the difficulty, and the possible solution, in distinguishing the risk-based and mispricing-based interpretations.

Finally, existing models of anomalies abstract away from the “discovery” aspect. That is, in existing models, the discovery of an anomaly does not inform more investors about the phenomenon. For example, Li, Livdan, and Zhang (2009) argue that many anomalies are consistent with a unified model of firms' optimal investment decisions. A large number of studies of value premium argue that value stocks are risky or investors are overly enthusiastic about growth stocks. In those models, a discovery does not change the set of investors who are aware of the phenomenon. In contrast, we take the discovery aspect seriously: by definition, a discovery should inform some investors of the phenomenon for the first time, unless the author who publishes the discovery turns out to be the last person

to discover the phenomenon. Our paper formally analyzes the consequence of this discovery aspect. As Cochrane (1999) commented, “...this view is (so far) the least stressed in academic analysis. In my opinion, it may end up being the most important.”

The rest of the paper is as follows. Sections 2 and 3 present a risk-based and a mispricing-based model, respectively. Section 4 compares the two cases and Section 5 concludes. The numerical algorithm and proofs are in the appendix.

2 A Model of Risk-Based Anomaly Discovery

We consider a two-period model, with time $t = 0, 1, 2$. Trading takes place at time 0 and 1, and consumption occurs at time 2. There is a continuum of identical investors, with a population size of one. There is one risk-free asset, and its interest rate is normalized to 0. There are two risky assets, asset 1 and asset 2, each of which is a claim to a single cash flow at $t = 2$. At time 0, investors are endowed with one unit of both assets, and k dollars cash.

The cash flows from assets 1 and 2 are independent and have the same *ex ante* distribution. Specifically, for $i = 1, 2$, and $t = 0, 1$, we have

$$D_{i,t+1} = D_{i,t} \times \mu_{i,t+1}, \tag{1}$$

where $D_{i,0} = 1$, and $\mu_{i,t+1}$ are independent across i and t and have the same binary distribution

$$\mu_{i,t+1} = \begin{cases} \mu + \sigma & \text{with probability } p, \\ \mu - \sigma & \text{with probability } 1 - p, \end{cases} \tag{2}$$

where $\mu > \sigma > 0$, and $0 < p < 1$. Asset i is a claim to cash flow $D_{i,2}$ at time $t = 2$. Therefore, the two cash flows (i.e., $D_{1,2}$ and $D_{2,2}$) are independent from each other and have the same distribution at $t = 0$. For $i = 1, 2$, and $t = 0, 1, 2$, we use $P_{i,t}$ to denote the price of asset i at time t , which will be determined endogenously in equilibrium. For

$t = 1, 2$, we denote the gross return of asset i at time t as

$$r_{i,t} = \frac{P_{i,t}}{P_{i,t-1}}.$$

At $t = 2$, asset prices are pinned down by the final cash flow: $P_{i,2} = D_{i,2}$, for $i = 1, 2$.

2.1 Anomaly discovery

In the literature, a phenomenon is labeled as an anomaly if the observed cross-sectional variation in average stock returns cannot be explained by existing standard models. For example, the evidence in Basu (1977) suggests that value stocks have higher average returns than growth stocks and the return difference cannot be explained by the CAPM. Hence, this phenomenon is viewed as an anomaly.

A large number of subsequent studies have tried to explore the cause of this anomaly. Broadly speaking, the explanations can be classified into two categories, risk-based explanations and mispricing-based ones. In risk-based explanations, value stocks offer low returns during “bad times,” and therefore demand a risk premium. On the other hand, mispricing-based explanations explore the idea that systematic behavioral biases may induce investors to overestimate the future profitability of growth stocks relative to value stocks.

We argue that these two approaches abstract away from the *discovery* aspect of an anomaly. By definition, a discovery should inform some investors of the phenomenon for the first time. However, in existing models of those two approaches, a discovery does not change the set of investors who are aware of the phenomenon. In the risk-based approach, investors knew the phenomenon before the discovery, and in the mispricing-based approach, biased investors do not recognize (or respond to) the phenomenon even after the discovery. Our paper attempts to fill this gap and formally analyze the effect from a discovery that informs some investors about the phenomenon for the first time. For convenience, we call

those newly informed investors “arbitrageurs.”

There is a continuum of identical arbitrageurs, with a population size of one. In aggregate, they have $W_0^a \geq 0$ dollars at $t = 0$. Upon the discovery of the anomaly, arbitrageurs start exploiting it. To capture this, we assume that once the arbitrageurs become aware of the anomaly, they will take a long-short strategy in the two assets so that they can exploit the anomaly and stay “market neutral.”² Specifically, we use $\theta_{i,t}^a$ to denote the fraction of arbitrageurs’ wealth that is invested in asset $i \in \{1, 2\}$ at date $t \in \{0, 1\}$, a market-neutral strategy is such that

$$\theta_{1,t}^a + \theta_{2,t}^a = 0. \quad (3)$$

In addition to the risk-free asset and assets 1 and 2, arbitrageurs also have access to another investment opportunity, which presumably exploits other existing anomalies and is not available to investors. We call this existing anomaly “asset e ,” and assume its gross return at $t = 1, 2$ is

$$r_{e,t} = \begin{cases} \mu_e + \sigma_e, & \text{with probability } p_e, \\ \mu_e - \sigma_e, & \text{with probability } 1 - p_e, \end{cases}$$

where $\mu_e > \sigma_e > 0$, and $0 < p_e < 1$. Moreover, $r_{e,t}$ is assumed to be independent from $D_{i,t}$. That is, assets 1 and 2 are independent from the existing anomaly—asset e .

If we use $\theta_{e,t}^a$ to denote the fraction of arbitrageurs’ wealth invested in asset e at date $t = 0, 1$, the arbitrageur’s wealth dynamic is given by

$$W_{t+1}^a = W_t^a \left[\sum_{i \in \{1, 2, e\}} \theta_{i,t}^a r_{i,t+1} + \left(1 - \sum_{i \in \{1, 2, e\}} \theta_{i,t}^a \right) \right], \quad (4)$$

²This assumption is made so that the arbitrageurs focus on exploiting the anomaly. Alternatively, we can simply assume that after the discovery, the arbitrageurs become aware of the existence of assets 1 and 2. Under this alternative assumption, however, arbitrageurs will not only take a long-short position in the two assets, but also start investing in both assets. The latter will simply push up the prices of both assets. Our paper is not interested in analyzing this latter effect. Moreover, in the value premium example, for instance, it seems more natural to think that, after the discovery of the value premium, hedge funds start buying value stocks and shorting growth stocks, rather than hedge funds become aware of the existence of both value and growth stocks and so start to buy all of them.

for $t = 0, 1$. Their objective is to choose $\theta_{i,t}^a$ for $i = 1, 2, e$, to

$$\max_{\theta_{i,t}^a} E_0 [\log (W_2^a)], \quad (5)$$

subject to (3) and (4).

2.2 Risk-based anomaly

We assume that investors have a hedging demand in one of the assets. Hence, in the equilibrium that we will characterize shortly, the prices of the two assets are different at $t = 0$, although their payoffs at $t = 2$ have the same *ex ante* distribution. We label this price difference as an anomaly. It is risk based because it is caused by investors' hedging demand.

Specifically, investors are endowed with a non-tradable asset, which is a claim to a cash flow $\rho D_{1,2}$ at $t = 2$ (e.g., labor income), with $\rho > 0$. That is, this non-cash endowment is perfectly correlated with the payoff from asset 1. Denote investors' wealth, excluding their non-cash endowment, at time t as W_t for $t = 0, 1, 2$. Let $\theta_{i,t}$ be the fraction of W_t that is allocated in asset $i \in \{1, 2\}$ at date $t \in \{0, 1\}$. Hence, investors' wealth dynamic is given by

$$W_{t+1} = W_t \left[\sum_{i \in \{1,2\}} \theta_{i,t} r_{i,t+1} + \left(1 - \sum_{i \in \{1,2\}} \theta_{i,t} \right) \right], \quad (6)$$

for $t = 0, 1$, with $W_0 = k + P_{1,0} + P_{2,0}$. Investors' objective is to choose $\theta_{i,t}$, for $i = 1, 2$, and $t = 0, 1$ to

$$\max_{\theta_{i,t}} E_0 [\log (W_2 + \rho D_{1,2})], \quad (7)$$

subject to (6).

This formulation represents the essence of risk-based interpretations of anomalies. In the value premium example, investors find value stocks risky and so reduce their demand

for those stocks. Similarly, investors in our model find asset 1 risky because its return is correlated with their endowment.

The competitive equilibrium is defined as asset prices ($P_{i,t}$ for $i = 1, 2$, and $t = 0, 1$) and the portfolios of investors and arbitrageurs ($\theta_{i,t}$ for $t = 0, 1$ and $i = 1, 2$, and $\theta_{i,t}^a$ for $t = 0, 1$, $i = 1, 2, e$), such that investors' portfolios optimize (7) and arbitrageurs' portfolios optimize (5), and markets clear, i.e., for $i = 1, 2$ and $t = 0, 1$,

$$W_t \theta_{i,t} + W_t^a \theta_{i,t}^a = P_{i,t}. \quad (8)$$

2.3 Equilibrium prices

Proposition 1 *Equilibrium prices $P_{i,t}$ and portfolio choices $\theta_{i,t}$, for $i = 1, 2$, $t = 1, 2$, and $\theta_{i,1}^a$, for $i = 1, 2, e$, can be characterized by equations (3), (8), and*

$$E_t \left[\frac{r_{i,t+1} - 1}{W_{t+1} + \rho P_{1,t+1}} \right] = 0, \quad (9)$$

$$E_t \left[\frac{r_{1,t+1} - r_{2,t+1}}{W_{t+1}^a} \right] = 0, \quad (10)$$

$$E_t \left[\frac{r_{a,t+1} - 1}{W_{t+1}^a} \right] = 0. \quad (11)$$

This equation system is highly non-linear and we have not been able to establish the existence and uniqueness of its solution.³ However, we have always been able to solve the equation system numerically, and the solution appears to be unique. Moreover, we can obtain some properties of the equilibrium analytically in special cases. For example, we show in the appendix that when W_0^a is sufficiently small, we have $P_{1,0} < P_{2,0}$. Due to the

³One might be somewhat surprised that the simple two-period structure in our model does not allow for a closed-form solution. In fact, the wealth effect in our model has similar complexity as that in the continuous-time model in Xiong (2001), which also heavily relies on numerical analysis. As noted in Gromb and Vayanos (2002), a two-period model of arbitrageurs and investors with a wealth effect is not as tractable as its appearance suggests (page 381). In a recent study, Konder and Vayanos (2013) gain more tractability by simplifying investors' decisions.

non-tradable asset, investors demand less of asset 1 than asset 2, leading to a lower price for asset 1, although both assets have the same fundamentals *ex ante*. This can be viewed as a risk-based anomaly, since it is caused by investors’ hedging demand.

To analyze the impact of the discovery of the anomaly, consider the following thought experiment. Before the anomaly is discovered, no arbitrageur is aware of the fact that asset 1 has a lower price and a higher expected return. This is represented by the case $W_0^a = 0$.

After the discovery, arbitrageurs buy asset 1 and sell asset 2. For convenience, we call the return from this long-short portfolio, $r_{1,1} - r_{2,1}$, the “anomaly return.” One of our goals is to compare the magnitude of the anomaly return before and after the discovery. That is, we will compare the anomaly return in the case of $W_0^a = 0$ (i.e., pre-discovery) with the anomaly return in the case of $W_0^a > 0$ (i.e., post-discovery).

We solve the equation system in Proposition 1 numerically, following the the algorithm in Appendix A. In the following numerical analysis, we use the parameter values in Table 1 as the baseline case. We then vary only one parameter at a time to examine its impact on the equilibrium.

Table 1: Baseline Parameterizations

Parameter	W_0^a	k	ρ	μ	σ	p	μ_e	σ_e	p_e
Value	5	1	1	1.2	0.6	0.5	1.4	0.5	0.5

Figure 1 illustrates the effects of arbitrageurs on anomaly returns. For example, Panel A shows that the discovery of an anomaly reduces its expected return. In the absence of arbitrageurs (i.e., $W_0^a = 0$), the expected anomaly return is around 5.5% and is due to investors’ hedging demand. When the anomaly is publicized, arbitrageurs start exploiting the opportunity, reducing the expected anomaly return. As shown in Panel A, the expected anomaly return is decreasing in arbitrageurs’ wealth level W_0^a . Moreover, Panel B shows

that the volatility of the anomaly return is increasing in arbitrageurs' wealth. This is because, through the fluctuation of arbitrageurs' wealth, their trading transmits the volatility from asset e to the prices of assets 1 and 2. Panels C and D demonstrate how arbitrageurs' existing investment opportunity (i.e., asset e) affects the anomaly return. If asset e is more attractive (i.e., μ_e increases or σ_e decreases), arbitrageurs allocate less wealth to exploit the newly discovered anomaly, leading to a higher expected anomaly return. As shown in Panels C and D, the expected anomaly return is increasing in μ_e and decreasing in σ_e .

2.4 Correlation among anomaly returns

By the construction of our model, in the absence of arbitrageurs, the anomaly return $r_{1,1} - r_{2,1}$ is independent of the existing anomaly return $r_{e,1}$. How does the discovery of an anomaly affect the correlation between its return and the return from the existing anomaly?

Intuitively, after the discovery of an anomaly, arbitrageurs start exploiting it, as well as the existing anomaly, asset e . This creates a correlation through the wealth effect. Suppose the return from asset e is unexpectedly high one period. This increases the wealth of the arbitrageurs. Everything else being equal, the arbitrageurs will allocate more investment to all of their opportunities, including the newly discovered anomaly. This higher investment pushes up the price of asset 1 and pushes down the price of asset 2, leading to a high anomaly return $r_{1,1} - r_{2,1}$ this period. That is, the wealth effect increases the correlation between the newly discovered anomaly return and the return from the existing anomaly. Interestingly, McLean and Pontiff (2013) not only find the post-discovery decay in the large number of anomalies they examine, they also find that after the discovery, the return of the new anomaly becomes more correlated with the returns from other existing anomalies.

The above intuition is illustrated in Figure 2. For example, Panel A plots the correlation coefficient between $r_{1,1} - r_{2,1}$ and $r_{e,1}$ against W_0^a . Before the discovery of the anomaly (i.e.,

$W_0^a = 0$), the correlation is 0. After the discovery, when arbitrageurs start exploiting the anomaly, the correlation becomes positive. The correlation is initially increasing in the size of the arbitrage capital. Interestingly, the impact of arbitrageurs' participation on the return correlation is not monotonic. This is because arbitrageurs have two effects. The first is the above-mentioned wealth effect, which increases the correlation. The second is that arbitrage capital reduces the effect of hedging demand on the prices of the two assets, so that the long-short return $r_{1,1} - r_{2,1}$ is driven more by the fundamental values of the two assets, which are independent of the existing anomaly. This reduces the correlation between $r_{1,1} - r_{2,1}$ and $r_{e,1}$. The plot shows that when the size of the arbitrage capital is sufficiently large, the second effect dominates and a further increase in arbitrage capital reduces the correlation.

The above intuition is further illustrated in Panels B through D. In particular, the wealth effect is stronger when arbitrageurs have a larger exposure to asset e . Naturally, when arbitrageurs have a larger position in asset e , their wealth is more sensitive to the realized return of $r_{e,1}$. Hence, asset e 's return has a stronger effect on the anomaly return, leading to a higher correlation. In Panel B, for example, as the expected return from asset e increases (i.e., a higher μ_e), it leads to a higher correlation. Similarly, in Panel C, as the volatility of asset e increases (i.e., a higher σ_e), it leads to a weaker wealth effect and a lower correlation. In Panel D, we vary investors' hedging demand ρ . In the case of $\rho = 0$, the long-short strategy return $r_{1,1} - r_{2,1}$ is driven purely by fundamental shocks and so is independent from the return of asset e . As ρ increases, it creates a spread between assets 1 and 2 and attracts arbitrageurs, leading to the above-mentioned wealth effect and correlation. The stronger the hedging demand (i.e., larger ρ), the higher the correlation.

3 Mispricing-based anomaly

To contrast mispricing with risk as the economic determinants of return anomalies, we introduce a version of our model with arbitrageurs and naive investors who have incorrect beliefs about the asset payoffs. Specifically, we modify the previous model by setting $\rho = 0$, that is, there is no hedging demand. Instead, the anomaly is now due to investors' misperception. The fundamentals of the two assets are still given by (1) and (2). However, investors have a misperception about asset 1, while their belief about asset 2 is correct. In particular, they believe that for $t = 0, 1$,

$$\mu_{1,t+1} = \begin{cases} \mu + \sigma & \text{with probability } p - b, \\ \mu - \sigma & \text{with probability } p + b, \end{cases} \quad (12)$$

where $0 \leq b < p$. That is, investors underestimate asset 1's expected cash flow, and their bias in belief is captured by b .

Similar to (7), investors' objective is to choose $\theta_{i,t}$, for $i = 1, 2$, and $t = 0, 1$ to

$$\max_{\theta_{i,t}} E_0^* [\log (W_2)], \quad (13)$$

subject to (6), where $E_0^* [\cdot]$ indicates that the expectation is taken under the biased belief in (12). Arbitrageurs have correct beliefs, and their objective is given by (5), as in the previous section.

This formulation is meant to capture the essence of mispricing-based interpretations of anomalies. For instance, in the value premium example, Lakonishok, Shleifer, and Vishny (1994) argue that investors are overly enthusiastic about the glamorous growth stocks and have a low demand for value stocks. Similarly, in our model, investors underestimate the payoff from asset 1 and so have a low demand.

The competitive equilibrium is defined as asset prices ($P_{i,t}$ for $i = 1, 2$, and $t = 0, 1$) and investors' and arbitrageurs' portfolio choices ($\theta_{i,t}$ for $t = 0, 1$, $i = 1, 2$, and $\theta_{i,t}^a$ for $t = 0, 1$,

and $i = 1, 2, e$), such that investors' portfolios optimize (13), and arbitrageurs' portfolios optimize (5), and markets clear as in (8).

Proposition 2 *Equilibrium prices $P_{i,t}$ and portfolio choices $\theta_{i,t}$, for $i = 1, 2$, $t = 1, 2$, and $\theta_{i,1}^a$, for $t = 0, 1$, and $i = 1, 2, e$, can be characterized by equations (3), (8), (10), (11), and*

$$E_t^* \left[\frac{r_{i,t+1} - 1}{W_{t+1}} \right] = 0. \quad (14)$$

Although we have not obtained the analytical solution to the above equation system, we can characterize some equilibrium properties analytically in special cases. For example, we show in the appendix that $P_{1,0} < P_{2,0}$ when W_0^a is sufficiently small. Due to the wrong belief in (12), investors demand less of asset 1 than asset 2. This naturally pushes down the price of asset 1.

Similar to the analysis in Section 2, the pre-discovery case is represented by $W_0^a = 0$. The post-discovery equilibrium is represented by the case $W_0^a > 0$. What is implicitly assumed here is that the discovery does not affect investors' bias b . That is, the bias is systematic and deeply rooted. These biased investors do not adjust their behavior after the discovery of the anomaly.

Comparing this mispricing-based model with the risk-based model in Section 2, one can see that in both cases the price of asset 1 is lower because of investors' low demand. Post-discovery, when arbitrageurs start exploiting the anomaly, they reduce its magnitude in both cases. This naturally leads to the question of whether we can distinguish a risk-based anomaly from a mispricing-based one from examining their post-discovery performance.

4 Compare Risk- and Mispricing-based Anomalies

4.1 Post-discovery performance

Can we distinguish risk-based and mispricing-based anomalies from the post-discovery changes in anomaly returns? To address this question, we compare the post-discovery equilibrium for a risk-based anomaly with that for a mispricing-based anomaly. In particular, we set $b = 0.065$ and adopt all the parameters (except for ρ) from Table 1. We choose this value for b so that, for $W_0^a = 0$, the prices of the two assets ($P_{1,0}$ and $P_{2,0}$) are the same across the risk-based model in Section 2 and the mispricing-based model in Section 3. That is, before the discovery, the risk-based anomaly and the mispricing-based anomaly are comparable. We now compare the post-discovery return dynamic across the two cases.

Figure 3 summarizes the comparisons. Panels A, B, and C show that it is very difficult to distinguish a risk-based anomaly from a mispricing-based one by examining post-discovery returns. The solid lines in these figures are for the risk-based anomaly, and the dashed lines the mispricing-based anomaly. In each of the panels, the two economies respond to increased arbitrageurs' participation in a similar fashion, both qualitatively and quantitatively. In particular, Panel A shows that after the discovery of an anomaly, its expected return decreases regardless of whether the anomaly is caused by risk or mispricing. Panel B shows that the post-discovery behavior of the anomaly volatility is also similar across the two cases, and the volatility of the anomaly return increases slightly after its discovery. Finally, for both the risk-based and mispricing-based cases, the discovery of an anomaly increases the correlation between its return and the existing anomaly return, as shown in Panel C. Even the non-monotonic pattern is similar across the two cases.

A number of empirical studies have analyzed the out-of-sample performance of anomaly returns. For example, McLean and Pontiff (2013) analyze the post-discovery performances

of 82 characteristics that have been identified in published academic studies. One of the goals of these studies is to distinguish risk-based and mispricing-based interpretations.⁴ The above analysis, however, casts doubt on the effectiveness of this approach. In essence, the discovery of an anomaly increases the amount of arbitrage capital that exploits the anomaly and so reduces its magnitude, regardless of whether the anomaly is caused by risk or mispricing in the first place. Therefore, the post-discovery decay of an anomaly does not necessarily imply that it was caused by mispricing.

4.2 New predictions

Our model also produces two new testable implications on asset prices. First, it shows that the discovery of an anomaly reduces the correlation coefficient between the returns of assets 1 and 2. The intuition is as follows. In the value premium example, after the discovery, arbitrageurs long value stocks and short growth stocks. The wealth effect reduces the correlation coefficient between the returns from value and growth stocks. Suppose value stock returns are higher than expected one period. It leads to a higher profit to the arbitrageurs, who then allocate more investment to their long-short portfolio to exploit the value premium. The short position in growth stocks pushes down their prices, leading to lower returns this period. That is, the wealth effect leads to a negative correlation between value and growth stocks.

This intuition is illustrated in Panel D of Figure 3. It shows that the correlation coefficient between assets 1 and 2 is decreasing in arbitrageurs' wealth level W_0^a . The mispricing-based case is shown by the dashed line. The correlation coefficient between the returns of the two assets is 4% before the discovery of the anomaly (i.e., $W_0^a = 0$). After the discovery, this correlation is smaller. It is decreasing in W_0^a and can even become negative

⁴Another important goal of this literature is to examine whether the anomalies are due to data mining, which is not analyzed in our model.

if W_0^a is large enough.⁵ The implication in the risk-based case is similar, and is captured by the solid line. The correlation between the returns of assets 1 and 2 decreases after the discovery of the anomaly.

The second new prediction is about the effect of the discovery on the market liquidity of assets 1 and 2. Specifically, we derive the following illiquidity measure in our model: the illiquidity of asset i is its price elasticity to its supply

$$IL_i = -\frac{\frac{dP_{i,0}}{P_{i,0}}}{d\epsilon_i}, \quad \text{for } i = 1, 2, \quad (15)$$

where ϵ_i is the extra supply of asset i to the economy at $t = 0$. That is, this illiquidity measure reflects the price impact of selling an extra unit of the asset to the economy. The higher the IL_i , the more illiquid the asset is. Hence, this measure corresponds to the liquidity measure in Amihud (2002).

Intuitively, in both the risk- and mispricing-based cases, asset 1 has a lower price at $t = 0$. After the discovery of the anomaly, arbitrageurs buy asset 1 and sell asset 2. Hence, if one sells asset 1 into the economy, its price impact will be smaller, due to arbitrageurs' purchase. Therefore, asset 1 becomes more liquid after the discovery, i.e., IL_1 is smaller. Similarly, asset 2 becomes less liquid after the discovery.

We illustrate this intuition in Panels E and F. Panel E plots the illiquidity measure of asset 1 against arbitrageurs' wealth W_0^a . The solid line is for the risk-based case, and it shows that the illiquidity measure of asset 1 decreases with arbitrageurs' wealth. When more arbitrageurs are attracted to the anomaly, their purchase of asset 1 supports its price, and hence selling asset 1 into the economy has a smaller price impact. The dashed line represents the mispricing-based case, and the result is similar. The liquidity measure for asset 2 is plotted in Panel F. Consistent with the above intuition, it shows that for both the

⁵Note that the fundamentals of the two assets are not correlated. The correlation between the returns of the two assets is positive because investors hold both of the assets, and their wealth fluctuation induces a positive correlation. The intuition is similar to the above wealth effect from arbitrageurs.

risk- and mispricing-based cases, asset 2 becomes less liquid when the arbitrageurs' wealth is larger.

4.3 One possible solution

Our analysis so far shows that the post-discovery behavior of anomaly returns is similar across the risk- and mispricing-based cases. More generally, even our model's new predictions on asset prices are similar across the two cases. These results highlight the difficulty of distinguishing risk- and mispricing-based anomalies by examining asset prices. So what is the solution?

We argue that although it is difficult to distinguish risk- and mispricing-based anomalies based on prices, it is more promising to analyze investors' *portfolios*. The idea is that investors' holdings might offer direct evidence on *why* they overweight one asset and underweight another. For example, Figure 4 plots investors' "portfolio tilt" against arbitrageurs' wealth. Specifically, the solid line represents the case of risk-based anomaly and plots the investors' exposure to asset 1 (including their non-tradable endowment) minus their holding in asset 2, denominated in dollars. The dashed line represents the case of mispricing-based anomaly and plots the investors' holding in asset 1 minus their holding in asset 2.

The figure illustrates the essential difference between risk- and mispricing-based anomalies. In the former, investors recognize the fact that asset 1's expected return is higher than asset 2's, and so they have a *higher* total exposure (including non-tradable endowment) to asset 1 than to asset 2. That is, the solid line is above zero. Due to the non-tradable assets, investors have a higher exposure to asset 1 than to asset 2, despite that they underweight asset 1 in the stock market. In the mispricing-based anomaly, however, investors underweight asset 1 because they mistakenly believe that it has a lower future payoff. That is, they have a lower total exposure to asset 1 than to asset 2. Therefore, the solid line (the

risk-based case) is above zero, while the dashed line (the mispricing-based case) is below zero. When the arbitrageurs' capital increases, investors tilt less for the risk-base case (i.e., the solid line is closer to zero), but tilt more for the mispricing-based case (i.e., the dashed line is further away from zero).

For example, Fama and French (1993, 1996) interpret the value premium as value stocks exposing investors to risks associated with economy-wide financial distress. To evaluate this risk-based explanation of value premium, one can examine whether the investors whose portfolios underweight value stocks are those who are more exposed to poor performances in times of financial distress (e.g., their labor income or other assets are more exposed to financial distress).

To be fair, while examining portfolio holdings is a direct approach in distinguishing risk- and mispricing-based anomalies, it is also very demanding in the dataset. It requires detailed information on investors' positions, including their non-tradable assets. Nevertheless, it is hopeful that as more micro-level data on investors' holdings become available, this test may eventually become feasible.

4.4 Welfare

How does the discovery of an anomaly affect investors' welfare? To address this question, we first need to clarify our welfare measures. In the risk-based case, we simply use investors' expected utility at $t = 0$. We then compare the pre-discovery welfare (i.e., for the case of $W_0^a = 0$) with the post-discovery welfare (i.e., for the case of $W_0^a > 0$). For the mispricing-based case, we use "subjective welfare" to refer to investors' subjective expected utility at $t = 0$, and use "objective welfare" to refer to investors' utility evaluated under the objective belief at $t = 0$.

Proposition 3 *The discovery of a risk-based anomaly increases investors' welfare, and the discovery of a mispricing-based anomaly increases investors' subjective welfare.*

In the case of a risk-based anomaly, arbitrageurs essentially offer better risk sharing to investors. Before the discovery, the endowment risk is narrowly shared among investors (i.e., arbitrageurs are not involved). After the discovery, this endowment risk is shared between investors and arbitrageurs: investors unload asset 1 to arbitrageurs to hedge against their endowment risk. Arbitrageurs' trading makes the hedging cheaper. As shown by the solid line in Panel A of Figure 5, arbitrageurs' involvement after the anomaly discovery improves investors' welfare, and investors' expected utility at $t = 0$ increases with arbitrageurs' wealth level.

What is the welfare impact in the mispricing-based anomaly? The above intuition suggests that investors' *subjective* expected utility increases after the discovery. That is, when arbitrageurs start exploiting the anomaly, naive investors think they are better off, since they can offload some of asset 1, which they are pessimistic about. As shown by the dashed line in Panel A, the pattern of investors' subjective expected utility is increasing in the arbitrageurs' wealth, similar to the pattern for the risk-based case.

What is the "true" effect on investors' welfare? In other words, how does the discovery of a mispricing-based anomaly affect naive investors' expected utility, evaluated under the objective belief? Naturally, as arbitrageurs take advantage of naive investors and exploit the anomaly, it should reduce naive investors' welfare. As shown by the dotted line in Panel A, naive investors' objective welfare decreases in arbitrageurs' wealth. In the value premium example, suppose it was caused by investors' misperception that the glamorous growth stocks will outperform. The discovery of this anomaly attracts arbitrageurs to buy value and sell growth stocks. Consequently, investors end up holding more growth stocks and less value stocks, and they will suffer from worse performances in the future. The

magnitude of the welfare impact naturally depends on the size of the bias. As shown in Panel B, when the size of investors' bias (i.e., b) increases, arbitrageurs can exploit investors' bias more heavily. Therefore, the discovery has a larger negative impact on naive investors' objective welfare.

5 Conclusion

We have analyzed a simple model of anomaly discovery. Our model contrasts risk- with mispricing-based anomalies, and analyzes the behavior of post-discovery anomaly returns. Our model shows that the discovery of an anomaly reduces its magnitude, regardless of whether the anomaly is due to risk or mispricing. Therefore, the post-discovery decay of an anomaly return does not necessarily imply that it was caused by mispricing. More generally, even our model's new predictions on asset prices are similar across the two cases. These results highlight the difficulty of distinguishing risk- and mispricing-based anomalies by examining asset prices.

We argue that to distinguish risk-based and mispricing-based anomalies, one should examine investors' portfolios directly. For value premium, for example, one can examine the exposures of those investors who underweight value stocks and overweight growth stocks. Do they choose growth stocks to hedge their risk? For instance, are they more exposed to the financial distress that coincides with poor performances of value stocks? While this direct approach is demanding for the required dataset, in this "big data" era, it might not be a fantasy, either.

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Appendix A. Numerical procedure

We follow the procedure described below to solve the model:

1. Take initial guesses for the total wealth for investors and arbitrageurs at $t = 1$: W_1 and W_1^a for the eight states at date 1.
2. For each of the eight states, take W_1 and W_1^a as given, solve for the portfolios ($\theta_{i,1}$ for $i = 1, 2$, and $\theta_{i,1}^a$ for $i = 1, 2, e$) and prices $P_{1,1}$ and $P_{2,1}$.
3. Take the prices $P_{1,1}$ and $P_{2,1}$ for the eight states in step 2 as given, solve for the $t = 0$ portfolios ($\theta_{i,0}$ for $i = 1, 2$, and $\theta_{i,0}^a$ for $i = 1, 2, e$) and prices $P_{1,0}$ and $P_{2,0}$.
4. Based on the portfolios in step 3 ($\theta_{i,0}$ for $i = 1, 2$, and $\theta_{i,0}^a$ for $i = 1, 2, e$) and the prices in steps 2 and 3 ($P_{1,0}$, $P_{2,0}$, and $P_{1,1}$, $P_{2,1}$ for all eight states at $t = 1$), calculate the investors' and arbitrageurs' updated wealth, W_1 and W_1^a , in the eight cases at $t = 1$.
5. Repeat steps 2 to 4 until the wealth, portfolios, and prices converge, i.e., for each variable, the difference between two iterations is no greater than 0.00005.

Appendix B. Proofs

Proof of Proposition 1

Due to the logarithmic preference, the maximization problem (7) is equivalent to maximizing the log wealth growth for each period. Hence, investors' first-order conditions are given by

$$E_t \left[\frac{r_{i,t+1} - 1}{W_{t+1} + \rho P_{1,t+1}} \right] = 0,$$

for $i = 1, 2, t = 0, 1$. Similarly, the arbitrageurs' optimization problem (5) can also be decomposed into a period-by-period optimization problem, and the first-order conditions are given by

$$\begin{aligned} E_t \left[\frac{r_{1,t+1} - r_{2,t+1}}{W_{t+1}^a} \right] &= 0, \\ E_t \left[\frac{r_{a,t+1} - 1}{W_{t+1}^a} \right] &= 0. \end{aligned}$$

Combining the above first-order conditions with the market-clearing conditions, we can characterize the equilibrium in Proposition 1.

We now prove $P_{1,0} < P_{2,0}$ when W_0^a is sufficiently small, by contradiction. Suppose $P_{1,0} \geq P_{2,0}$. Note that in the case when W_0^a goes to 0, investors' optimal portfolio in equilibrium is to hold one unit of both assets. Suppose investor sell ϵ unit asset 1 and buy ϵ unit asset 2. Define their expected utility as

$$U(\epsilon) \equiv E_0[\text{Log}(k + (1 + \rho - \epsilon)D_{1,2} + (1 + \epsilon)D_{2,2})].$$

It is easy to see that $\frac{dU}{d\epsilon}|_{\epsilon=0} > 0$. That is, investors can strictly improve their portfolio by selling ϵ unit asset 1 and buying ϵ unit asset 2. This leads to a contradiction.

Proof of Propositions 2

The first-order condition to the maximization problem (13) is given by

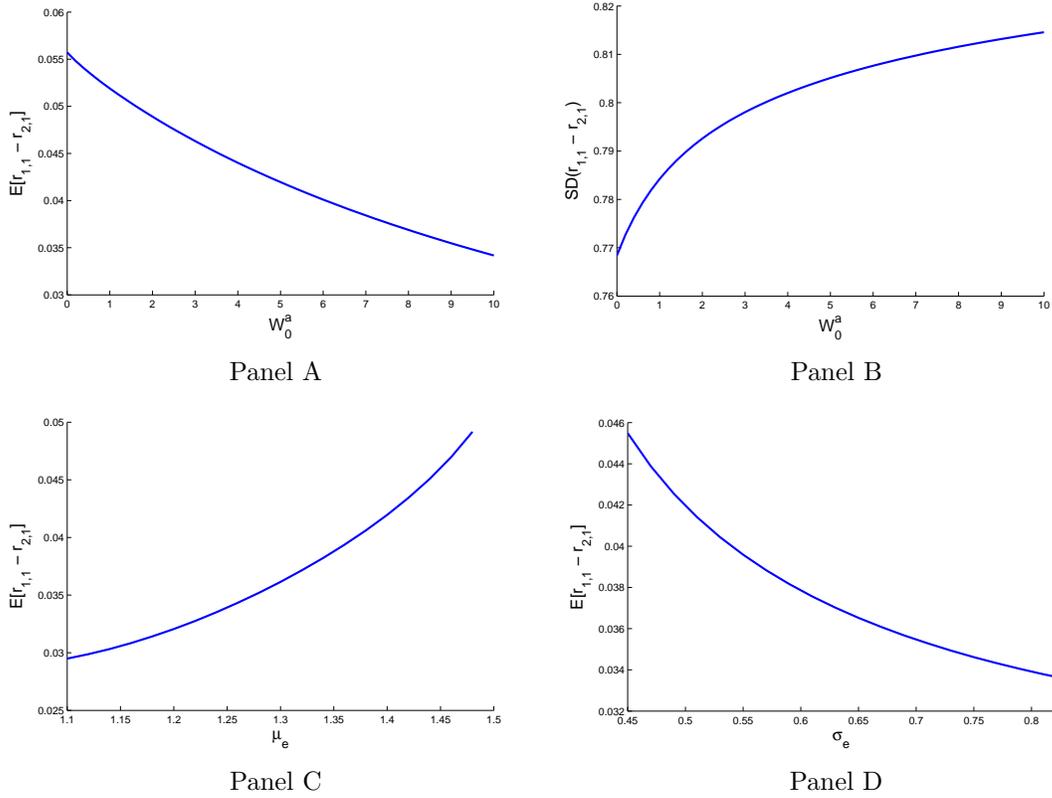
$$E_t^* \left[\frac{r_{i,t+1} - 1}{W_{t+1}} \right] = 0, \tag{16}$$

for $i = 1, 2, t = 0, 1$. The first-order conditions for arbitrageurs are still given by (10) and (11). An equilibrium can be characterized by these optimality and market-clearing conditions: i.e., equations (3), (8), (10), (11), and (14). Similar to the proof for Proposition 1, we have $P_{1,0} < P_{2,0}$ when W_0^a is sufficiently small,

Proof of Proposition 3

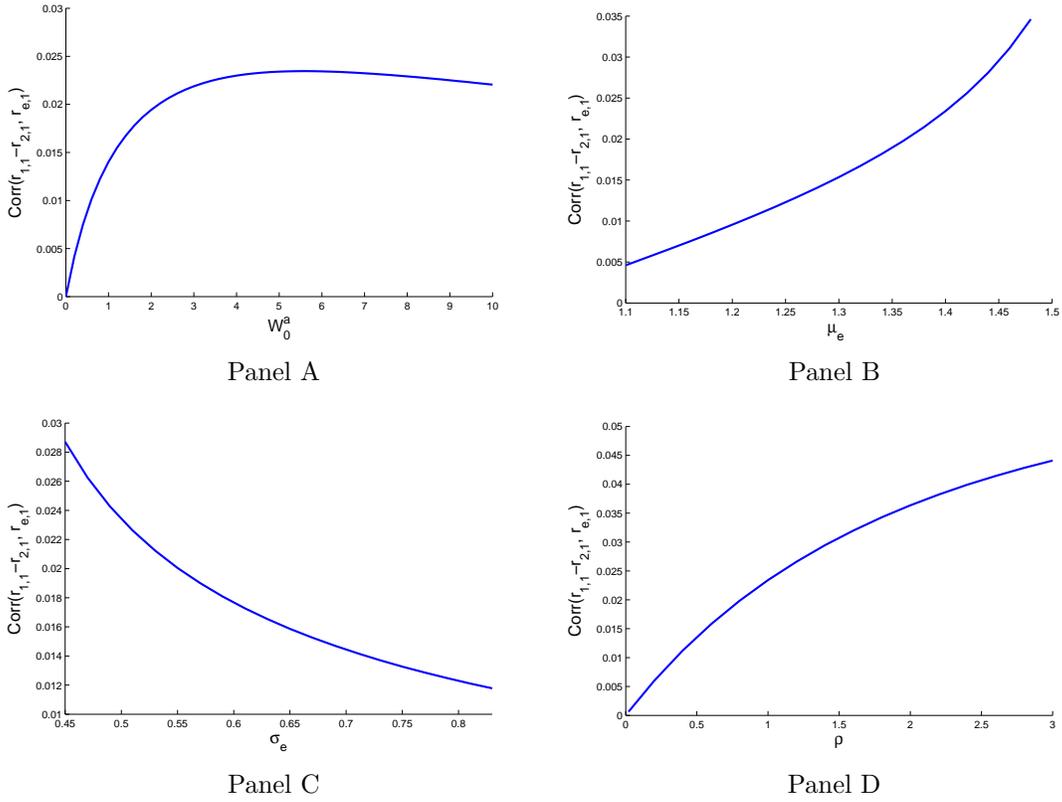
In both the risk-based and mispricing-based cases, investors have the option not to trade. The participation constraint implies that the investors' expected utility cannot be lower than that in the pre-discovery case. Moreover, investors' concave utility function and convex budget constraint implies that the investors' optimization problem has a unique solution. It is easy to see that in the case of $W_0^a > 0$, the portfolio characterized in Proposition 1 is strictly different from the non-participation portfolio. Hence, the discovery strictly increases investors' welfare.

Figure 1: Anomaly Return



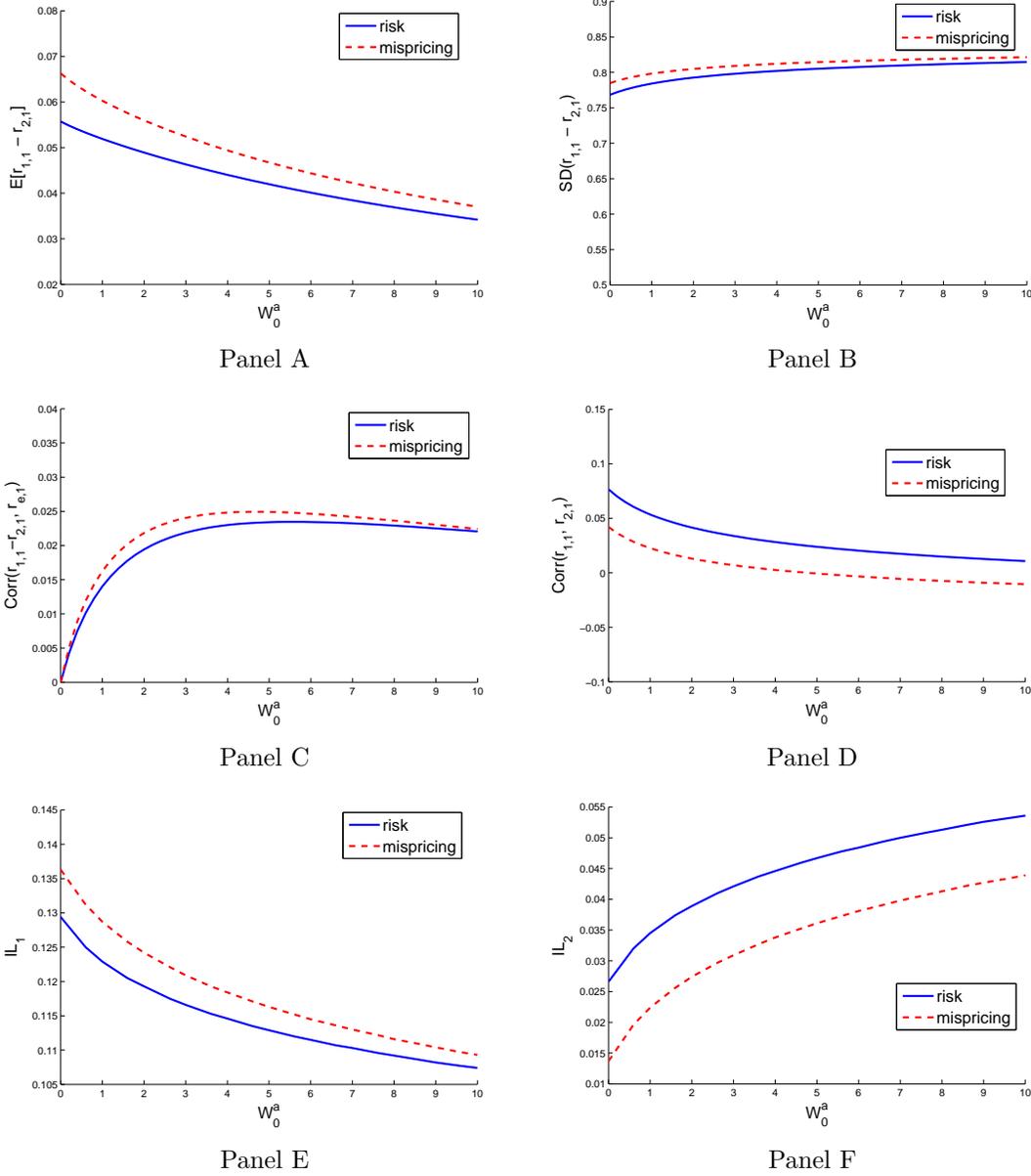
Panels A and B plot the expected anomaly return ($E[r_{1,1} - r_{2,1}]$) and its standard deviation ($SD(r_{1,1} - r_{2,1})$) on the arbitrageurs' initial wealth (W_0^a), respectively. Panels C and D plot the expected anomaly return on asset e 's expected return (μ_e) and volatility (σ_e), respectively. The parameter values are given by Table 1.

Figure 2: Correlation Among Anomaly Returns



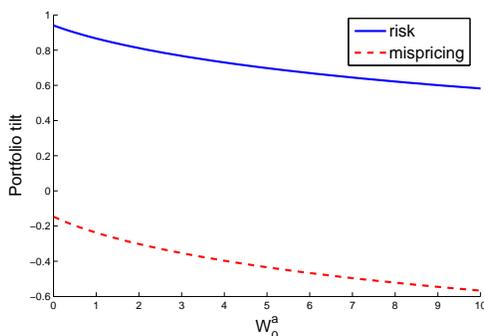
Panels A through D plot the correlation coefficient between the anomaly return and asset e 's return ($\text{Corr}(r_{1,1} - r_{2,1}, r_{e,1})$) on the arbitrageurs' initial wealth (W_0^a), asset e 's expected return (μ_e), volatility (σ_e), and investors' endowment (ρ), respectively. The parameter values are given by Table 1.

Figure 3: Comparison: Asset Prices



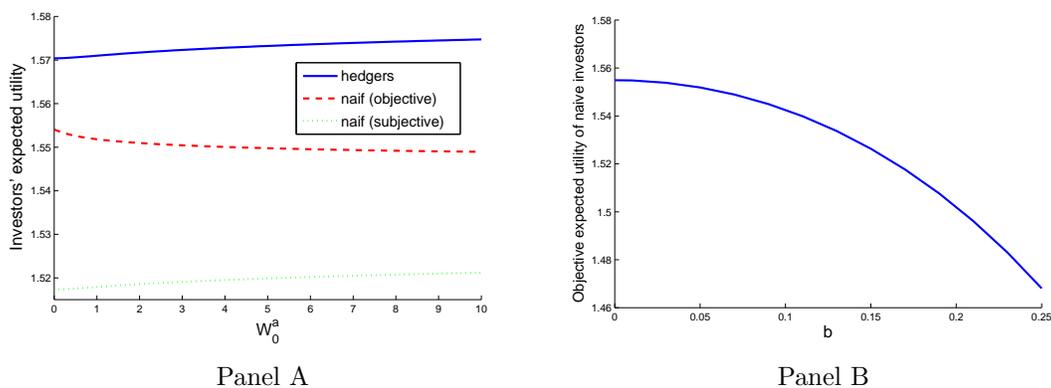
Panels A through F plot the expected anomaly return ($E[r_{1,1} - r_{2,1}]$), its standard deviation ($SD(r_{1,1} - r_{2,1})$), its correlation coefficient with asset e 's return ($Corr(r_{1,1} - r_{2,1}, r_{e,1})$), the correlation coefficient between assets 1 and 2 ($Corr(r_{1,1}, r_{2,1})$), and the illiquidity of assets 1 and 2 (IL_1 and IL_2), defined in (15), on the arbitrageurs' initial wealth (W_0^a), respectively. The solid line is for the case of risk-based anomaly, and the dashed line the mispricing-based anomaly. Parameter values: In the mispricing case, investors' belief bias is $b = 0.065$. Other parameter values are given by Table 1.

Figure 4: **Comparison: Portfolios**



This figure plots investors’ “portfolio tilt” on the arbitrageurs’ initial wealth (W_0^a). The portfolio tilt is investors’ exposure to asset 1 (including their non-tradable endowment in the risk-based case) minus their exposure to asset 2, denominated in dollars. The solid line is for the case of risk-based anomaly, and the dashed line the mispricing-based anomaly. Parameter values: In the mispricing case, investors’ belief bias is $b = 0.065$. Other parameter values are given by Table 1.

Figure 5: **Comparison: expected utility of hedgers and naive investors**



Panel A plots investors’ expected utility at $t = 0$ on the arbitrageurs’ initial wealth W_0^a . The solid line is for the case of risk-based anomaly. In the mispricing-based case, the dashed line is for investors’ subjective expected utility at $t = 0$, and the dotted line is for their utility evaluated under the objective belief. Panel B plots the naive investors’ expected utility at $t = 0$ (evaluated under the objective belief) on their belief bias b . Parameter values: In the mispricing case, investors’ belief bias is $b = 0.065$. Other parameter values are given by Table 1.