# Corporate Innovation and Returns*

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**Abstract**

Among U.S. public firms, technological innovation is concentrated in a small set of large players, with innovation "leaders" having considerably lower market betas than "laggards." To understand this fact, we build a model to study how competition in innovation affects rival firms' expected returns. In the model, a firm's expected return decreases in its innovation output and increases in the innovation output of its rival. We find strong support for these predictions using a comprehensive firm-level panel of information on patenting activity in the 1976–2006 period. Our findings suggest that the imperfect nature of competition has important implications for firms' expected returns.



*JEL Classification Codes*: G12, G14

*Keywords*: Innovation, Systematic risk, Cross-section of returns, Expected returns

# 1 Introduction

Technological progress is a major determinant of economic growth and has recently been used to explain several empirical patterns in asset prices.[1] In most technology fields, innovations are created by a small number of firms that compete to acquire exclusive rents protected by patents. The exclusivity of these rents generates cash flows interdependence that induces firms to invest strategically in innovation. Imperfect competition is therefore a fundamental feature of the innovation process.[2] Since firms active in technological innovation account for close to one half of the U.S. market's capitalization, understanding the impact of imperfect competition in innovation on firms' cost of capital is of the first order of importance for understanding cross sectional properties of stock returns.

In this paper, we study, both empirically and theoretically, how imperfect competition in innovation affects firms' expected returns. Using information on U.S. public firms' patenting activities in the 1976–2006 period, we first provide evidence suggesting that competition in technological innovation is indeed imperfect: on average, only 303 firms are actively innovating in about 100 fields of technology in any given year, and these innovating firms account for 40.8% of the total market capitalization. Moreover, the top tercile of innovating firms—measured by the share of awarded patents—represent 25.9% of the market capitalization. These findings are consistent with those in Kogan, Papanikolaou, Seru, and Stoffman (2012) who argue that "a few large firms are very important for the aggregate rate of innovation in the economy" (p. 17).

Next, we find that the portfolio of innovating firms with high shares of patents in any given field of technology—"leaders"—has a lower market beta than the portfolio of firms with low shares of patents—"laggards"—implying a difference in expected returns between the two portfolios of about half the size of the market risk premium. We further find that innovation leaders and laggards have different loadings (betas) with respect to commonly used size and value risk factors, as well as with respect to a technology risk factor that we construct from our data.

To understand how imperfect competition in innovation affects firms' exposure to systematic risk, we develop a model in which firms decide when to make an irreversible investment to acquire

[1] See, for example, Pastor and Veronesi (2009); Gârleanu, Panageas, and Yu (2011); Gârleanu, Kogan, and Panageas (2011); and Hsu (2009). We discuss the literature later in the introduction.

[2] The economics of innovation focuses on imperfect competition that is of the "winner-takes-all" type; see Tirole (1988), Aghion and Howitt (1998), Hall and Rosenberg (2010), or Acemoglu (2009).

exclusive rents protected by patents in a field of technology. The value of the rents is subject to market-wide systematic risk, and each firm's innovation activity is subject to a firm-specific technological risk. Technological risk implies that, upon investment in innovation, a firm does not immediately make a discovery and, hence, its investment does not preclude investments and discoveries by rivals. The model predicts that, relative to their rivals, firms with lower (higher) innovation output have higher (lower) exposure to systematic risk. This obtains because the winner-takes-all nature of innovation implies that decisions by more innovative firms to invest erode the value of the innovation options of the rival firms making them riskier. Further, the model predicts that an increase in a firm's innovation output is associated with a decrease in its own beta and an increase in its rival's beta. Since this mechanism is independent of the specific sources of systematic risk in the economy, our model generates a cross-sectional relationship between firms' relative innovation output and factor loadings that can be reliably tested.

We test the model's predictions using a comprehensive firm-level panel of information on patenting activity in all fields of technology in the U.S. over the 1976–2006 period. We find that a firm's exposure to systematic risk decreases with the firm's relative innovation output, measured as the share of patents in any given technology field. This relationship holds using three alternative measures of systematic risk at both annual and monthly data frequencies, and obtains regardless of whether we use cross-sectional or within-firm data variation. Further, we show that changes in firms' positions in innovation relative to their rivals are associated with changes in their rivals' betas. Importantly, we find that the increase in a firm's beta is larger the larger is the increase in the relative innovation output of its rivals. This suggests that competing firms' betas are interdependent through strategic investing, as predicted by our theory.

Our results are robust to modifications in the construction of the relative innovation output measure, such as the time window over which it is measured, the definition of technology fields of innovation, or the use of citation-weighted patent counts. Also, different ways of aggregating firms' innovativeness across multiple technology fields have little effect on our findings. Finally, our results are robust to a variety of empirical specifications and control variables.

The empirical literature that analyzes the market structure of innovation is limited and does not focus on firms' expected returns.[3] Hou and Robinson (2006) show that portfolios of firms

[3]Austin (1993) uses an event-study methodology to estimate the effect of a patent award on rival firms' market values relative to the effect on its recipient. Schroth and Szalay (2010) estimate the winning probabilities of incumbents and entrants in pharmaceutical innovation races as a function of the incumbents' and entrants'

in highly concentrated industries, when measured by firms' product market shares, earn lower returns, and who conjecture that this might be because firms in such industries engage less in innovation. To the best of our knowledge, we are the first to provide firm-level empirical evidence that the imperfect nature of competition in innovation has an impact on firms' expected returns.

Our empirical analysis is based on the literature in economics that uses patent data to study firm performance[4] and belongs to a growing literature in finance on innovation and equity returns. Hsu (2009) finds that aggregate patent and R&D shocks predict market returns and premia in the U.S. as well as in other G7 countries. Building on this result, Hsu and Huang (2010) construct a technology risk factor and find that it helps to explain the variation of Fama and French (1993) portfolio returns. Hirshleifer, Hsu, and Li (2011) show that a firm's innovation efficiency, defined as patents or citations scaled by R&D expenditure, predicts future returns, and they suggest an explanation based on limited investors' attention. We add to these studies by showing that a firm's position in innovation relative to its rival firms is a key determinant of its systematic risk and consequently returns in the cross-section.

Our theory builds on the industrial organization literature on patent races[5] and the finance literature on real options and investment-based asset pricing.[6] Specifically, we follow Weeds (2002) to model the strategic nature of R&D investments. Since we study firm-level decisions with a goal of deriving cross sectional predictions, we follow Berk, Green, and Naik (1999, 2004) and work within a partial equilibrium framework. Garlappi (2004) derives the risk premia dynamics of two firms engaged in a multi-stage R&D game and numerically documents that risk premia increase when a firm lags behind; we are able to derive closed-form solutions for the dynamics of firms' beta. Carlson, Dockner, Fisher, and Giammarino (2011) analyze the risk of firms that compete in quantities in a product market with options to expand and contract production. They show that a competitor's option to adjust capacity acts as a natural hedge for a

financial wealth. Lerner (1997) examines innovation in the disk drive industry and shows that strategic interactions between firms explain the decision to innovate.

[4]This literature documents that a firm's innovation output measured by patents and patent citations is associated with higher stock market valuation (e.g., Pakes (1985, 1986); Griliches (1990); Bloom and Van Reenen (2002); or Hall, Jaffe, and Trajtenberg (2005)).

[5]A partial list of early work includes Loury (1979); Lee and Wilde (1980); Reinganum (1981b,c); Fudenberg, Gilbert, Stiglitz, and Tirole (1983); Fudenberg and Tirole (1985); Harris and Vickers (1985, 1987); and Grossman and Shapiro (1987).

[6]For example, Smets (1993); Grenadier (1996, 1999, 2002); Huisman and Kort (2003, 2004); Thijssen, Huisman, and Kort (2002); Huisman (2001); Boyer, Lasserre, Mariotti, and Moreaux (2004); Lambrecht (2000); Aguerrevere (2003, 2009); Meng (2008); and Pawlina and Kort (2006). See Grenadier (2000) for a survey of the strategic real option literature. Relevant works in the large investment-based asset pricing literature are the early paper by Cochrane (1996) and the more recent contribution of Liu, Whited, and Zhang (2009).

firm's risk. Their model predicts that before (after) capacity expansion/contraction the leader's risk is higher (lower) than that of the laggard. Since we model competition in innovation to be winner-takes-all, the predictions of our model about firms' risk do not depend on investment timing, demand elasticities, or adjustment costs.

A recent theoretical literature studies the asset pricing effects of technological innovation in a general equilibrium setting. Pastor and Veronesi (2009) show how technology adoption can explain the rise of stock price bubbles. Gârleanu, Panageas, and Yu (2011) examine the link from infrequent technological shocks embodied in new capital vintages to excess return predictability and stylized cross-sectional return patterns, complementing existing endowment models of Campbell and Cochrane (1999) or Bansal and Yaron (2004). Using an overlapping generation model, Gârleanu, Kogan, and Panageas (2011) argue that innovation introduces an unhedgeable "displacement" risk due to lack of intergenerational risk-sharing. Greenwood and Jovanovic (1999) and Hobijn and Jovanovic (2001) link the stock market drop in the 1970s and its rebound in the 1980s to the information technology revolution. These papers do not study asset pricing implications of the cash flow interdependence and strategic investing that are inherent to innovation. We show theoretically that imperfect market structure of the innovation process affects expected returns, and we document empirically that such considerations are statistically significant and economically meaningful for firms that account for 30% to 50% of the total U.S. market capitalization over the last three decades.

The rest of the paper proceeds as follows. In the next section, we describe our data, sample, and variables, and present the portfolio returns results. In Section 3, we develop a model of competition in innovation and derive testable predictions about firms' betas. Section 4 presents tests of the model's predictions. Variable definitions are in Appendix A, and tables with robustness results are in Appendix B. The separate Internet Appendix contains details of the model and proofs.

## 2 Innovation and portfolio returns

In this section, we use portfolio analysis to establish a link between corporate innovation activity and returns. The goal is to provide initial evidence on whether imperfect competition in innovation is associated with firms' cost of capital. We choose data and construct variables so that we can determine which firms compete in innovation and measure their relative positions in such competition.

## 2.1 Data

We rely on data from four sources: (i) the NBER Patent Data Project (January 2011); (ii) the Worldwide Patent Statistical Database (PATSTAT, April 2008), compiled by the European Patent Office; (iii) the CRSP/Compustat Merged Database; and (iv) the CRSP Daily and Monthly Stock Files.[7]

The NBER Patent Data Project provides data about all utility patents[8] awarded by the U.S. Patent and Trademark Office (USPTO) over the period 1976–2006. Among other variables, the NBER project contains, for each patent, a unique patent number and patent assignee names matched to firms in Compustat (a patent number-GVKEY link). Hall, Jaffe, and Trajtenberg (2001) originally matched patent assignees by name to firms in Compustat. Since then, the matching has been updated using multiple manual and computer-generated matches (see Bessen (2009) for details). The PATSTAT database contains information from patent documents submitted to and issued by the USPTO, including the exact day when an application for each patent was filed and the day when each patent was awarded. We merge the NBER and PATSTAT databases to create a day-by-day time series of patent filing and award events for each firm.

Patents are classified according to the standards of the International Patent Classification (IPC) system,[9] which cuts though traditional industry classifications such as Standard Industry Classification (SIC) codes or Fama and French industry definitions. We use the second level of the IPC classification as our main proxy for the technology field of innovation, and we refer

[7]We use delisting returns from CRSP. To address delisting bias documented by Shumway (1997), we follow the procedure in CRSP (2001): for delisting returns that are missing, we set them to −30%. For partial delisting returns (delisting payment date prior to or at the delisting date), we compound them with −30%. These adjustments, however, have no effect on our results.

[8]According to the U.S. Patent Law (35 U.S.C. §101), utility is a necessary requirement for patentability and is used to prevent the patenting of inoperative devices. In our analysis, we do not use plant patents, i.e., patents for new varieties of plants.

[9]The IPC classification system was created under the Strasbourg Agreement (1971) and is updated on a regular basis by a committee of experts, consisting of representatives of the contracting states of that agreement. It is used as a search tool for the retrieval of patent documents by intellectual property offices, and it serves as a basis for investigating the state of the art in a given area of technology by patent examiners. A patent examiner assigns a classification to a patent (or a patent application) at the most detailed level of the IPC hierarchy that is applicable to its content. See IPC (2009) for details (`http://www.wipo.int/export/sites/www/classifications/ipc/en/guide/guide_ipc_2009.pdf`).

to it as "class."[10] The ability to use patent data is key to our analysis because it allows us to observe in which classes a firm is innovating. We use this information to determine which firms compete in innovation and measure a firm's innovation output relative to its rivals. This cannot be achieved using R&D expenses data, since they are reported as an aggregate dollar amount without distinguishing between research and development stages, and without specifying the innovations towards which R&D expenses are directed.[11]

### 2.2 Sample of innovating firms

We construct our sample of innovating firms by using a variable that tracks the classes in which a firm is innovating at any given point in time. Specifically, we denote a firm to be "innovating" in class $k$ at month $t$ if the firm has been awarded at least one patent in class $k$ during the last $\tau$ months (including $t$) and has filed one or more patent applications in class $k$ in at least $\tau_a$ of the last $\tau$ months. Specifically, we create a dummy variable

$$D_{ikt}^{\text{innovating}} = \begin{cases} 1 & \text{if } \sum_{s=t-\tau+1}^{t} D_{iks}^{\text{application}} \geq \tau_a \text{ and } \sum_{s=t-\tau+1}^{t} D_{iks}^{\text{award}} \geq 1 \\ 0 & \text{otherwise} \end{cases}, \tag{1}$$

where $D_{iks}^{\text{application}}$ ($D_{iks}^{\text{award}}$) is equal to 1 if firm $i$ files one or more patent applications (is awarded one or more patents) in class $k$ at month $s$, and is equal to 0 otherwise.

In definition (1), we use application dates because they represent the earliest time in which one can observe a firm innovating. This is important as, on average in our data, patents are awarded two to three years after patent applications are filed and, upon filing, applications have to contain a detailed description of completed innovations. Hence, firms' investment in innovation typically occurs a long time before they are awarded a patent. Since there are about 3 million patents issued by the USPTO over the period covered by the NBER patent data, filing a single patent application is typically not a significant event. Therefore, to classify a firm

[10]The hierarchical structure of IPC is made up of a section, class, subclass, main group, and subgroup. There are eight sections in the first level of IPC, about 400 classes in the second level, and about 650 subclasses in the third level. The IPC class describes, "as precisely as is possible in a small number of words, the main characteristic of a portion of the whole body of knowledge covered by the IPC." See IPC (2009, p. 13) for details. We use IPC sections or subclasses for robustness.

[11]In addition, R&D is a noisy measure of inputs for the innovation process as it may not include indirect costs. Further, because firms covered by Compustat do not have to disclose R&D, only about one-third of firms do so. As a result, R&D data are unreliable for the purpose of our study.

as innovating, we require the firm to be applying for patents repeatedly. We achieve this by imposing threshold $\tau_a$ in (1) which ensures that rarely innovating firms do not enter our sample.

To form our sample, we begin with all firms in the CRSP/Compustat Merged Database in the 1976–2006 period, when the NBER patent data are available. We then form a subset of firms that satisfy definition (1) in at least one class-month in this period. Using this subset of firms, we create a firm-class-month dataset that contains, for each firm, all class-month pairs in which the firm is innovating. Following these steps, we create two samples of innovating firms: The *3-Year Sample*, where in definition (1) $\tau = 36$ , and the *1-Year Sample*, where $\tau = 12$. In both samples, we set threshold $\tau_a = 0.2 \times \tau$.

Table 1 Panel A reports summary statistics of characteristics of innovating firms present in the *3-Year Sample* and compares them to those of all non-innovating firms. Relative to the median non-innovating firm, the median innovating firm is considerably bigger, is more profitable, has lower book-to-market equity and leverage ratios, and spends more on R&D.

## 2.3 A firm's share of patents

We identify firms that compete for the same monopoly rents in a technology field to be those that actively pursue patents in the same class at the same time, i.e., firms with $D_{ikt}^{\text{innovating}} = 1$ for a given class-month pair.

We use a firm's share of awarded patents to measure its innovation output relative to competitors. This is analogous to using sales market shares to identify leaders and laggards in product markets. Specifically, for each innovating firm $i$ in class $k$, we compute the relative amount of patenting output $Y_{ikt}$ that firm $i$ has achieved in class $k$ over a moving window of $\tau$-months

$$Y_{ikt} = \frac{\sum_{s=t-\tau+1}^{t} P_{iks}}{\sum_{\{j:D_{jkt}^{\text{innovating}}=1\}} \sum_{s=t-\tau+1}^{t} P_{jks}}, \tag{2}$$

where $P_{iks}$ is the number of patents awarded to firm $i$ in class $k$ at month $s$. The share of patents (2) indicates how close, relative to its competitors, a firm is to have a monopoly right to rents stemming from patents in a given field of technology.

Table 1 Panel B reports summary statistics of characteristics of three subsamples of innovating firms obtained by dividing them according to their share of patents into a sample of "leaders" (the top tercile), "middle," and "laggards" (the bottom tercile). Firms closer to a leadership position in innovation are significantly larger and more profitable. Consistent with Bloom and Van Reenen (2002), our evidence suggests that higher innovation output is associated with higher performance and market valuations. Interestingly, the book-to-market equity ratio does not vary across subsamples, and the R&D-to-sales ratio is the lowest for the leaders. The latter is consistent with Phillips and Zhdanov (2012), who show that large firms may optimally decide to acquire smaller innovative firms and conduct less R&D themselves, and with Bena and Li (2012), who find that firms with large patent portfolios and low R&D expenses acquire R&D intensive firms with slow growth in patenting output.

### 2.4 A firm's share of patents and portfolio returns

In Table 2 Panel A, we compare the key characteristics of the portfolio of innovating firms, constructed using firms in the *3-Year Sample*, to those of the portfolio of non-innovating firms. We form portfolios in June and rebalance annually. While on average the number of innovating firms represents only 6.3% (303 firms) of the entire cross section, their share of total market capitalization is 40.8%. The excess return and volatility of the portfolio of innovating firms is similar to that of non-innovating firms. The returns of both portfolios have CAPM alphas close to zero and insignificant with unconditional beta estimates close to one.

Next, we divide the portfolio of innovating firms into tercile portfolios according to each firm's share of patents. Specifically, the portfolio of leaders (laggards) consists of firms that belong to the top (bottom) tercile of firms in the *3-Year Sample* by patent share defined in (2) as of June of each year. The relative share of market capitalization of leaders is considerably higher than that of laggards. On average, leaders represent 25.9% of the market capitalization, compared to only 4.1% for laggards. These findings suggest that, since the majority of innovation output is provided by a small number of firms with large market capitalization, innovation takes place in an imperfect competitive setting.

The monthly excess return and volatility of the laggards portfolio is higher than that of the leaders portfolio, although the differences are small and not statistically significant. Interestingly, the portfolio of leaders exhibits a considerably lower market beta (0.881) compared to that of the laggards portfolio (1.387), and the difference is statistically significant. This difference implies that the laggards' expected return is bigger by about half of the market risk premium and suggests that the relative position of a firm in the competitive innovation process has implications for the exposure of its returns to systematic risk.

In Table 2 Panel B, we show that the pattern of market betas from the market model also obtains in the three-factor Fama and French (1993) and the four-factor Carhart (1997) model. In addition, we find that both the leaders and laggards portfolio returns have negative loadings on the value factor. Interestingly, the returns of the laggards portfolio have, in absolute value, bigger loading on the value factor compared to the leaders portfolio. This explains why, despite having significantly higher market beta, the laggards do not earn higher returns. Finally, we note that the leaders (laggards) portfolio has negative (positive) loading on the size factor.

To see whether the above results are indeed due to firms being active and having different relative positions in innovation, in Table B-1 in Appendix B, we report analogous factor loading estimates obtained using portfolios of "matched leaders," "matched middle," and "matched laggards." Specifically, for each firm in the leaders, middle, and laggards portfolio, respectively, we determine in which NYSE size and book-to-market decile the firm belongs. We then use the value-weighted excess return on the portfolio of all firms in the intersection of this NYSE size and book-to-market decile, instead of the excess return on the firm, to compute the matched leaders, matched middle, and matched laggards portfolio returns. We show that, while the matched laggards portfolio has higher market beta compared to the matched leaders portfolio, the difference between the betas of the two matched portfolios (0.062) is about ten times smaller compared to the difference between laggards and leaders portfolio betas (0.506). Furthermore, in contrast to results presented in Table 2 Panel B, matched leaders have, in absolute value, bigger loading on the value factor compared to matched laggards.

In summary, the above findings suggest that, for firms active in technological innovation, relative positions in innovation, measured by their shares of patents, are associated with firms' exposure to systematic risk.

## 3 A model of competition in innovation

In this section, we develop a simple model that captures some of the key features of competition in technological innovation: the exclusivity of rents protected by patents, the technological risk of R&D investments, and the presence of a small number of firms acting strategically.

### 3.1 The setup

Two all-equity financed firms, $i = 1, 2$, have an opportunity to make an irreversible risky investment in innovation. The first firm to make a discovery, which is a random event, obtains monopoly rents in a technology field (i.e., the "winner takes all"). The market value of the rents $x(t)$, i.e., the market value of future cash flows from commercialization of innovations in the technology field, evolves stochastically over time according to the geometric Brownian motion process

$$dx(t) = \mu x(t)dt + \sigma x(t)dW(t), \tag{3}$$

where $dW(t)$ is the increment of a standard Brownian motion, $\mu$ is the constant drift, and $\sigma$ is the constant volatility. The firms take $x(t)$ as exogenous and are thus subject to both systematic (market-wide) and idiosyncratic (technological) risk. Each firm exercises its one-time option to invest by deploying a fixed amount of capital $K > 0$. Once the capital has been deployed, the discovery happens randomly according to a Poisson distribution with constant hazard rate $h_i > 0$.[12]

A firm's strategy is a mapping from the history of the innovation game to the possible actions: invest or wait. The history at time $t$ contains the path of the process $x$ and the actions of both firms up to and including time $t$. We restrict our attention to Markov strategies, in which actions depend only on the current level of $x$. A firm's Markov strategy is an investment rule characterized by a threshold $x^*$ for the process $x$ such that the firm invests when $x$ crosses $x^*$ from below for the first time. Solving the game involves finding a *Markov perfect equilibrium*—a set of strategies such that, at each time, each firm's strategy is value maximizing conditional on the rival's strategy.

In the Internet Appendix, we formally derive firm values and investment thresholds, and characterize the Markov perfect equilibria of the game. Here, we focus on the implications of the game for firms' systematic risk and describe the link between the model's predictions and our empirical analysis.

[12] For simplicity, we assume that $h_i$ is not a function of $K_i$. Propositions IA.1 and IA.2 and Lemma IA.3 in the Internet Appendix show that firms' values are monotonically increasing in their own hazard rate $h_i$ and decreasing in the investment $K_i$. Hence, all else being equal, a higher $h_i$ is equivalent to a lower $K_i$. Without loss of generality, we keep $K$ constant between firms.

### 3.2 Firm values and betas

There are three possible outcomes of the innovation game: (i) firm 1 invests first and firm 2 invests at a later date; (ii) firm 2 invests first and firm 1 invests at a later date; or (iii) both firms invest simultaneously. The solution of the game involves two steps. First, we derive the firms' values and investment thresholds associated with these three possibilities, taking the timing of investment as given (Propositions IA.1, IA.2, and IA.3 in the Internet Appendix). To value firms, we assume the existence of a pricing kernel in which only the systematic risk $x$ is priced directly, i.e., agents are risk neutral with respect to each firm's technological risk. As we will see, because the decision to invest in innovation is made contingent on the realization of $x$, this imparts a risk premium for technological risk, despite its purely idiosyncratic nature, as in Berk, Green, and Naik (2004).

Second, we endogeneize the investment timing decisions and derive equilibrium strategies by comparing the value of investing first, the value of waiting and investing second, and the value of investing simultaneously (Proposition IA.4). Depending on the difference between firms' hazard rates $h_1$ and $h_2$, two types of equilibria emerge: (i) *sequential/preemptive* equilibria, in which the firm with a higher hazard rate invests first, and (ii) *simultaneous* equilibria, in which both firms invest at the same time.

In Proposition IA.5, we derive closed-form expressions for the firms' values $V_i(x)$ in each of the above equilibria. The instantaneous expected return of firm $i$ can then be expressed as

$$E\left[r_i\right] = r + \beta_i \lambda, \;\; \beta_i = \frac{dV_i(x)}{dx}\frac{x}{V_i(x)}, \tag{4}$$

where $\beta_i$ is the systematic risk of firm $i$, $r$ is the riskless rate, and $\lambda$ is the risk premium of $x$.[13]

Using the expressions for the firms' values in the different equilibria, we obtain the following characterization of the firms' betas.[14]

**Proposition 1.** *Let $h_1 > h_2$ and let $\beta_i$ $i = 1, 2$ be firm $i$'s beta, defined in equation (4).*

[13] Equation (4) is obtained from the valuation equation under the risk-neutral measure, $E[dV_i(x)] = rV_i(x)dt$. Applying Itô's Lemma with $x$ following the process (IA.1), we get $1/2V''x^2\sigma^2 + V'(r-\delta)x = rV$. The instantaneous expected return is $E[r_i] = E[dV_i]/V_i$, where the expectation is taken under the physical measure. Using the valuation equation to express $1/2V''x^2\sigma^2$, we obtain $E[r_i] = r + \beta_i\lambda$, where $\beta_i$ is given by (4) and $\lambda = \mu - (r - \delta)$.

[14] Section IA.5 in the Internet Appendix discusses how one can generalize the result of Proposition 1 to the case of sequential equilibria with $N$ firms (see Proposition IA.6).

1. *In a preemptive/sequential equilibrium, firms' betas,* $\beta_1^{\mathrm{PS}}$ *and* $\beta_2^{\mathrm{PS}}$*, are:*

$$(\,\beta_1^{\mathrm{PS}}\,,\,\beta_2^{\mathrm{PS}}\,) = \begin{cases} (\,\phi_0\,,\,\phi_0\,) & \text{if } x < x_1^P \\ (\,1-\omega(x)(\phi_1-1)\,,\,\phi_1\,) & \text{if } x_1^P < x < x_2^G \\ (\,1\,,\,1) & \text{if } x > x_2^G \end{cases}, \tag{5}$$

*where* $\phi_0 > 1$ *is given in (IA.10),* $\phi_1 > \phi_0$ *is given in (IA.5),* $\omega(x) = \frac{b(x)}{a(x)-b(x)} > 0$*, with* $a(x) > 0$ *and* $b(x) > 0$ *defined in equation (IA.14) of Proposition IA.5. Threshold* $x_2^G$ *is defined in (IA.4), threshold* $x_1^P = \min\{x_1^D, x_2^P\}$ *in a preemptive equilibrium, and* $x_1^P = x_1^D$ *in a sequential equilibrium, with* $x_1^D$ *and* $x_2^P$ *given by (IA.36) and (IA.12), respectively.*

2. *In a simultaneous equilibrium, firms' betas,* $\beta_1^{\mathrm{S}}$ *and* $\beta_2^{\mathrm{S}}$*, are:*

$$(\,\beta_1^{\mathrm{S}}\,,\,\beta_2^{\mathrm{S}}\,) = \begin{cases} (\,\phi_0\,,\,\phi_0\,) & \text{if } x < x_1^C \\ (\,1\,,\,1\,) & \text{if } x > x_1^C \end{cases}, \tag{6}$$

*where* $x_1^C$ *is defined in (IA.11).*

**Proof:** The proof is a direct application of the definition of beta in equation (4) to the equilibrium values derived in Proposition IA.5. ∎

The proposition formalizes the interdependence between betas of competing firms. Figure 1 plots betas for the case of preemptive/sequential equilibria in which firm 1 is the Leader and firm 2 is the Laggard ($h_1 > h_2$). At the Leader's investment threshold, $x_1^P$, the Leader's beta *decreases* from $\phi_0 > 1$ to $1-\omega(x)(\phi_1-1) < 1$, while the Laggards's beta *increases* from $\phi_0$ to $\phi_1 > \phi_0$. At the Laggard's investment threshold, $x_2^G$, the opposite occurs: the Leader's beta *increases* and the Laggard's beta *decreases*, and both take value 1, which is the beta of an unlevered claim on $x$. The figure highlights the two effects of innovation options on the dynamics of firms' betas. The first is an "own-beta" effect: when a firm invests, its beta drops. This happens because the option to innovate is a levered asset whose riskiness is reduced when it is exercised. The second effect is a "rival-beta" effect: when a firm invests, the beta of its rival increases.

To understand the rival-beta effect, consider the impact of the Laggard's investment on the Leader's beta. From equation (IA.14) in Proposition IA.5, the Leader's value derives from a long position in assets in place, worth $a(x)$, and a *short* position in an innovation option, worth

$b(x)$. Since assets in place have beta 1 and the innovation option has beta larger than 1, the short position in the option lowers the Leader's beta below 1. When the Laggard invests at the threshold $x_2^G$, the short position disappears from the Leader's value, causing its beta to jump to 1. For the Laggard, the rival-beta effect occurs when the Leader invests at the threshold $x_1^P$. The investment reduces the probability of the Laggard to obtain $x$, making its innovation option less valuable and more "levered." As a result, the Laggard's beta jumps from $\phi_0$ to $\phi_1 > \phi_0$.

To link the model to empirically observable quantities, we define the *relative innovation output* $Y_i$ of firm $i$ as the probability of receiving $x$.[15] For every given investment time $\tau_2^G$ of firm 2, the relative innovation output of firm 1 is

$$Y_1 = \int_0^{\tau_2^G} e^{-h_1 t} h_1 dt + \int_{\tau_2^G}^{\infty} e^{-(h_1+h_2)t} h_1 dt = 1 - e^{-h_1 \tau_2^G} + e^{-(h_1+h_2)\tau_2^G} \frac{h_1}{h_1 + h_2}, \tag{7}$$

and the relative innovation output of firm 2 is $Y_2 = 1 - Y_1$. The first integral in (7) represents the probability that firm 1 receives $x$ before firm 2 invests, while the second represents the probability that firm 1 receives $x$ when both firms have invested. For every $\tau_2^G$, if $h_1 > h_2$ then $Y_1 > Y_2$,[16] and hence the Leader is a firm with higher innovation output. Because, by Proposition 1, the Leader has lower beta than the Laggard, we obtain the following relationship between firms' relative innovation output and betas:

**Corollary 1.** *Among firms competing in innovation for the same monopoly rents, a firm with higher relative innovation output has (weakly) lower beta than a firm with lower relative innovation output.*

The dynamics of betas described in Proposition 1 allow us to characterize the relationship between changes in firms' relative innovation output and betas. Due to the own-beta effect, when a firm invests and its rival does not, the firm's relative innovation output increases and its beta decreases. Due to the rival-beta effect, when a firm invests and its rival does not, the relative innovation output of the rival decreases while its beta increases. Furthermore, a larger

[15] Since a discovery occurs according to the Poisson distribution with hazard rate $h_i$, its arrival time $\tau$ has a negative exponential distribution, $Pr(\tau < t) = 1 - e^{-h_i t}$, which represents the probability of innovating before time $t$.

[16] To see this, for $\tau_2^G = 0$, $Y_1 = \frac{h_1}{h_1+h_2}$ and $Y_2 = \frac{h_2}{h_1+h_2}$ and hence $Y_1 > Y_2$. Moreover, $\partial Y_1/\partial \tau_2^G = h_1 e^{-(h_1+h_2)\tau_2^G}\left(e^{h_2\tau_2^G} - 1\right) > 0$ and $\partial Y_2/\partial \tau_2^G = h_1 e^{-h_1\tau_2^G}\left(e^{-h_2\tau_2^G} - 1\right) < 0$. Hence, $Y_1 > Y_2$ for all $\tau_2^G \geq 0$.

increase in a firm's relative innovation output ($\Delta Y_i$) is associated with a larger increase in its rival's beta ($\Delta\beta_j$). We summarize these predictions in the following corollary:[17]

**Corollary 2.** *Among firms competing in innovation for the same monopoly rents, an increase (decrease) in a firm's relative innovation output is associated with a decrease (increase) in its beta. The increase in the beta of a firm losing relative innovation output is larger the larger is the gain in its rival's relative innovation output.*

### 3.3 Theoretical versus empirical betas

Before describing our empirical methodology, we explain how our model betas are connected to commonly used empirical factor loadings. In our single-factor model, equation (4) shows that expected returns are determined by exposure of firms' values to the market value of rents from innovation $x$. Note that $\beta_i$ in equation (4) is the "cash flow beta" of firm $i$, and, unless $x$ coincides with the return on the market portfolio, it is *not* the empirically observed market beta.

We implicitly assume that $x$ is a determinant of the stochastic discount factor (SDF) of the economy. Let us denote the SDF by $\tilde{m}$ and the return of a generic asset $i$ by $R_i$. By definition, $\tilde{m}$ is an SDF if and only if $E[\tilde{m}R_i] = 1$ for all assets. This implies that asset $i$'s excess expected return over the risk-free rate $R_f = 1/E[\tilde{m}]$ is

$$E[R_i] - R_f = -\frac{cov(\tilde{m}, R_i)}{E[\tilde{m}]}. \tag{8}$$

In any economy where $x$ is correlated with $\tilde{m}$, we have

$$E[R_i] - R_f = -\beta_i^x corr(\tilde{m}, x)\, \sigma(x)\, SR. \tag{9}$$

In equation (9), $\beta_i^x = cov(R_i, x)/var(x)$, $corr(\tilde{m}, x)$ is the correlation between $\tilde{m}$ and $x$, $\sigma(x)$ is the volatility of $x$, and $SR = \sigma(\tilde{m})/E(\tilde{m})$ is the Hansen-Jagannathan bound, i.e., the highest Sharpe ratio attainable in the economy.[18]

[17] The proof is a direct extension of the results derived in Proposition 1 for the relative innovation output of competing firms and is provided in Section IA.6 of the Internet Appendix.

[18] To see this, consider the projection of $\tilde{m}$ onto $x$, $\tilde{m} = a + b\,x + \epsilon$, with $\epsilon$ orthogonal to the space of returns. From (8) we obtain

$$E[R_i] - R_f = -\frac{cov(a + b\,x + \epsilon, R_i)}{\sigma^2(x)} \frac{\sigma^2(x)}{\sigma(\tilde{m})} \frac{\sigma(\tilde{m})}{E[\tilde{m}]} = -\beta_i^x \frac{b\,\sigma^2(x)}{\sigma(\tilde{m})\sigma(x)} \sigma(x)\, SR = -\beta_i^x\, corr(\tilde{m}, x)\sigma(x)\, SR. \tag{10}$$

Suppose our one-factor model is a model of an economy in which the Capital Asset Pricing Model (CAPM) holds. In this case, $\tilde{m} = a + b\,x$, where $a > 0$, $b < 0$,[19] and $x$ is the return on the Market Portfolio. Condition (9) holds with $corr(\tilde{m}, x) = -1$ and $SR = \frac{E[x]-R_f}{\sigma(x)}$, i.e.,

$$E[R_i] - R_f = \beta_i^x\,\sigma(x)\,SR. \tag{11}$$

If $x$ is perfectly captured by the observable value-weighted return on a broad equity portfolio $R_M$ (i.e., $x = R_M$), knowing $R_M$ is equivalent to knowing $\tilde{m}$, and the "empirical" beta $\beta_i^{mkt} = cov(R_M, R_i)/var(R_M)$ coincides with the "theoretical" beta $\beta_i^x = cov(R_i, x)/var(x)$. If, instead, $R_M$ is a noisy proxy for $x$ (i.e., $x = R_M + \epsilon$, with $\epsilon$ orthogonal to $R_M$), $\tilde{m} = a + bR_M + b\epsilon$, and from (8) we have

$$E[R_i] - R_f = -\beta_i^{mkt} corr(\tilde{m}, R_M)\sigma(R_M)SR. \tag{12}$$

Comparing (11) and (12), we conclude that

$$\beta_i^x = -\beta_i^{mkt} corr(\tilde{m}, R_M)\,\frac{\sigma(R_M)}{\sigma(x)}, \tag{13}$$

which shows that any cross sectional pattern in the unobserved theoretical $\beta_i^x$ is preserved in the empirical $\beta_i^{mkt}$, as long as $corr(\tilde{m}, R_M) < 0$. In our CAPM example, $b < 0$ and therefore $corr(\tilde{m}, R_M) < 0$, as long as $R_M$ is positively correlated with $x$.

In general, without knowing the structural form of $\tilde{m}$, one cannot predict whether empirical factor loadings and our model betas are positively or negatively related. However, from equation (13) we note that any cross sectional pattern in the *absolute values* of the model betas is preserved in the empirical factor loadings. The argument also applies to an economy where the highest Sharpe ratio asset is a multi-factor portfolio instead of the Market Portfolio. In this case, the cross sectional relationship between the model betas and individual empirical factor loadings is analogous to that derived in equation (13).

[19] Specifically, $a = 1/R_f$, $b = -\frac{E[x]-R_f}{R_f\sigma^2(x)} < 0$. See, e.g., Back (2010), p. 111.

# 4 Empirical analysis

In this section, we test the model's predictions summarized in Corollaries 1 and 2 above. We introduce our beta measures and regression specifications, present the results, and discuss robustness of the main findings.

## 4.1 Beta measures

We consider three proxies for the market value of rents from innovation $x$. The first proxy is the value-weighted return on the market portfolio. To estimate corresponding betas, we use time-series regressions $R_{it} = \alpha_i + \beta_i R_{Mt} + \zeta_{it}$, where $R_{it}$ is firm $i$'s excess stock return and $R_{Mt}$ is the excess return on the CRSP value-weighted index. We calculate excess returns using returns on the one-month U.S. Treasury Bills obtained from Kenneth French's website. We use daily returns from CRSP Daily Stock File to estimate the equity beta for each firm-month (firm-year) using separate short-window regressions (see Lewellen and Nagel (2006)).[20] We refer to these estimates as *Market Betas*. Next, we correct monthly betas for the potential intervalling-effect bias, introduced by Dimson (1979), using the methodology of Cohen, Hawawini, Maier, Schwartz, and Whitcomb (1983).[21] We refer to this refinement of *Market Beta* estimates as *Sum Betas*.

A possible concern with using the market portfolio as a proxy for the market value of innovation rents is that not all firms benefit equally from the adoption of new technologies. For example, growth firms that successfully innovate may benefit more than value firms that do not innovate (see Hobijn and Jovanovic (2001); Gârleanu, Panageas, and Yu (2011); and Gârleanu, Kogan, and Panageas (2011)). Therefore, it is not clear whether a broad market index captures the different degree to which firms benefit from adoption of new technologies. To address this concern, we use the excess return on the value-weighted portfolio of innovating firms present in the *1-Year Sample* as our second proxy for the market value of innovation rents. We again use daily returns to estimate the equity beta for each firm-month (firm-year) using separate short-window regressions, and we refer to these estimates as *Technology Betas*.

[20] To reduce the impact of outliers, we use betas that are estimated using at least 19 (100) daily observations for monthly (annual) data frequency, and we also winsorize betas at the 1% level.

[21] We follow Proposition 3 in Cohen, Hawawini, Maier, Schwartz, and Whitcomb (1983). This involves summing up the contemporaneous with the one-day-lead and the one-day-lag equity betas, which we estimate one at a time, using analogous short-window regressions.

Finally, we use the spread portfolio of growth and value firms as our third proxy for the market value of innovation rents. Specifically, the proxy we use is minus the returns to the value factor, $-R_{HML}$, obtained from Kenneth French's website (see, Kogan and Papanikolaou (2011, 2012)). Using this proxy, we estimate the *Growth-Value Beta* for each firm-month (firm-year) as the coefficient on $-R_{HML}$ in a one-factor model short-window regression analogous to the one introduced above for the *Market Betas*.

## 4.2 Relative innovation output and betas

To remove across-class differences in the mean and dispersion of the firms' share of patents $Y_{ikt}$ defined in (2), in the following analysis, we use the relative share of patents defined as

$$y_{ikt} = \frac{Y_{ikt} - \overline{Y}_{kt}}{\text{std}(Y_{kt})}, \tag{14}$$

where $\overline{Y}_{kt}$ and $\text{std}(Y_{kt})$ are, respectively, the average and standard deviation of $Y_{ikt}$ computed across all innovating firms in class $k$.[22] The relative share of patents $y_{ikt}$ measures a firm's position in each class in units that correspond to standard deviations from the class average, and hence allows comparison of firms' patent shares in different classes and over time. It also allows the aggregation of patent shares across classes for those firms in our sample that are innovating in multiple classes at the same time.

Our first set of tests follows from Corollary 1. According to this corollary, firms with higher relative innovation output have lower beta than firms with lower relative innovation output. We perform two tests of this prediction. In the first test, we estimate a firm-level regression

$$\beta_{it} = \lambda_0 + \lambda_1 \overline{y}_{it} + \lambda_2 X_{it} + FE_{(\cdot)} + \epsilon_{it}, \tag{15}$$

where $\beta_{it}$ is the estimated beta of firm $i$, and the main explanatory variable is the average relative share of patents $\overline{y}_{it}$ computed across all classes in which firm $i$ is innovating.[23] A negative and significant estimate of $\lambda_1$ would be consistent with the prediction of Corollary 1.

[22] Specifically, $\overline{Y}_{kt} = \frac{1}{N_{kt}} \sum_{i=1}^{N_{kt}} Y_{ikt}$ and $\text{std}(Y_{kt}) = \left( \frac{1}{N_{kt}-1} \sum_{i=1}^{N_{kt}} (Y_{ikt} - \overline{Y}_{kt})^2 \right)^{1/2}$, where $N_{kt}$ is the number of innovating firms in class $k$.

[23] Let $K_{it} = \{k : D_{ikt}^{\text{innovating}} = 1\}$ be the set of classes $k$ in which firm $i$ is innovating at time $t$, according to definition (1). Then, $\overline{y}_{it} = \frac{1}{|K_{it}|} \sum_{k=1}^{|K_{it}|} y_{ikt}$, where $y_{ikt}$ is defined in (14) and $|K_{it}|$ denotes the cardinality of the set $K_{it}$.

In regression (15), $X_{it}$ stands for firm-level control variables, and $FE_{(\cdot)}$ denotes either SIC three-digit industry interacted with period fixed effects, $FE_{SIC\times t}$, or firm fixed effects, $FE_i$. Our choice of firm-level control variables is motivated by structural models of investment and asset pricing that link conditional betas to firm characteristics. The firm-level control variables capture: size (market capitalization), growth options (book-to-market equity ratio and profitability), operating leverage, tangibility, financial leverage (book leverage and cash holding), and investment policy (capital and R&D expenditure). Detailed definitions of the control variables are provided in Appendix A.

In the second test of Corollary 1, we consider an alternative way to account for the fact that firms in our sample are innovating in multiple classes at the same time[24] and estimate a firm-class-level regression

$$\beta_{it} = \lambda_0 + \lambda_1 y_{ikt} + \lambda_2 N_{kt} + \lambda_3 X_{it} + FE_k + FE_t + \epsilon_{ikt}, \tag{16}$$

where, for each firm-time period, the dataset we use has as many observations as the number of classes in which a firm is innovating according to definition (1). $N_{kt}$ is the number of innovating firms in class $k$, and $FE_k$ and $FE_t$ denote class and time period fixed effects, respectively. Corollary 1 implies a negative and significant estimate of $\lambda_1$ in regression (16).

We estimate both regressions above using the *3-Year Sample* at annual data frequency and the *1-Year Sample* at monthly frequency. The advantage of using monthly data frequency is that it allows betas to be conditional on innovation characteristics $\overline{y}_{it}$, $y_{ikt}$, and $N_{kt}$, which change month by month. The benefits of using annual data frequency are that yearly beta estimates are less affected by market microstructure issues and that the frequency of all variables in the regression matches that of the firm-level control variables. At annual data frequency, the sample used in regression (15) consists of all firm-years in which a firm is innovating in at least one month in a given year. In this case, we compute the average relative share of patents $\overline{y}_{it}$ as follows. First, for each firm-month, we take the average of $y_{ikt}$ across all classes in which firm $i$ is innovating at $t$. Second, we convert this monthly average to annual frequency by further averaging across the monthly observations for each firm-year. The sample for regression (16)

[24]For example, in the *1-Year Sample* at monthly frequency, 44.5% of innovating firms are active (according to definition (1)) in two or more classes in at least one month during our sample period.

at annual frequency is defined analogously, and variables $y_{ikt}$ and $N_{kt}$ are respective annual averages of monthly observations in which firm $i$ is innovating.

Table 3 Panel A presents estimates of the firm-level regression (15) using the *3-Year Sample* at annual data frequency. The dependent variables are the *Market Beta*, *Technology Beta*, and *Growth-Value Beta*. For each beta measure, we present four specifications that differ in the firm-level control variables and in the set of fixed effects that we include in the regression. Across all twelve specifications, negative and significant coefficient estimates of $\lambda_1$ show that higher average relative share of patents is associated with lower betas, as predicted by Corollary 1.

To assess the magnitude of the effect, consider, for example, the second specification in Table 3 Panel A. An increase of the average relative share of patents by one, which corresponds to an increase of the share of patents by one standard deviation, decreases an innovating firm's *Market Beta* by 0.092. This is about one-sixth of the in-sample standard deviation of *Market Beta*, or, assuming a market risk premium of 5%, it represents a decrease in an innovating firm's expected return by 0.46%. The magnitude of the effect for the other two beta measures is similar.

Table 3 Panel B presents estimates of regression (15) using the *1-Year Sample* at monthly data frequency with the same independent variables and specifications as in Panel A. In eleven out of twelve specifications, we estimate the negative and significant effect of the average relative share of patents on innovating firms' betas. These results show that modifying the definition of the sample of innovating firms and changing the data frequency has little impact on the estimates.

Table 4 reports results from estimating the firm-class-level regression (16). For each of the three beta measures, we present two specifications. In the first specification, we control for market capitalization. The second specification contains the full set of firm-level control variables introduced in Table 3. For brevity, we only report estimates of coefficients on the relative share of patents and the number of innovating firms. With this alternative data structure, we continue to find a negative and significant effect of a firm's relative share of patents on the firm's beta.

In most specifications, we estimate a positive and significant coefficient on the number of innovating firms, suggesting that classes with more innovating firms command higher betas on average. This is consistent with the fact that the share of patents variable is skewed (as shown in Table 1 Panel B), hence classes with a large number of innovating firms contain disproportionately more firms with small values of the relative share of patents. Table 4 shows that our results are unaffected when we explicitly allow for possible differences in a firm's share of patents across multiple classes.

In summary, our empirical findings support the predictions of Corollary 1: firms with higher relative innovation output have lower exposure to systematic risk.

### 4.3 Interdependence between betas of rival firms

Our second set of tests follows from Corollary 2 and concerns the relationship between changes in firms' relative innovation output and betas. We estimate a firm-level regression

$$\Delta\beta_{it} = \lambda_0 + \lambda_1 D_{it}^{\text{top}} + \lambda_2 D_{it}^{\text{bottom}} + \lambda_3 X_{it} + FE_{\text{SIC}} + FE_t + \epsilon_{it}, \tag{17}$$

where $\Delta\beta_{it}$ is the annual change in a firm's beta, and $D_{it}^{\text{top}}$ ($D_{it}^{\text{bottom}}$) is equal to one if a firm experiences an annual change in the relative share of patents in the top (bottom) tercile of the distribution of changes, and zero otherwise. We estimate the distribution of changes separately for each class-year. $X_{it}$ stands for firm-level control variables as in regression (15), and $FE_{\text{SIC}}$ and $FE_t$ denote SIC three-digit industry and time period fixed effects, respectively. The results reported in Table 5 Panel A show that a large decrease in a firm's relative innovation output is associated with an increase in its beta. We find no effect of an increase in a firm's relative innovation output on beta.

To capture more closely the theoretical concepts of innovation leaders and laggards, we then estimate a firm-level regression

$$\begin{aligned}\Delta\beta_{it} = \lambda_0 + \lambda_1 D_{it}^{\text{Middle-to-Leader}} + \lambda_2 D_{it}^{\text{Middle-to-Middle}} + \lambda_3 D_{it}^{\text{Middle-to-Laggard}} + \\ + \lambda_4 X_{i,t} + FE_{\text{SIC}} + FE_t + \epsilon_{it},\end{aligned} \tag{18}$$

where $D_{it}^{\text{Middle-to-Leader}}$ is equal to one if a firm is in the middle tercile of the distribution of the relative share of patents at year $t-1$ and transitions to the top tercile of the distribution of the relative share of patents in the same class by year $t$, and zero otherwise. The indicator variables $D_{it}^{\text{Middle-to-Middle}}$ and $D_{it}^{\text{Middle-to-Laggard}}$ are defined analogously. The three indicator variables capture large changes in a firm's position in innovation relative to its rivals. We show that becoming laggard is associated with an increase in beta and becoming leader is associated with a decrease in beta, although the relationship is statistically significant at conventional levels only for *Growth-Value Beta*. These results are consistent with the prediction of the first part of Corollary 2: changes in firms' relative innovation output lead to changes in their betas.

The results in Table 5 Panel A show that, upon a drop in the relative innovation output, a firm's beta increases. Because a drop in the relative innovation output of a firm implies an increase in the innovation output of its rivals, this finding suggests the presence of the rival-beta effect. According to Corollary 2, due to the rival-beta effect, the increase in the beta of a firm losing relative innovation output is larger the larger is the gain in its rivals' relative innovation output. To test this prediction, we estimate a class-level regression

$$\Delta\beta_{kt}^{-} = \lambda_0 + \lambda_1 \Delta y_{kt}^{+} + \lambda_2 \Delta y_{kt}^{-} + \lambda_3 N_{kt} + FE_t + \epsilon_{kt}, \tag{19}$$

where $\Delta\beta_{kt}^{-}$ denotes the average of the annual beta changes of firms that, in class $k$ and year $t$, experienced a decline from year $t-1$ in their relative innovation output bigger than 10% of one standard deviation of the share of patents (i.e., $y_{ik,t} - y_{ik,t-1} < -0.1$). We refer to such firms as "retreating," and we use them to proxy for firms that, in our model, do not invest in innovation at $t$.

The main explanatory variable in regression (19), $\Delta y_{kt}^{+}$, is the average of the annual changes in the relative share of patents of firms that, in class $k$ and year $t$, increased their relative innovation output from year $t-1$ by at least 10% of one standard deviation of the share of patents (i.e., $y_{ik,t} - y_{ik,t-1} > 0.1$). We refer to such firms as "advancing," and we use them to proxy for firms that, in our model, invest in innovation at $t$. According to Corollary 2, a positive and significant estimate of $\lambda_1$ indicates the presence of the rival-beta effect.

We estimate regression (19) using a class-year panel that we create using firms in the *3-Year Sample*. The dependent variable is the average of the annual changes in *Market Beta*, *Technology Beta*, and *Growth-Value Beta* of retreating firms. We control for the average of the annual changes in the relative share of patents of retreating firms, $\Delta y_{kt}^{-}$, the number of innovating firms in class $k$, and year fixed effects. Table 5 Panel B presents estimates of regression (19). The coefficient estimates on the change in the relative innovation output of advancing firms are always positive and significant, confirming the prediction of Corollary 2 that the dynamics of firms' betas is affected by the innovation output of their rivals.

In summary, the evidence we present in Table 5 supports the key mechanism of our theory that, among firms competing in innovation, a firm's beta increases when its rival invests. Together with the results on beta levels reported in Tables 3 and 4, our empirical analysis therefore shows that strategic considerations are an important determinant of firms' exposure to systematic risk.

### 4.4 Robustness

We conduct robustness checks along three dimensions. First, we explore whether changes in our sample construction procedures impact the results. Second, we consider several refinements of our key explanatory variable. Third, we provide results using alternative estimates of firms' betas.

In our first set of robustness checks, we examine whether our results are robust to changing the definition of the fields of technology—classes—in which firms compete in innovation. In our main analysis, a class is proxied by the second level of the IPC classification. This proxy is prominent in our empirical analysis as both the definition of the sample of innovating firms (1), as well as the definition of the firm's share of patents (2), rely on measurement of the class.

To assess the extent to which our results are affected by changing this proxy, we consider a definition of class based on more aggregated "sections of IPC" (the first level of the IPC hierarchy), as well as a definition of class based on more disaggregated "subclasses of IPC" (the third level of the IPC hierarchy). In both cases, we re-estimate regression (15) using the *3-Year Sample* at annual data frequency (*1-Year Sample* at monthly data frequency) with the same dependent variables and specifications as in Table 3 Panel A (Panel B). The results of this robustness check, reported in Table 6, are analogous to the main results of Table 3.

Next, we re-estimate regression (19) using the above alternative definitions of class. The results are reported in Table B-2 Panel A in Appendix B. When we use sections of IPC, the coefficients on the $\Delta$ *Relative share of patents of advancing firms* are all positive but not statistically significant, suggesting that eight fields of technology are too broad to capture competitive interactions among innovating firms. Importantly, when we use subclasses of IPC, we obtain stronger results compared to those in Table 5 Panel B. Overall, broadening or narrowing the definition of the fields of technology in which firms compete in innovation has little material effect on our findings.

In our second set of robustness checks, we account for heterogeneity in the importance of the fields of technology and patent quality. Specifically, when estimating the firm-level regression (15), we use the weighted average of a firm's relative share of patents to measure the firm's overall relative position in innovation. We use two weighting schemes. In the first, the weights are, for each class, the number of innovating firms in this class divided by the sum of the number of innovating firms in all classes in which the firm is innovating. In the second, the weights are defined based on the market capitalization of firms that are innovating in each class.[25]

In both cases, we re-estimate regression (15) using the *3-Year Sample* at annual data frequency (*1-Year Sample* at monthly data frequency) with the same dependent variables and specifications as in Table 3 Panel A (Panel B). The results, reported in Table 7, suggest that our findings do not depend on how we aggregate the relative share of patents for firms that are innovating in multiple classes. These tests are not applicable to regression (19) because it is at the class level.

To account for heterogeneity in the quality of patents, we compute the share of patents (2) using the citation-weighted number of patents (instead of the raw number of patents) following the methodology introduced by Hall, Jaffe, and Trajtenberg (2001, 2005). They argue that a patent that is more often cited by other patents is more important than a patent with few citations. With this new measure, we re-estimate regressions (15) and (19). The results, reported in Table B-3 and in Table B-2 Panel B in Appendix B are analogous to our main findings.

Our third set of robustness checks uses different measures of firms' exposure to systematic risk. First, to address concerns that our beta estimates at monthly data frequency are affected by market microstructure biases and/or noise, we re-estimate regressions (15) and (16) using the *Sum Betas* (the specifications are otherwise identical to those in Tables 3 and 4). Table B-4 in Appendix B shows that our results are unchanged. Second, to account for the fact that firms in our model are all-equity financed, we re-estimate regressions (15) and (19) using "unlevered betas" computed as in Bharath and Shumway (2008). Table B-5 and Table B-2 Panel C in Appendix B show that our results are again unaffected.

In summary, we conclude that our main findings obtain in all robustness checks we consider.

[25]Specifically, let $K_{it} = \{k : D_{ikt}^{\text{innovating}} = 1\}$ be the set of classes $k$ in which firm $i$ is innovating according to definition (1) at time $t$. Then, $\overline{y}_{it}^{MKT} = \sum_{k=1}^{|K_{it}|} w_{kt}^{MKT} y_{ikt}$, where $w_{kt}^{MKT} = M_{kt} / \sum_{k=1}^{|K_{it}|} M_{kt}$, $M_{kt}$ is the market capitalization of all innovating firms in class $k$ at $t$, and $|K_{it}|$ denotes the cardinality of the set $K_{it}$.

# 5 Conclusion

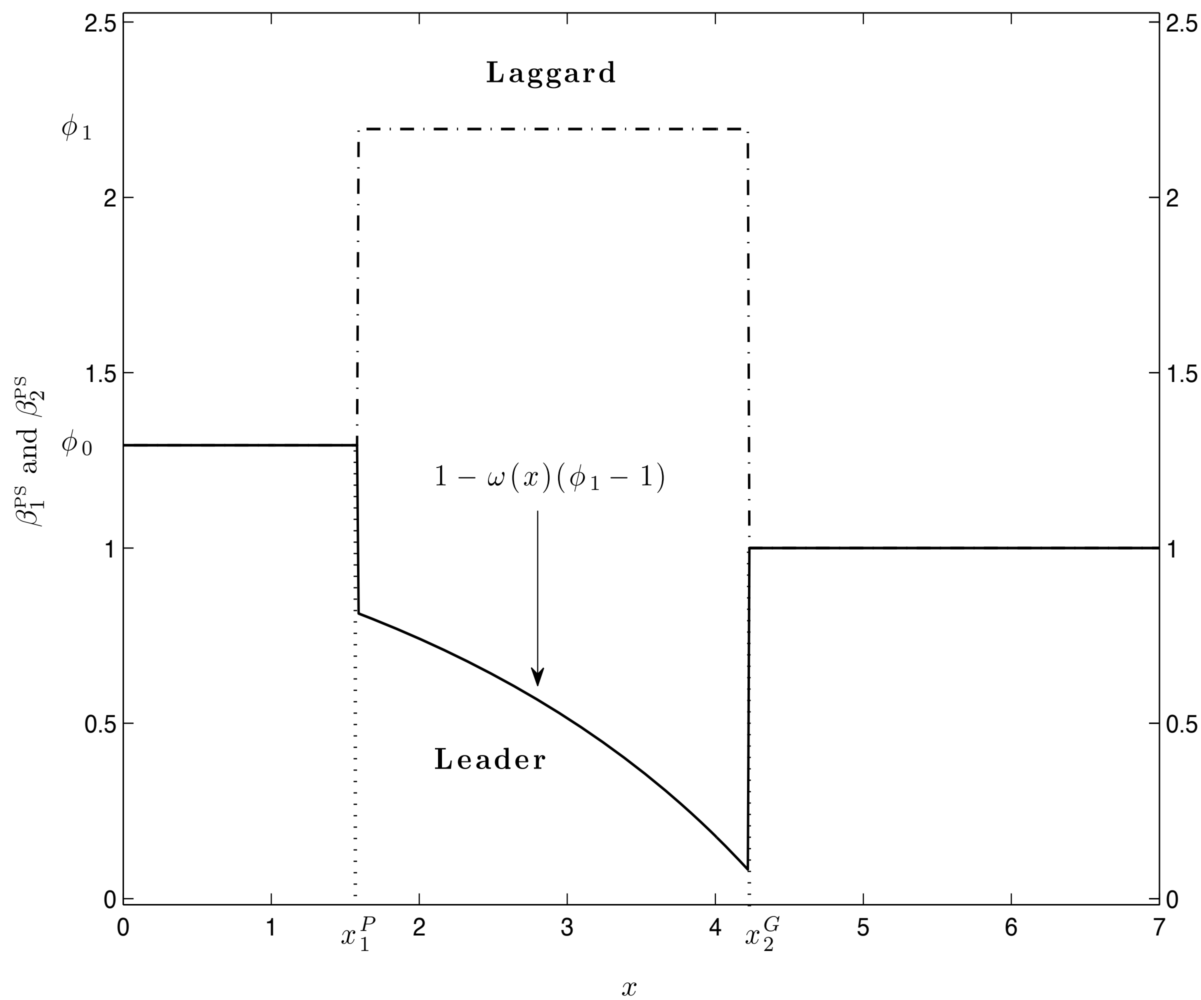
We study the relationship between corporate innovation activity and stock returns. We model an industry in which firms compete for exclusive technology field rents that are subject to market-wide risk. When firms' investments are irreversible, and discoveries are idiosyncratic events, the winner-takes-all nature of innovation implies that the leader's decision to invest erodes the value of the laggard's innovation option, making it riskier. The model predicts that a firm's beta is decreasing in its innovation output and increasing in the innovation output of its rivals.

We test the model's predictions using a comprehensive firm-level panel of information on patenting activity in all fields of technology in the 1976–2006 period and find that both predictions are supported in the data. We contribute to the literature by providing direct firm-level evidence of the impact of imperfect competition in innovation on rival firms' expected returns.

Our findings have both practical and theoretical implications. The pattern of within-industry heterogeneity of equity betas we discover challenges the commonly followed practice of using industry peer betas to estimate a firm's cost of capital. From a theoretical perspective, our results highlight the importance of studying the effects of imperfect competition in innovation on firms' risk dynamics in order to understand cross sectional properties of stock returns.

**Figure 1: Leader's and laggard's betas**

The figure plots the beta of the Leader (firm 1), $\beta_1^{\mathrm{PS}}$, and the Laggard (firm 2), $\beta_2^{\mathrm{PS}}$, in preemptive/sequential equilibria, derived in Proposition 1. $x_1^P$ denotes the entry threshold of the Leader and $x_2^G$ denotes the entry threshold of the Laggard. Parameter values: $\delta = 2\%$, $K = 1$, and $\sigma = 30\%$.



## Table 1: Summary statistics of innovating firms in 1976–2006

This table reports summary statistics of the characteristics of innovating firms in the *3-Year Sample* (see Section 2.2 for sample formation details). *Market capitalization* (item prcc_f×csho) and *Total assets* (item at) are measured in USD billions. Market leverage is total long-term debt plus debt in current liabilities (item dltt+dlc) scaled by the sum of total long-term debt, debt in current liabilities, and market value of common equity (item dltt+dlc+prcc_f×csho). We require all reported characteristics to be non-missing in order for a firm-year to be included in the table. *Share of patents* is computed as the average of the share of patents defined in Section 2.3 taken, first, across classes in which a firm is innovating in each month, and, second, across months in each year. *Relative share of patents* is the average relative share of patents, defined in Section 4.2. The remaining firm-level variables are defined in Appendix A. We winsorize all variables at the 1% level before computing the summary statistics.

*Panel A: Innovating and non-innovating firms*

This panel compares characteristics of innovating firms (9,412 firm-year observations; 1,170 firms) to those of all non-innovating firms covered by CRSP/Compustat (102,972 firm-year observations; 13,904 firms).

| | Innovating firms | | | | | Non-innovating firms | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Mean | S.D. | 10th Percentile | Median | 90th Percentile | Mean | S.D. | 10th Percentile | Median | 90th Percentile |
| Market capitalization | 6.30 | 16.25 | 0.11 | 1.14 | 14.24 | 0.50 | 1.81 | 0.01 | 0.06 | 0.91 |
| Total assets | 4.97 | 11.50 | 0.11 | 1.19 | 12.24 | 0.60 | 2.56 | 0.01 | 0.07 | 1.01 |
| Profitability | 0.13 | 0.13 | 0.03 | 0.15 | 0.25 | 0.08 | 0.18 | -0.09 | 0.11 | 0.24 |
| B/M | 0.58 | 0.42 | 0.17 | 0.48 | 1.14 | 0.76 | 0.62 | 0.18 | 0.59 | 1.58 |
| Tangibility | 0.29 | 0.16 | 0.10 | 0.27 | 0.51 | 0.29 | 0.22 | 0.05 | 0.23 | 0.63 |
| Cash | 0.15 | 0.18 | 0.01 | 0.08 | 0.41 | 0.15 | 0.19 | 0.01 | 0.07 | 0.43 |
| Book leverage | 0.19 | 0.14 | 0.00 | 0.19 | 0.37 | 0.23 | 0.19 | 0.00 | 0.20 | 0.49 |
| Market leverage | 0.19 | 0.18 | 0.00 | 0.15 | 0.44 | 0.25 | 0.24 | 0.00 | 0.19 | 0.62 |
| R&D | 0.09 | 0.19 | 0.01 | 0.04 | 0.19 | 0.05 | 0.15 | 0.00 | 0.00 | 0.12 |
| Capex | 0.07 | 0.07 | 0.02 | 0.05 | 0.13 | 0.11 | 0.22 | 0.01 | 0.04 | 0.22 |
| Operating leverage | 0.95 | 0.41 | 0.46 | 0.90 | 1.48 | 1.23 | 0.82 | 0.31 | 1.10 | 2.25 |
| Share of patents | 0.037 | 0.070 | 0.001 | 0.010 | 0.101 | | | | | |
| Relative share of patents | -0.313 | 0.490 | -0.773 | -0.443 | 0.363 | | | | | |

**Table 1 (cont.): Summary statistics of innovating firms in 1976–2006**

*Panel B: Innovating firms by the average share of patents*

This panel reports summary statistics of three sub-samples of innovating firms. Each year, we split the innovating firms present in the *3-Year Sample* into terciles based on the average share of patents. We then pool firms in the same tercile across years and compute the summary statistics.

| | Leaders | | | Middle | | | Laggards | | |
|---|---|---|---|---|---|---|---|---|---|
| | Mean | S.D. | Median | Mean | S.D. | Median | Mean | S.D. | Median |
| Market capitalization | 12.10 | 23.13 | 2.84 | 4.64 | 12.25 | 1.30 | 2.04 | 6.77 | 0.47 |
| Total assets | 9.98 | 16.97 | 3.31 | 3.44 | 6.95 | 1.42 | 1.39 | 3.96 | 0.38 |
| Profitability | 0.16 | 0.08 | 0.16 | 0.13 | 0.12 | 0.15 | 0.10 | 0.16 | 0.14 |
| B/M | 0.58 | 0.42 | 0.47 | 0.59 | 0.41 | 0.48 | 0.58 | 0.42 | 0.47 |
| Tangibility | 0.33 | 0.15 | 0.31 | 0.29 | 0.16 | 0.27 | 0.25 | 0.16 | 0.22 |
| Cash | 0.09 | 0.11 | 0.05 | 0.15 | 0.18 | 0.08 | 0.21 | 0.21 | 0.14 |
| Book leverage | 0.22 | 0.12 | 0.22 | 0.20 | 0.14 | 0.19 | 0.16 | 0.15 | 0.14 |
| Market leverage | 0.22 | 0.17 | 0.19 | 0.20 | 0.18 | 0.16 | 0.16 | 0.17 | 0.10 |
| R&D | 0.05 | 0.09 | 0.03 | 0.09 | 0.18 | 0.05 | 0.14 | 0.26 | 0.07 |
| Capex | 0.07 | 0.06 | 0.05 | 0.07 | 0.07 | 0.05 | 0.08 | 0.08 | 0.05 |
| Operating leverage | 0.96 | 0.39 | 0.92 | 0.95 | 0.40 | 0.91 | 0.93 | 0.43 | 0.88 |
| Share of patents | 0.095 | 0.096 | 0.058 | 0.012 | 0.008 | 0.009 | 0.003 | 0.002 | 0.002 |
| Relative share of patents | 0.038 | 0.606 | -0.004 | -0.391 | 0.330 | -0.420 | -0.594 | 0.171 | -0.584 |

## Table 2: Innovation output and portfolio returns

This table reports estimates from asset pricing models. The portfolio of innovating firms is constructed using firms in the *3-Year Sample*, while the portfolio of non-innovating firms consists of all firms covered by CRSP/Compustat that are not included in the portfolio of innovating firms. $I-N$ denotes the return spread between innovating and non-innovating firms portfolios. The Leaders/Middle/Laggards portfolios are constructed using firms in the *3-Year Sample* that belong to the top/middle/bottom tercile of firms by the average share of patents as of June of each year. Firms are kept in the same tercile portfolio for twelve months. $G-L$ denotes the return spread between Laggards and Leaders portfolios. All portfolios are formed in June and rebalanced annually. We use monthly return data from the 1976–2006 period. Excess returns are computed as the difference between respective portfolio returns and the one-month U.S. Treasury Bill rates obtained from Kenneth French's website. Standard errors for Excess return and Volatility, reported in parentheses, are computed by bootstrapping. Standard errors for empirical asset pricing models are computed using the Newey-West estimator allowing for 1 lag of serial correlation. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

*Panel A: Descriptive statistics*

This panel reports summary statistics on minimum, mean, and maximum of the number of firms in each portfolio, and minimum, mean, and maximum of the share of total market capitalization taken by the firms in each portfolio. The statistics are computed across 360 monthly observations in our sample period. Next, the panel reports the average and standard deviation of value-weighted excess return (monthly in percent) on each portfolio. Finally, the panel reports the estimated intercepts ($\alpha$) and slopes ($\beta_{MKT}$) from regressions of each portfolio's excess returns on the excess returns of the CRSP value-weighted index.

| | Number of firms | | | Share of market capitalization | | | Excess return | Volatility | Market model | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Min | Mean | Max | Min | Mean | Max | (% monthly) | (% monthly) | $\alpha$(%) | $\beta_{MKT}$ |
| *Portfolios of innovating and non-innovating firms* | | | | | | | | | | |
| Innovating firms ($I$) | 129 | 303 | 488 | 31.2% | 40.8% | 52.8% | 0.562** | 4.556 | -0.033 | 1.001*** |
| | | | | | | | (0.246) | | (0.017) | (0.017) |
| Non-innovating firms ($N$) | 3,350 | 4,539 | 6,379 | 47.2% | 59.2% | 68.8% | 0.585** | 4.478 | -0.010 | 1.001*** |
| | | | | | | | (0.238) | | (0.047) | (0.013) |
| $I-N$ | | | | | | | -0.023 | 2.009 | -0.023 | 0.004 |
| | | | | | | | (0.101) | | (0.109) | (0.029) |
| *Innovating firms sorted into tercile portfolios using the average share of patents* | | | | | | | | | | |
| Leaders ($L$) | 42 | 101 | 161 | 17.3% | 25.9% | 36.3% | 0.554** | 4.141 | 0.003 | 0.881*** |
| | | | | | | | (0.219) | | (0.077) | (0.019) |
| Middle | 44 | 103 | 165 | 4.5% | 10.7% | 20.6% | 0.595** | 5.595 | -0.107 | 1.157*** |
| | | | | | | | (0.298) | | (0.128) | (0.037) |
| Laggards ($G$) | 43 | 100 | 162 | 2.4% | 4.1% | 8.1% | 0.605* | 6.832 | -0.089 | 1.387*** |
| | | | | | | | (0.322) | | (0.179) | (0.056) |
| $G-L$ | | | | | | | 0.051 | 4.139 | -0.091 | 0.506*** |
| | | | | | | | (0.199) | | (0.194) | (0.059) |

**Table 2 (cont.): Innovation output and portfolio returns**

*Panel B: Factor loadings*

This panel reports, for each portfolio, the estimated intercepts ($\alpha$) and factor loadings obtained by regressing portfolio excess returns on factor returns. $MKT$ refers to the excess return on the value-weighted CRSP index, $SMB$ and $HML$ refer to Fama and French (1993) factors, and $UMD$ refers to the momentum factor (Carhart (1997)), all obtained from Kenneth French's website.

| | Fama-French three-factor model | | | | Cahart four-factor model | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $\alpha(\%)$ | $\beta_{MKT}$ | $\beta_{SMB}$ | $\beta_{HML}$ | $\alpha(\%)$ | $\beta_{MKT}$ | $\beta_{SMB}$ | $\beta_{HML}$ | $\beta_{UMD}$ |
| Leaders ($L$) | 0.100 | 0.906*** | -0.233*** | -0.044 | 0.162** | 0.901*** | -0.223*** | -0.056 | -0.064** |
| | (0.075) | (0.018) | (0.027) | (0.039) | (0.073) | (0.020) | (0.028) | (0.037) | (0.026) |
| Middle | 0.195* | 1.025*** | -0.110** | -0.434*** | 0.228** | 1.023*** | -0.105** | -0.440*** | -0.034 |
| | (0.109) | (0.031) | (0.051) | (0.054) | (0.112) | (0.030) | (0.053) | (0.053) | (0.044) |
| Laggards ($G$) | 0.124 | 1.157*** | 0.134* | -0.593*** | 0.216 | 1.149*** | 0.149** | -0.610*** | -0.096* |
| | (0.153) | (0.044) | (0.072) | (0.083) | (0.144) | (0.042) | (0.073) | (0.079) | (0.050) |
| $G-L$ | 0.024 | 0.251*** | 0.367*** | -0.549*** | 0.054 | 0.249*** | 0.372*** | -0.554*** | -0.031 |
| | (0.165) | (0.049) | (0.072) | (0.089) | (0.156) | (0.048) | (0.072) | (0.089) | (0.052) |

## Table 3: Beta and the average relative share of patents

This table reports estimates from firm-level OLS regressions of equity beta on firm characteristics (equation (15) in Section 4.2). The average relative share of patents is computed as the average of the relative share of patents (equation (14) in Section 4.2) across all classes in which the firm is innovating. *Market Beta*, *Technology Beta*, and *Growth-Value Beta* measure a firm's exposure to systematic risk and are defined in Section 3.3. Definitions of the firm-level control variables are provided in Appendix A. We winsorize all variables at the 1% level. Robust standard errors (clustered at the firm level) are reported in parentheses; *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

*Panel A: 3-Year Sample at annual frequency*

This panel is based on the *3-Year Sample* at annual data frequency. We include the 3-digit SIC industry interacted with year fixed effects or the firm fixed effects.

| | Market Beta | | | | Technology Beta | | | | Growth-Value Beta | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) |
| Relative share of patents | -0.130*** | -0.092*** | -0.042*** | -0.043*** | -0.102*** | -0.071*** | -0.034*** | -0.024** | -0.258*** | -0.157*** | -0.069*** | -0.062* |
| | (0.025) | (0.022) | (0.011) | (0.014) | (0.022) | (0.019) | (0.010) | (0.012) | (0.043) | (0.037) | (0.021) | (0.034) |
| Ln(Market capitalization) | 0.062*** | 0.075*** | 0.017*** | 0.005 | 0.073*** | 0.087*** | 0.020*** | 0.040*** | 0.115*** | 0.119*** | 0.031*** | -0.045** |
| | (0.008) | (0.009) | (0.005) | (0.009) | (0.007) | (0.008) | (0.005) | (0.008) | (0.014) | (0.016) | (0.009) | (0.020) |
| Profitability | | -0.181* | -0.166** | 0.424*** | | -0.147 | -0.129* | 0.347*** | | 0.151 | 0.039 | 1.633*** |
| | | (0.102) | (0.077) | (0.084) | | (0.091) | (0.072) | (0.075) | | (0.188) | (0.181) | (0.180) |
| Ln(B/M) | | -0.051*** | -0.065*** | -0.021 | | -0.028* | -0.054*** | 0.030** | | -0.134*** | -0.129*** | 0.059** |
| | | (0.018) | (0.013) | (0.013) | | (0.016) | (0.012) | (0.012) | | (0.032) | (0.027) | (0.029) |
| Tangibility | | -0.255** | -0.098 | -0.267*** | | -0.184* | -0.078 | -0.246*** | | -0.468** | -0.331** | 0.339* |
| | | (0.110) | (0.062) | (0.081) | | (0.101) | (0.055) | (0.074) | | (0.208) | (0.130) | (0.198) |
| Cash | | 0.493*** | 0.156*** | 0.190*** | | 0.485*** | 0.152*** | 0.200*** | | 1.147*** | 0.483*** | 0.403*** |
| | | (0.082) | (0.055) | (0.071) | | (0.074) | (0.048) | (0.063) | | (0.146) | (0.107) | (0.146) |
| Book leverage | | 0.175** | 0.037 | -0.216*** | | 0.150* | 0.025 | -0.167*** | | 0.187 | 0.048 | -0.142 |
| | | (0.085) | (0.050) | (0.061) | | (0.078) | (0.044) | (0.054) | | (0.153) | (0.097) | (0.140) |
| R&D | | 0.051 | -0.043 | 0.068 | | 0.048 | -0.028 | 0.000 | | 0.144 | -0.040 | 0.277* |
| | | (0.061) | (0.047) | (0.060) | | (0.056) | (0.044) | (0.052) | | (0.130) | (0.101) | (0.158) |
| Capex | | 0.752*** | 0.091 | 0.713*** | | 0.657*** | 0.098 | 0.577*** | | 1.739*** | 0.696*** | 2.816*** |
| | | (0.151) | (0.136) | (0.121) | | (0.141) | (0.124) | (0.110) | | (0.293) | (0.262) | (0.275) |
| Operating leverage | | -0.045 | -0.041** | 0.096*** | | -0.022 | -0.028 | 0.109*** | | -0.054 | -0.022 | 0.195*** |
| | | (0.033) | (0.020) | (0.028) | | (0.030) | (0.018) | (0.025) | | (0.060) | (0.038) | (0.067) |
| Beta 1-lag | | | 0.449*** | | | | 0.472*** | | | | 0.414*** | |
| | | | (0.018) | | | | (0.019) | | | | (0.019) | |
| Beta 2-lag | | | 0.184*** | | | | 0.186*** | | | | 0.166*** | |
| | | | (0.018) | | | | (0.018) | | | | (0.017) | |
| SIC 3-digit × Year FEs | Yes | Yes | Yes | | Yes | Yes | Yes | | Yes | Yes | Yes | |
| Firm FEs | | | | Yes | | | | Yes | | | | Yes |
| $R^2$ | 0.52 | 0.59 | 0.75 | 0.61 | 0.55 | 0.62 | 0.78 | 0.64 | 0.61 | 0.68 | 0.78 | 0.50 |
| N | 10,791 | 9,412 | 6,985 | 9,412 | 10,791 | 9,412 | 6,985 | 9,412 | 10,791 | 9,412 | 6,985 | 9,412 |

# Table 3 (cont.): Beta and the average relative share of patents

*Panel B: 1-Year Sample at monthly frequency*

This panel is based on the *1-Year Sample* at monthly data frequency. Market capitalization comes from the CRSP Monthly Stock File (item abs(prc)×shrout) and is entered at monthly frequency. The other firm-level control variables come from Compustat and take the same value for all months in a given year. We include the 3-digit SIC industry interacted with month fixed effects or the firm fixed effects.

| | Market Beta | | | | Technology Beta | | | | Growth-Value Beta | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) |
| Relative share of patents | -0.117*** | -0.086*** | -0.060*** | -0.018** | -0.097*** | -0.069*** | -0.047*** | -0.013** | -0.229*** | -0.139*** | -0.093*** | -0.023 |
| | (0.024) | (0.021) | (0.014) | (0.007) | (0.021) | (0.018) | (0.012) | (0.006) | (0.042) | (0.034) | (0.026) | (0.018) |
| Ln(Market capitalization) | 0.086*** | 0.106*** | 0.069*** | 0.041*** | 0.091*** | 0.110*** | 0.071*** | 0.058*** | 0.154*** | 0.167*** | 0.117*** | -0.033*** |
| | (0.008) | (0.009) | (0.007) | (0.005) | (0.007) | (0.008) | (0.006) | (0.004) | (0.015) | (0.016) | (0.012) | (0.011) |
| Profitability | | -0.181* | -0.079 | 0.344*** | | -0.136 | -0.066 | 0.254*** | | 0.237 | 0.212 | 1.631*** |
| | | (0.103) | (0.080) | (0.065) | | (0.090) | (0.069) | (0.056) | | (0.188) | (0.158) | (0.135) |
| Ln(B/M) | | -0.028 | -0.020 | 0.032*** | | -0.015 | -0.011 | 0.055*** | | -0.100*** | -0.079*** | 0.119*** |
| | | (0.018) | (0.013) | (0.008) | | (0.016) | (0.012) | (0.007) | | (0.031) | (0.025) | (0.018) |
| Tangibility | | -0.158 | -0.100 | -0.137** | | -0.085 | -0.049 | -0.118** | | -0.218 | -0.109 | 0.456*** |
| | | (0.114) | (0.083) | (0.054) | | (0.103) | (0.074) | (0.047) | | (0.211) | (0.165) | (0.127) |
| Cash | | 0.659*** | 0.435*** | 0.416*** | | 0.616*** | 0.407*** | 0.379*** | | 1.390*** | 1.067*** | 0.446*** |
| | | (0.086) | (0.060) | (0.048) | | (0.076) | (0.053) | (0.042) | | (0.147) | (0.111) | (0.105) |
| Book leverage | | 0.183** | 0.118* | -0.093** | | 0.162** | 0.100* | -0.080** | | 0.218 | 0.187 | -0.039 |
| | | (0.087) | (0.063) | (0.042) | | (0.079) | (0.056) | (0.036) | | (0.155) | (0.122) | (0.091) |
| R&D | | 0.021 | 0.083 | 0.070 | | 0.011 | 0.061 | -0.011 | | 0.041 | 0.088 | 0.117 |
| | | (0.073) | (0.060) | (0.064) | | (0.065) | (0.052) | (0.053) | | (0.134) | (0.113) | (0.130) |
| Capex | | 0.651*** | 0.410*** | 0.665*** | | 0.571*** | 0.359*** | 0.531*** | | 1.680*** | 1.157*** | 3.482*** |
| | | (0.163) | (0.127) | (0.096) | | (0.146) | (0.110) | (0.082) | | (0.334) | (0.273) | (0.205) |
| Operating leverage | | -0.010 | -0.005 | 0.143*** | | 0.005 | 0.006 | 0.125*** | | 0.014 | 0.013 | 0.353*** |
| | | (0.034) | (0.025) | (0.019) | | (0.030) | (0.022) | (0.016) | | (0.060) | (0.049) | (0.042) |
| Beta 1-lag | | | 0.174*** | | | | 0.179*** | | | | 0.137*** | |
| | | | (0.007) | | | | (0.007) | | | | (0.006) | |
| Beta 2-lag | | | 0.149*** | | | | 0.153*** | | | | 0.127*** | |
| | | | (0.008) | | | | (0.008) | | | | (0.007) | |
| SIC 3-digit × Month FEs | Yes | Yes | Yes | | Yes | Yes | Yes | | Yes | Yes | Yes | |
| Firm FEs | | | | Yes | | | | Yes | | | | Yes |
| $R^2$ | 0.35 | 0.40 | 0.45 | 0.24 | 0.37 | 0.42 | 0.47 | 0.26 | 0.45 | 0.49 | 0.52 | 0.17 |
| N | 114,711 | 100,798 | 90,309 | 100,798 | 113,625 | 99,739 | 89,308 | 99,739 | 114,711 | 100,798 | 90,309 | 100,798 |

**Table 4: Beta and the relative share of patents in each technology class**

This table reports estimates from firm-class-level OLS regressions of equity beta on firm and technology class characteristics (equation (16) in Section 4.2). The relative share of patents is given by equation (14) in Section 4.2. The number of innovating firms stands for the number of firms that repeatedly innovate in a given class (definition (1) in Section 2.2). *Market Beta*, *Technology Beta*, and *Growth-Value Beta* measure a firm's exposure to systematic risk and are defined in Section 3.3. The top panel is based on the *3-Year Sample* at annual data frequency, and we include the technology class and year fixed effects in the regression. The bottom panel is based on the *1-Year Sample* at monthly data frequency, and we include the technology class and month fixed effects in the regression. Definitions of the firm-level control variables (not reported) are provided in Appendix A. We winsorize all variables at the 1% level. Robust standard errors (clustered at the technology class level) are reported in parentheses; *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

| | Market Beta | | Technology Beta | | Growth-Value Beta | |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| | *3-Year Sample at annual frequency* | | | | | |
| Relative share of patents | -0.019*** | -0.016*** | -0.015** | -0.012*** | -0.040*** | -0.023** |
| | (0.006) | (0.005) | (0.006) | (0.004) | (0.013) | (0.010) |
| Number of innovating firms | 0.187** | 0.053 | 0.206** | 0.076 | 0.359** | 0.073 |
| | (0.081) | (0.072) | (0.081) | (0.069) | (0.154) | (0.129) |
| Class, Year FEs | Yes | Yes | Yes | Yes | Yes | Yes |
| $R^2$ | 0.18 | 0.33 | 0.20 | 0.37 | 0.37 | 0.54 |
| N | 38,281 | 34,096 | 38,281 | 34,096 | 38,281 | 34,096 |
| | *1-Year Sample at monthly frequency* | | | | | |
| Relative share of patents | -0.015*** | -0.013*** | -0.011*** | -0.009*** | -0.026*** | -0.015** |
| | (0.003) | (0.003) | (0.003) | (0.003) | (0.007) | (0.006) |
| Number of innovating firms | 0.270*** | 0.117*** | 0.266*** | 0.124*** | 0.415*** | 0.128*** |
| | (0.018) | (0.017) | (0.016) | (0.015) | (0.034) | (0.031) |
| Class, Month FEs | Yes | Yes | Yes | Yes | Yes | Yes |
| $R^2$ | 0.09 | 0.16 | 0.11 | 0.19 | 0.28 | 0.36 |
| N | 390,683 | 349,096 | 387,034 | 345,603 | 390,683 | 349,096 |

**Table 5: Interdependence between betas of rival firms**

This table reports tests of Corollary 2, described in Section 4.3, using firms in the *3-Year Sample* at annual data frequency. *Market Beta*, *Technology Beta*, and *Growth-Value Beta* measure a firm's exposure to systematic risk and are defined in Section 3.3. We winsorize all variables at the 1% level. Robust standard errors are reported in parentheses; *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

*Panel A: Firm-level*

Columns (1), (3), and (5) (columns (2), (4), and (6)) report estimates from firm-level OLS regression (17) (regression (18)) in Section 4.3. The dependent variable is the annual change in a firm's beta. Δ *Relative share of patents: top (bottom) tercile* is equal to one if a firm experiences an annual change in the relative share of patents in the top (bottom) tercile of the distribution of changes, and zero otherwise. *Middle-to-Leader* is equal to one if a firm is in the middle tercile of the distribution of the relative share of patents at year $t-1$, and transitions to the top tercile of the distribution of the relative share of patents in the same class by year $t$, and zero otherwise. The indicator variables *Middle-to-Middle* and *Middle-to-Laggard* are defined analogously. Definitions of the firm-level control variables are provided in Appendix A.

| | Δ Market Beta | | Δ Technology Beta | | Δ Growth-Value Beta | |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| Δ Relative share of patents: top tercile | -0.001 | | -0.002 | | 0.012 | |
| | (0.010) | | (0.008) | | (0.017) | |
| Δ Relative share of patents: bottom tercile | 0.028*** | | 0.024*** | | 0.064*** | |
| | (0.010) | | (0.008) | | (0.017) | |
| Middle-to-Leader | | -0.018 | | -0.019 | | -0.050** |
| | | (0.014) | | (0.012) | | (0.024) |
| Middle-to-Middle | | 0.015* | | 0.010 | | 0.034** |
| | | (0.009) | | (0.008) | | (0.016) |
| Middle-to-Laggard | | 0.025* | | 0.025** | | 0.048* |
| | | (0.013) | | (0.012) | | (0.027) |
| Firm-level controls | Yes | Yes | Yes | Yes | Yes | Yes |
| SIC 3-digit FEs, Year FEs | Yes | Yes | Yes | Yes | Yes | Yes |
| $R^2$ | 0.12 | 0.12 | 0.12 | 0.12 | 0.39 | 0.39 |
| N | 7,871 | 7,871 | 7,871 | 7,871 | 7,871 | 7,871 |

## Table 5 (cont.): Interdependence between betas of rival firms

*Panel B: Technology class-level*

This panel reports estimates from class-level OLS regression (19) in Section 4.3. The dependent variable is the average of the annual beta changes of firms that, in class $k$ and year $t$, experienced a decline from year $t-1$ in their relative innovation output bigger than 10% of one standard deviation of the share of patents (*retreating firms*). $\Delta$ *Relative share of patents of advancing firms* stands for the average of the annual changes in the relative share of patents of firms that, in class $k$ and year $t$, increased their relative innovation output from year $t-1$ by at least 10% of one standard deviation of the share of patents. $\Delta$ *Relative share of patents of retreating firms* stands for the average of the annual changes in the relative share of patents of retreating firms.

| | Δ Market Beta of retreating firms | | Δ Technology Beta of retreating firms | | Δ Growth-Value Beta of retreating firms | |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| Δ Relative share of patents of advancing firms | 0.113** | 0.097* | 0.112*** | 0.101** | 0.229*** | 0.194** |
| | (0.048) | (0.052) | (0.041) | (0.046) | (0.067) | (0.093) |
| Δ Relative share of patents of retreating firms | | -0.083 | | -0.058 | | -0.219* |
| | | (0.063) | | (0.059) | | (0.130) |
| Number of innovating firms | | 0.007 | | 0.005 | | 0.024** |
| | | (0.007) | | (0.006) | | (0.011) |
| Year FEs | Yes | Yes | Yes | Yes | Yes | Yes |
| $R^2$ | 0.29 | 0.29 | 0.21 | 0.21 | 0.70 | 0.70 |
| N | 942 | 942 | 942 | 942 | 942 | 942 |

## Table 6: Competition in broad/narrow fields of technology

This table reports estimates from firm-level OLS regressions of equity beta on firm characteristics (equation (15) in Section 4.2). The table is based on alternative definitions of the fields of technology—classes—in which firms compete in innovation. The samples and regression specifications are analogous to those in Table 3. Definitions of the firm-level control variables (not reported) are provided in Appendix A. We winsorize all variables at the 1% level. Robust standard errors (clustered at the firm level) are reported in parentheses; *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

*Panel A: Classes are defined using "sections" (the first level) of the IPC hierarchical classification of patents*

| | Market Beta | | | | Technology Beta | | | | Growth-Value Beta | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) |
| | *3-Year Sample at annual frequency* | | | | | | | | | | | |
| Relative share of patents | -0.168*** | -0.126*** | -0.039*** | -0.111*** | -0.125*** | -0.093*** | -0.028*** | -0.083*** | -0.350*** | -0.218*** | -0.083*** | -0.064 |
| | (0.030) | (0.026) | (0.012) | (0.019) | (0.026) | (0.023) | (0.010) | (0.016) | (0.049) | (0.044) | (0.022) | (0.049) |
| SIC 3-digit × Year FEs | Yes | Yes | Yes | | Yes | Yes | Yes | | Yes | Yes | Yes | |
| Firm FEs | | | | Yes | | | | Yes | | | | Yes |
| $R^2$ | 0.50 | 0.58 | 0.74 | 0.60 | 0.54 | 0.60 | 0.77 | 0.63 | 0.60 | 0.67 | 0.77 | 0.49 |
| N | 13,087 | 11,486 | 8,612 | 11,486 | 13,087 | 11,486 | 8,612 | 11,486 | 13,087 | 11,486 | 8,612 | 11,486 |
| | *1-Year Sample at monthly frequency* | | | | | | | | | | | |
| Relative share of patents | -0.164*** | -0.134*** | -0.092*** | -0.054*** | -0.123*** | -0.098*** | -0.067*** | -0.039*** | -0.328*** | -0.216*** | -0.156*** | -0.033 |
| | (0.030) | (0.027) | (0.019) | (0.010) | (0.026) | (0.023) | (0.016) | (0.009) | (0.050) | (0.045) | (0.034) | (0.024) |
| SIC 3-digit × Month FEs | Yes | Yes | Yes | | Yes | Yes | Yes | | Yes | Yes | Yes | |
| Firm FEs | | | | Yes | | | | Yes | | | | Yes |
| $R^2$ | 0.34 | 0.39 | 0.43 | 0.23 | 0.36 | 0.41 | 0.46 | 0.26 | 0.43 | 0.47 | 0.50 | 0.16 |
| N | 137,238 | 121,340 | 109,707 | 121,340 | 135,784 | 119,931 | 108,369 | 119,931 | 137,238 | 121,340 | 109,707 | 121,340 |

## Table 6 (cont.): Competition in broad/narrow fields of technology

*Panel B: Classes are defined using "subclasses" (the third level) of the IPC hierarchical classification of patents*

| | Market Beta | | | | Technology Beta | | | | Growth-Value Beta | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) |
| | *3-Year Sample at annual frequency* | | | | | | | | | | | |
| Relative share of patents | -0.116*** | -0.099*** | -0.037*** | -0.028** | -0.099*** | -0.084*** | -0.032*** | -0.009 | -0.232*** | -0.176*** | -0.064*** | -0.100*** |
| | (0.022) | (0.021) | (0.011) | (0.012) | (0.021) | (0.020) | (0.010) | (0.011) | (0.040) | (0.036) | (0.022) | (0.031) |
| SIC 3-digit × Year FEs | Yes | Yes | Yes | | Yes | Yes | Yes | | Yes | Yes | Yes | |
| Firm FEs | | | | Yes | | | | Yes | | | | Yes |
| $R^2$ | 0.52 | 0.60 | 0.76 | 0.62 | 0.55 | 0.63 | 0.79 | 0.65 | 0.61 | 0.69 | 0.80 | 0.51 |
| N | 8,855 | 7,717 | 5,707 | 7,717 | 8,855 | 7,717 | 5,707 | 7,717 | 8,855 | 7,717 | 5,707 | 7,717 |
| | *1-Year Sample at monthly frequency* | | | | | | | | | | | |
| Relative share of patents | -0.091*** | -0.069*** | -0.051*** | -0.008 | -0.080*** | -0.060*** | -0.043*** | -0.000 | -0.199*** | -0.133*** | -0.098*** | -0.051*** |
| | (0.021) | (0.020) | (0.014) | (0.007) | (0.019) | (0.018) | (0.013) | (0.006) | (0.037) | (0.036) | (0.028) | (0.016) |
| SIC 3-digit × Month FEs | Yes | Yes | Yes | | Yes | Yes | Yes | | Yes | Yes | Yes | |
| Firm FEs | | | | Yes | | | | Yes | | | | Yes |
| $R^2$ | 0.37 | 0.42 | 0.47 | 0.24 | 0.39 | 0.44 | 0.49 | 0.27 | 0.47 | 0.52 | 0.55 | 0.18 |
| N | 95,279 | 83,651 | 74,380 | 83,651 | 94,481 | 82,876 | 73,669 | 82,876 | 95,279 | 83,651 | 74,380 | 83,651 |

## Table 7: Heterogeneity in the fields of technology

This table reports estimates from firm-level OLS regressions of equity beta on firm characteristics (equation (15) in Section 4.2). The table is based on the weighted average of a firm's relative share of patents in the fields of technology—classes—in which the firm competes in innovation. The samples and regression specifications are analogous to those in Table 3. Definitions of the firm-level control variables (not reported) are provided in Appendix A. We winsorize all variables at the 1% level. Robust standard errors (clustered at the firm level) are reported in parentheses; *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

*Panel A: Weights computed using the number of innovating firms in each class*

$\overline{y}_{it}^N$ denotes the weighted average of a firm's relative share of patents. The weights are, for each class, the number of innovating firms in this class divided by the sum of the number of innovating firms in all classes in which the firm is innovating.

| | Market Beta | | | | Technology Beta | | | | Growth-Value Beta | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) |
| | *3-Year Sample at annual frequency* | | | | | | | | | | | |
| $\overline{y}_{it}^N$ | -0.136*** | -0.088*** | -0.033*** | -0.039*** | -0.105*** | -0.066*** | -0.026*** | -0.022* | -0.268*** | -0.150*** | -0.056*** | -0.051 |
| | (0.024) | (0.021) | (0.010) | (0.013) | (0.021) | (0.018) | (0.009) | (0.011) | (0.042) | (0.036) | (0.020) | (0.033) |
| SIC 3-digit × Year FEs | Yes | Yes | Yes | | Yes | Yes | Yes | | Yes | Yes | Yes | |
| Firm FEs | | | | Yes | | | | Yes | | | | Yes |
| $R^2$ | 0.52 | 0.59 | 0.75 | 0.61 | 0.55 | 0.62 | 0.78 | 0.64 | 0.61 | 0.68 | 0.78 | 0.50 |
| N | 10,791 | 9,412 | 6,985 | 9,412 | 10,791 | 9,412 | 6,985 | 9,412 | 10,791 | 9,412 | 6,985 | 9,412 |
| | *1-Year Sample at monthly frequency* | | | | | | | | | | | |
| $\overline{y}_{it}^N$ | -0.124*** | -0.084*** | -0.057*** | -0.013* | -0.099*** | -0.065*** | -0.043*** | -0.009 | -0.232*** | -0.129*** | -0.086*** | -0.012 |
| | (0.023) | (0.020) | (0.014) | (0.007) | (0.020) | (0.017) | (0.012) | (0.006) | (0.039) | (0.033) | (0.025) | (0.017) |
| SIC 3-digit × Month FEs | Yes | Yes | Yes | | Yes | Yes | Yes | | Yes | Yes | Yes | |
| Firm FEs | | | | Yes | | | | Yes | | | | Yes |
| $R^2$ | 00.35 | 0.40 | 0.45 | 0.24 | 0.37 | 0.42 | 0.47 | 0.26 | 0.45 | 0.49 | 0.52 | 0.17 |
| N | 114,711 | 100,798 | 90,309 | 100,798 | 113,625 | 99,739 | 89,308 | 99,739 | 114,711 | 100,798 | 90,309 | 100,798 |

## Table 7 (cont.): Heterogeneity in the fields of technology

*Panel B: Weights computed using the market capitalization of innovating firms in each class*

$\overline{y}_{it}^{MKT}$ denotes the weighted average of a firm's relative share of patents. The weights are, for each class, the sum of the market capitalization of innovating firms in this class divided by the sum of the market capitalization of innovating firms in all classes in which the firm is innovating.

| | Market Beta | | | | Technology Beta | | | | Growth-Value Beta | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) |
| | *3-Year Sample at annual frequency* | | | | | | | | | | | |
| $\overline{y}_{it}^{MKT}$ | -0.136*** | -0.090*** | -0.034*** | -0.038*** | -0.106*** | -0.068*** | -0.027*** | -0.021* | -0.269*** | -0.153*** | -0.058*** | -0.050 |
| | (0.025) | (0.022) | (0.011) | (0.013) | (0.021) | (0.019) | (0.009) | (0.012) | (0.043) | (0.036) | (0.021) | (0.034) |
| SIC 3-digit × Year FEs | Yes | Yes | Yes | | Yes | Yes | Yes | | Yes | Yes | Yes | |
| Firm FEs | | | | Yes | | | | Yes | | | | Yes |
| $R^2$ | 0.52 | 0.59 | 0.75 | 0.61 | 0.55 | 0.62 | 0.78 | 0.64 | 0.61 | 0.68 | 0.78 | 0.50 |
| N | 10,791 | 9,412 | 6,985 | 9,412 | 10,791 | 9,412 | 6,985 | 9,412 | 10,791 | 9,412 | 6,985 | 9,412 |
| | *1-Year Sample at monthly frequency* | | | | | | | | | | | |
| $\overline{y}_{it}^{MKT}$ | -0.124*** | -0.085*** | -0.058*** | -0.014** | -0.100*** | -0.066*** | -0.044*** | -0.009 | -0.230*** | -0.129*** | -0.084*** | -0.010 |
| | (0.024) | (0.021) | (0.014) | (0.007) | (0.020) | (0.018) | (0.012) | (0.006) | (0.040) | (0.033) | (0.025) | (0.017) |
| SIC 3-digit × Month FEs | Yes | Yes | Yes | | Yes | Yes | Yes | | Yes | Yes | Yes | |
| Firm FEs | | | | Yes | | | | Yes | | | | Yes |
| $R^2$ | 0.35 | 0.40 | 0.45 | 0.24 | 0.37 | 0.42 | 0.47 | 0.26 | 0.45 | 0.49 | 0.52 | 0.17 |
| N | 114,711 | 100,798 | 90,309 | 100,798 | 113,625 | 99,739 | 89,308 | 99,739 | 114,711 | 100,798 | 90,309 | 100,798 |

# A Definitions of firm-level control variables

| | |
|---|---|
| Ln(Market capitalization) | Natural logarithm of market capitalization of a firm's common equity in USD millions. We use the market capitalization reported in Compustat as of the fiscal year-end (item prcc_f×csho) in our annual regressions, while we use market capitalization from the CRSP Monthly Stock File (item abs(prc)×shrout) in our monthly regressions. |
| Profitability | Earnings before interest, taxes, depreciation, and amortization (item oibdp) scaled by total assets (item at). |
| Ln(B/M) | Natural logarithm of the book value of common equity (item ceq) scaled by the market value of common equity (item prcc_f×csho). |
| Tangibility | Total net property, plant, and equipment (item ppent) scaled by total assets (item at). |
| Cash | Cash and short-term investment (item che) scaled by total assets (item at). |
| Book leverage | Total long-term debt plus debt in current liabilities (item dltt+dlc) scaled by total assets (item at). |
| R&D | Research and development expenses (item xrd) scaled by sales (item sale). |
| Capex | Capital expenditures (item capx) scaled by sales (item sale). |
| Operating leverage | Cost of goods sold (item cogs) plus selling, general, and administrative expenses (item xsga) scaled by total assets (item at). See Novy-Marx (2011). |

# B Robustness tables

**Table B-1: Innovation output and portfolio returns: Portfolios of firms matched on size and B/M**

This table reports estimates from asset pricing models using portfolios matched by size and book-to-market to the leaders, middle, and laggards portfolios introduced in Table 2, respectively. To construct matched portfolios, for each firm in the leaders, middle, and laggards portfolio, we determine in which NYSE size and book-to-market deciles the firm belongs. We then use the value-weighted excess return on the portfolio of all firms in the intersection of these NYSE size and book-to-market deciles, instead of the excess return on the firm, to compute the matched leaders, matched middle, and matched laggards portfolio returns. We obtain NYSE size and book-to-market decile breakpoints as well as the value-weighted returns on the portfolios of firms in each intersection of the NYSE size and book-to-market deciles from Kenneth French's website. Standard errors, reported in parentheses, are computed using the Newey-West estimator allowing for 1 lag of serial correlation. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

| | Market model | | Fama-French three-factor model | | | | Cahart four-factor model | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\alpha(\%)$ | $\beta_{MKT}$ | $\alpha(\%)$ | $\beta_{MKT}$ | $\beta_{SMB}$ | $\beta_{HML}$ | $\alpha(\%)$ | $\beta_{MKT}$ | $\beta_{SMB}$ | $\beta_{HML}$ | $\beta_{UMD}$ |
| Matched Leaders ($L^M$) | 0.046 | 0.953*** | 0.162*** | 0.961*** | -0.267*** | -0.110*** | 0.170*** | 0.960*** | -0.266*** | -0.111*** | -0.008 |
| | (0.059) | (0.014) | (0.040) | (0.010) | (0.017) | (0.022) | (0.043) | (0.011) | (0.017) | (0.021) | (0.015) |
| Matched Middle | -0.006 | 0.982*** | 0.089** | 0.979*** | -0.180*** | -0.099*** | 0.112*** | 0.977*** | -0.176*** | -0.104*** | -0.024* |
| | (0.050) | (0.014) | (0.043) | (0.012) | (0.016) | (0.022) | (0.042) | (0.011) | (0.016) | (0.023) | (0.013) |
| Matched Laggards ($G^M$) | 0.081* | 1.015*** | 0.120*** | 1.007*** | -0.051 | -0.048 | 0.122*** | 1.007*** | -0.050* | -0.049 | -0.003 |
| | (0.048) | (0.013) | (0.045) | (0.016) | (0.031) | (0.031) | (0.046) | (0.016) | (0.030) | (0.032) | (0.018) |
| $G^M - L^M$ | 0.035 | 0.062*** | -0.043 | 0.046*** | 0.216*** | 0.062* | -0.048 | 0.046*** | 0.216*** | 0.063* | 0.005 |
| | (0.063) | (0.017) | (0.047) | (0.015) | (0.028) | (0.033) | (0.049) | (0.015) | (0.027) | (0.035) | (0.020) |

## Table B-2: Robustness on the interdependence between betas of rival firms

This table reports estimates from class-level OLS regression (19) in Section 4.3 using firms in the *3-Year Sample* at annual data frequency. *Market Beta*, *Technology Beta*, and *Growth-Value Beta* measure a firm's exposure to systematic risk and are defined in Section 3.3. We winsorize all variables at the 1% level. Robust standard errors (clustered at the technology class level) are reported in parentheses; *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

*Panel A: Competition in broad/narrow fields of technology*

This panel is based on alternative definitions of the fields of technology—classes—in which firms compete in innovation. In the top (bottom) part, the dependent variable is the average of the annual beta changes of firms that, in class $k$ and year $t$, experienced a decline from year $t-1$ in their relative innovation output bigger than 5% (20%) of one standard deviation of the share of patents (*retreating firms*). Δ *Relative share of patents of advancing firms* stands for the average of the annual changes in the relative share of patents of firms that, in class $k$ and year $t$, increased their relative innovation output from year $t-1$ by at least 5% (20%) of one standard deviation of the share of patents. Δ *Relative share of patents of retreating firms* stands for the average of the annual changes in the relative share of patents of retreating firms.

| | Δ Market Beta of retreating firms | | Δ Technology Beta of retreating firms | | Δ Growth-Value Beta of retreating firms | |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| *Classes are defined using "sections" (the first level) of the IPC hierarchical classification of patents* | | | | | | |
| Δ Relative share of patents of advancing firms | 0.246 | 0.169 | 0.305 | 0.226 | 0.506 | 0.292 |
| | (0.274) | (0.256) | (0.247) | (0.219) | (0.515) | (0.467) |
| Δ Relative share of patents of retreating firms | | -0.445** | | -0.459*** | | -1.295*** |
| | | (0.119) | | (0.075) | | (0.313) |
| Number of innovating firms | | 0.006 | | 0.008 | | 0.034 |
| | | (0.011) | | (0.009) | | (0.032) |
| Year FEs | Yes | Yes | Yes | Yes | Yes | Yes |
| $R^2$ | 0.69 | 0.70 | 0.66 | 0.67 | 0.88 | 0.89 |
| N | 118 | 118 | 118 | 118 | 118 | 118 |
| *Classes are defined using "subclasses" (the third level) of the IPC hierarchical classification of patents* | | | | | | |
| Δ Relative share of patents of advancing firms | 0.127*** | 0.104** | 0.114*** | 0.099** | 0.208*** | 0.160** |
| | (0.042) | (0.044) | (0.038) | (0.041) | (0.077) | (0.079) |
| Δ Relative share of patents of retreating firms | | -0.096* | | -0.067 | | -0.134 |
| | | (0.049) | | (0.042) | | (0.126) |
| Number of innovating firms | | 0.003 | | 0.003 | | -0.022 |
| | | (0.010) | | (0.009) | | (0.018) |
| Year FEs | Yes | Yes | Yes | Yes | Yes | Yes |
| $R^2$ | 0.16 | 0.17 | 0.11 | 0.11 | 0.53 | 0.53 |
| N | 1,295 | 1,295 | 1,295 | 1,295 | 1,295 | 1,295 |

## Table B-2 (cont.): Robustness on the interdependence between betas of rival firms

*Panel B: Citation-weighted average relative share of patents*

In this panel, the average relative share of patents is computed using citation-weighted number of patents following the methodology introduced by Hall, Jaffe, and Trajtenberg (2001) and Hall, Jaffe, and Trajtenberg (2005). The dependent variable is the average of the annual beta changes of firms that, in class $k$ and year $t$, experienced a decline from year $t-1$ in their relative innovation output bigger than 10% of one standard deviation of the share of patents (*retreating firms*). Δ *Relative share of patents of advancing firms* stands for the average of the annual changes in the relative share of patents of firms that, in class $k$ and year $t$, increased their relative innovation output from year $t-1$ by at least 10% of one standard deviation of the share of patents. Δ *Relative share of patents of retreating firms* stands for the average of the annual changes in the relative share of patents of retreating firms.

| | Δ Market Beta of retreating firms | | Δ Technology Beta of retreating firms | | Δ Growth-Value Beta of retreating firms | |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| Δ Relative share of patents of advancing firms | 0.129*** | 0.116** | 0.118*** | 0.102** | 0.217** | 0.155 |
| | (0.042) | (0.047) | (0.037) | (0.043) | (0.083) | (0.094) |
| Δ Relative share of patents of retreating firms | | -0.053 | | -0.057 | | -0.213 |
| | | (0.075) | | (0.070) | | (0.131) |
| Number of innovating firms | | 0.003 | | 0.003 | | 0.009 |
| | | (0.007) | | (0.006) | | (0.011) |
| Year FEs | Yes | Yes | Yes | Yes | Yes | Yes |
| $R^2$ | 0.28 | 0.28 | 0.20 | 0.20 | 0.72 | 0.73 |
| N | 954 | 954 | 954 | 954 | 954 | 954 |

*Panel C: Unlevered betas*

In this panel, the dependent variables are unlevered *Market Beta*, *Technology Beta*, and *Growth-Value Beta* computed following methodology of Bharath and Shumway (2008). The dependent variable is the average of the annual beta changes of firms that, in class $k$ and year $t$, experienced a decline from year $t-1$ in their relative innovation output bigger than 10% of one standard deviation of the share of patents—*retreating firms*. Δ *Relative share of patents of advancing firms* stands for the average of the annual changes in the relative share of patents of firms that, in class $k$ and year $t$, increased their relative innovation output from year $t-1$ by at least 10% of one standard deviation of the share of patents. Δ *Relative share of patents of retreating firms* stands for the average of the annual changes in the relative share of patents of retreating firms.

| | Δ Market Beta of retreating firms | | Δ Technology Beta of retreating firms | | Δ Growth-Value Beta of retreating firms | |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| Δ Relative share of patents of advancing firms | 0.168*** | 0.120** | 0.161*** | 0.120** | 0.178* | 0.095 |
| | (0.048) | (0.057) | (0.044) | (0.051) | (0.094) | (0.115) |
| Δ Relative share of patents of retreating firms | | -0.133* | | -0.115* | | -0.278** |
| | | (0.066) | | (0.062) | | (0.123) |
| Number of innovating firms | | 0.001 | | 0.001 | | 0.013 |
| | | (0.007) | | (0.007) | | (0.011) |
| Year FEs | Yes | Yes | Yes | Yes | Yes | Yes |
| $R^2$ | 0.29 | 0.29 | 0.21 | 0.21 | 0.61 | 0.61 |
| N | 802 | 802 | 802 | 802 | 802 | 802 |

## Table B-3: Beta and citation-weighted average relative share of patents

This table reports estimates from firm-level OLS regressions of equity beta on firm characteristics (equation (15) in Section 4.2). The average relative share of patents is computed using citation-weighted number of patents following the methodology introduced by Hall, Jaffe, and Trajtenberg (2001) and Hall, Jaffe, and Trajtenberg (2005). The samples and regression specifications are analogous to those in Table 3. Definitions of the firm-level control variables (not reported) are provided in Appendix A. We winsorize all variables at the 1% level. Robust standard errors (clustered at the firm level) are reported in parentheses; *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

| | Market Beta | | | | Technology Beta | | | | Growth-Value Beta | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) |
| | | | | | *3-Year Sample at annual frequency* | | | | | | | |
| Relative share of patents | -0.095*** | -0.074*** | -0.035*** | -0.026* | -0.072*** | -0.053*** | -0.026*** | -0.009 | -0.182*** | -0.114*** | -0.051** | -0.046 |
| | (0.025) | (0.022) | (0.011) | (0.014) | (0.022) | (0.020) | (0.010) | (0.012) | (0.044) | (0.037) | (0.020) | (0.034) |
| SIC 3-digit × Year FEs | Yes | Yes | Yes | | Yes | Yes | Yes | | Yes | Yes | Yes | |
| Firm FEs | | | | Yes | | | | Yes | | | | Yes |
| $R^2$ | 0.51 | 0.59 | 0.75 | 0.61 | 0.54 | 0.62 | 0.78 | 0.64 | 0.60 | 0.68 | 0.78 | 0.50 |
| N | 10,791 | 9,412 | 6,985 | 9,412 | 10,791 | 9,412 | 6,985 | 9,412 | 10,791 | 9,412 | 6,985 | 9,412 |
| | | | | | *1-Year Sample at monthly frequency* | | | | | | | |
| Relative share of patents | -0.081*** | -0.064*** | -0.045*** | -0.004 | -0.065*** | -0.050*** | -0.033*** | -0.000 | -0.148*** | -0.092*** | -0.054** | -0.002 |
| | (0.023) | (0.021) | (0.014) | (0.007) | (0.020) | (0.017) | (0.012) | (0.006) | (0.040) | (0.033) | (0.025) | (0.017) |
| SIC 3-digit × Month FEs | Yes | Yes | Yes | | Yes | Yes | Yes | | Yes | Yes | Yes | |
| Firm FEs | | | | Yes | | | | Yes | | | | Yes |
| $R^2$ | 0.35 | 0.40 | 0.45 | 0.24 | 0.37 | 0.42 | 0.47 | 0.26 | 0.45 | 0.49 | 0.52 | 0.17 |
| N | 114,814 | 100,899 | 90,421 | 100,899 | 113,723 | 99,835 | 89,416 | 99,835 | 114,814 | 100,899 | 90,421 | 100,899 |

**Table B-4: Sum betas**

The top (bottom) panel of this table reports estimates from firm-level (firm-class-level) OLS regressions of equity beta on firm characteristics, equation (15) (equation (16)) in Section 4.2. The dependent variables are *Sum Betas* obtained following the methodology of Cohen, Hawawini, Maier, Schwartz, and Whitcomb (1983), using the excess returns on the CRSP value-weighted and equally-weighted index; see Section 3.3 for details. The samples and regression specifications are analogous to those in Table 3 (top panel of this table) and Table 4 (bottom panel of this table). Definitions of the firm-level control variables (not reported) are provided in Appendix A. We winsorize all variables at the 1% level. Robust standard errors are reported in parentheses; *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

| | Value-weighted market portfolio | | | | Equally weighted market portfolio | | | |
|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| | *Firm's beta and the average relative share of patents* | | | | | | | |
| Relative share of patents | -0.128*** | -0.089*** | -0.078*** | -0.024** | -0.150*** | -0.103*** | -0.097*** | -0.025 |
| | (0.026) | (0.023) | (0.020) | (0.012) | (0.033) | (0.029) | (0.026) | (0.016) |
| SIC 3-digit × Month FEs | Yes | Yes | Yes | | Yes | Yes | Yes | |
| Firm FEs | | | | Yes | | | | Yes |
| $R^2$ | 0.30 | 0.34 | 0.36 | 0.13 | 0.30 | 0.34 | 0.36 | 0.11 |
| N | 114,673 | 100,766 | 90,234 | 100,766 | 114,673 | 100,766 | 90,234 | 100,766 |
| | *Firm's beta and the relative share of patents in each technology class* | | | | | | | |
| Relative share of patents | -0.012*** | -0.010** | | | -0.008* | -0.007* | | |
| | (0.004) | (0.004) | | | (0.005) | (0.005) | | |
| Number of innovating firms | 0.307*** | 0.159*** | | | 0.373*** | 0.168*** | | |
| | (0.021) | (0.021) | | | (0.027) | (0.026) | | |
| Class, Month FEs | Yes | Yes | | | Yes | Yes | | |
| $R^2$ | 0.06 | 0.10 | | | 0.11 | 0.14 | | |
| N | 390,560 | 348,984 | | | 390,560 | 348,984 | | |

## Table B-5: Unlevered betas

This table reports estimates from firm-level OLS regressions of equity beta on firm characteristics (equation (15) in Section 4.2). Dependent variables are unlevered *Market Beta*, *Technology Beta*, and *Growth-Value Beta* computed following methodology of Bharath and Shumway (2008). The samples and regression specifications are analogous to those in Table 3. Definitions of the firm-level control variables (not reported) are provided in Appendix A. We winsorize all variables at the 1% level. Robust standard errors (clustered at the firm level) are reported in parentheses; *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

| | Unlevered Market Beta | | | | Unlevered Technology Beta | | | | Unlevered Growth-Value Beta | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) |
| | *3-Year Sample at annual frequency* | | | | | | | | | | | |
| Relative share of patents | -0.152*** | -0.087*** | -0.039*** | -0.048*** | -0.122*** | -0.067*** | -0.031*** | -0.028*** | -0.285*** | -0.153*** | -0.066*** | -0.069** |
| | (0.023) | (0.020) | (0.011) | (0.012) | (0.020) | (0.017) | (0.010) | (0.011) | (0.042) | (0.033) | (0.020) | (0.030) |
| SIC 3-digit × Year FEs | Yes | Yes | Yes | | Yes | Yes | Yes | | Yes | Yes | Yes | |
| Firm FEs | | | | Yes | | | | Yes | | | | Yes |
| $R^2$ | 0.53 | 0.63 | 0.77 | 0.65 | 0.56 | 0.65 | 0.80 | 0.68 | 0.60 | 0.69 | 0.79 | 0.54 |
| N | 10,757 | 9,411 | 6,966 | 9,411 | 10,757 | 9,411 | 6,966 | 9,411 | 10,757 | 9,411 | 6,966 | 9,411 |
| | *1-Year Sample at monthly frequency* | | | | | | | | | | | |
| Relative share of patents | -0.123*** | -0.082*** | -0.058*** | -0.020*** | -0.101*** | -0.066*** | -0.045*** | -0.015** | -0.233*** | -0.134*** | -0.090*** | -0.025 |
| | (0.023) | (0.019) | (0.013) | (0.007) | (0.020) | (0.016) | (0.011) | (0.006) | (0.040) | (0.031) | (0.024) | (0.016) |
| SIC 3-digit × Month FEs | Yes | Yes | Yes | | Yes | Yes | Yes | | Yes | Yes | Yes | |
| Firm FEs | | | | Yes | | | | Yes | | | | Yes |
| $R^2$ | 0.36 | 0.41 | 0.46 | 0.24 | 0.38 | 0.43 | 0.48 | 0.27 | 0.45 | 0.50 | 0.53 | 0.17 |
| N | 114,282 | 100,774 | 90,256 | 100,774 | 113,196 | 99,715 | 89,258 | 99,715 | 114,282 | 100,774 | 90,256 | 100,774 |

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Internet Appendix for:

# Corporate Innovation and Returns

(Not for Publication)

This separate Internet Appendix formally describes the structure and solution of the innovation game discussed in Section 3 of the main text. The proofs of all propositions are contained in subsection IA.6.

## Equilibria in the innovation game

In Section 3 of the main text, we model competition in innovation as a *stochastic stopping time game* and restrict our attention to *Markov strategies*, i.e., time invariant strategies in which actions depend only on the current level of the variable $x(t)$.[1] Specifically, a firm's strategy is a *stopping rule* characterized by a threshold $x^*$ for $x(t)$ such that the firm invests when $x(t)$ crosses $x^*$ from below for the first time.[2] A *Markov perfect equilibrium* is the set of strategies such that, in every state, each firm's strategy is value maximizing conditional on the rival's strategy. Unlike Reinganum (1981a,b,c), the firms in our model do not precommit ex-ante to a specific investment date. Instead, as in Fudenberg and Tirole (1985), we allow the firms to observe and respond immediately to their rival's investment decisions. As we will show later, this can lead to (subgame perfect) equilibria in which the firms may try to "preempt" each other. Preemptive strategies are not feasible when the firms precommit to investment dates.

### IA.1 Firms' values and investment strategies

Firms' values are the net present values (NPV) of their risky profits. To evaluate these profits, we assume the existence of a pricing kernel. Following a standard argument (e.g., Duffie (1996)), we construct a risk-neutral probability measure under which the process $x(t)$ evolves as

$$dx(t) = (r-\delta)x(t)dt + \sigma x(t)d\widehat{W}(t), \quad \delta > 0, \tag{IA.1}$$

[1] See Dutta and Rustichini (1993) for a formal treatment of stopping time games. Smets (1993) and Grenadier (1996) are early applications of stochastic stopping time games to foreign direct investments and real estate development, respectively.

[2] Because $x(t)$ is a Markov process, Markov strategies contain all payoff-relevant information. In general, one cannot exclude the existence of non-Markovian strategies. However, if one firm follows a Markov strategy, the opponent's best response is also Markov (see Fudenberg and Tirole (1991), Chapter 13, for a formal treatment of Markov equilibria).

where $d\widehat{W}(t)$ is the increment of a standard Brownian motion under the risk-neutral probability measure implied by the pricing kernel, $r$ is the risk-free rate, and $\delta$ is the opportunity cost of keeping the option to invest in innovation alive.[3] From equation (3) in the main text and (IA.1) we infer that the constant risk-premium associated with the process $x(t)$ is $\lambda \equiv \mu - (r - \delta)$. We assume that $\mu > r - \delta$, implying a positive risk-premium $\lambda > 0$.

To ensure that no firm has already invested in innovation at the beginning of the game, we require that the initial value of the patent $x(0)$ is sufficiently low so that the NPV of investing at time zero is negative (ignoring any strategic interactions among firms):[4]

$$E\left[\int_0^\infty e^{-(r+h_i)t} h_i x(t) dt\right] - K < 0, \quad \text{i.e.,} \quad \frac{h_i\, x(0)}{h_i + \delta} - K < 0, \quad i = 1, 2. \tag{IA.2}$$

This assumption rules out the multiplicity of equilibria with simultaneous immediate investment considered in Grenadier (1996), although, in general, it does not prevent the existence of other types of simultaneous equilibria, as we will show in Proposition IA.4.[5] Furthermore, if (IA.2) is violated, then it would be optimal for at least one firm to invest immediately, and mixed-strategies equilibria could occur. Thijssen, Huisman, and Kort (2002) analyze this situation in the case of symmetric firms.

In principle, there are three possible outcomes of the innovation game: (i) firm 1 invests first (leader), and firm 2 invests at a later date (laggard); (ii) firm 2 invests first as a leader, and firm 1 follows; or (iii) both firms invest simultaneously. As is standard in dynamic games, we first derive the firms' values and investment strategies associated with these three possibilities, taking the roles of leader and laggard as given. We then endogeneize the firms' roles and construct equilibrium investment strategies by comparing the value of investing as a leader, the value of waiting and being a laggard, and the value of investing simultaneously. As we will show, in the case of an asymmetric game in which firms differ in their hazard rates, $h_i \neq h_j$, the more

[3] By assuming an exogenous pricing kernel exogenously, we implicitly rule out the possibility that any firm's innovation activity alters the state prices in the economy.

[4] Note that, because the discovery occurs according to the Poisson distribution with hazard rate $h_i$, its arrival time $\tau$ has a negative exponential distribution, $Pr(\tau < t) = 1 - e^{-h_i t}$. Hence, $e^{-h_i t} h_i$ in condition (IA.2) is the density of a negative exponential random variable, i.e., it represents the probability at $t$ of making the discovery in the next $dt$ instant, conditional on no discovery occurring until time $t$.

[5] In particular, condition (IA.2) rules out equilibria in which firms invest when $x(t)$ decreases, as in the "recession-induced construction booms" studied in Grenadier (1996).

innovative firm (high hazard rate) endogenously takes the leadership role, unless firms invest simultaneously.[6]

**Laggard**

The problem of the laggard is to determine the optimal time to invest, given that its opponent has already invested. Let firm $i$ be the laggard and firm $j$ be the leader, and denote by $x_i^G$ the laggard's investment threshold. The value of firm $i$ is the solution of the optimal stopping time problem:

$$V_i^G(x) = \sup_{\tau_i^G} E\left[e^{-(r+h_j)\tau_i^G}\left(\int_{\tau_i^G}^{\infty} e^{-(r+h_i+h_j)(t-\tau_i^G)} h_i x(t) dt - K\right)\right], \quad i \neq j, \tag{IA.3}$$

where $\tau_i^G = \inf\{t > 0 : x(t) \geq x_i^G\}$ is the stopping time, and the expectation is taken under the risk-neutral measure. Since the discovery by firm $j$ occurs with a Poisson arrival rate $h_j$, $e^{-h_j \tau_i^G}$ in (IA.3) represents the probability that firm $j$ does not make the discovery in the time period $[0, \tau_i^G]$. Moreover, as the discoveries are independent, $e^{-(h_i+h_j)(t-\tau_i^G)} h_i$ is the probability that firm $i$ makes the discovery in the next $dt$ instant, given that neither firm was successful before time $t$.

Notice that the hazard rate of the leader $h_j$ increases the discount rate used to discount future profits of the laggard in (IA.3). This is common in R&D models involving a constant Poisson arrival process (e.g., Loury (1979)). Furthermore, the expected profits $h_i x(t)$ for firm $i$ upon investing are discounted at a rate that includes the hazard rate of both firms, $r + h_i + h_j$. The following proposition characterizes the solution to the laggard's stopping time problem.

**Proposition IA.1.** *Conditional on firm $j$ having already invested in innovation, the optimal strategy of firm $i$ is to invest at the threshold*

$$x_i^G = \frac{\phi_j}{\phi_j - 1} \frac{h_i + h_j + \delta}{h_i} K, \quad i \neq j, \tag{IA.4}$$

[6] In a symmetric innovation game, one needs a selection mechanism to determine the role of firms in the game. This selection mechanism requires an enlargement of the strategy space to allow for mixed strategies in continuous time (see Fudenberg and Tirole (1985)).

*where*

$$\phi_j = \frac{1}{2} - \frac{r-\delta}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{r-\delta}{\sigma^2}\right)^2 + \frac{2(r+h_j)}{\sigma^2}} > 1. \tag{IA.5}$$

*The value of the Laggard is*

$$V_i^G(x) = \begin{cases} \left(\frac{x}{x_i^G}\right)^{\phi_j} \left[\frac{h_i x_i^G}{h_i+h_j+\delta} - K\right] & \text{if } x < x_i^G \\ \frac{h_i x}{h_i+h_j+\delta} - K & \text{if } x \geq x_i^G \end{cases}. \tag{IA.6}$$

The proposition highlights several aspects of the interactions between the leader's and laggard's decisions. First, firm $i$'s NPV, $\frac{h_i x}{h_i+h_j+\delta} - K$, is decreasing in firm $j$'s hazard rate, $h_j$. Second, as shown in Lemma IA.2 below (Section IA.6), firm $i$'s investment threshold $x_i^G$ is increasing in $h_j$, i.e., the more innovative the leader (firm $j$) is, the later the laggard (firm $i$) invests. Finally, the value of the option to wait, determined via the price $(x/x_i^G)^{\phi_j}$ of the Arrow-Debreu security—which pays one dollar when the process $x$ first hits the threshold $x_i^G$ (conditional on firm $j$ not succeeding before)—depends on the hazard rate $h_j$ of the leader. Figure IA.1 Panel A illustrates the results of the above proposition

**Leader**

We now determine the value of a firm conditional on investing as the leader and anticipating that the laggard will respond optimally according to Proposition IA.1. The value of firm $i$ when it invests as a leader is

$$V_i^L(x) = E\left[\int_0^{\tau_j^G} e^{-(r+h_i)t} h_i x(t)dt\right] + E\left[e^{-(r+h_i)\tau_j^G} \int_{\tau_j^G}^{\infty} e^{-(r+h_i+h_j)(t-\tau_j^G)} h_i x(t)dt\right] - K, \tag{IA.7}$$

where $\tau_j^G$ is the time at which the laggard (firm $j$) invests in innovation. The first term captures the expected profits that firm $i$ receives before firm $j$ invests, while the second term captures the expected profits it receives after firm $j$ invests. The next proposition characterizes the value of the leader $V_i^L(x)$.

**Proposition IA.2.** *Conditional on firm $j$ investing as a laggard at the threshold $x_j^G$ derived in Proposition IA.1, the value of the Leader at the time it invests is*

$$V_i^L(x) = \begin{cases} \frac{h_i x}{h_i+\delta} - \left(\frac{x}{x_j^G}\right)^{\phi_i} \left[\frac{h_i x_j^G}{h_i+\delta} - \frac{h_i x_j^G}{h_i+h_j+\delta}\right] - K & \text{if } x < x_j^G \\ \frac{h_i x}{h_i+h_j+\delta} - K & \text{if } x \geq x_j^G \end{cases}. \tag{IA.8}$$

When the laggard has not invested yet, $x < x_j^G$, the value of the leader in (IA.8) has three parts. The first part is the present value of a perpetuity with expected profits $h_i x$ and discount rate $h_i + \delta$. The second part in (IA.8) represents the loss of monopoly rights of the leader upon investment of the laggard. It helps to think of this part as having the value of a "short" position in $\left[\frac{h_i x_j^G}{h_i+\delta} - \frac{h_i x_j^G}{h_i+h_j+\delta}\right]$ options, each paying one dollar when $x$ first hits $x_j^G$. The third part is the fixed investment cost. Lemma IA.3 below (Section IA.6) shows that the value of the leader is increasing in its innovativeness. Figure IA.1 Panel B illustrates the results of the above proposition.

**Simultaneous investment**

A possible outcome of the investment timing game is simultaneous investment. Under the conditions discussed later in Proposition IA.4, it is possible to sustain equilibria in which the two firms agree to invest at the same threshold. This happens if the value of each firm from investing simultaneously dominates the value of investing as a leader. The following proposition characterizes the value of firm $i$ when both firms invest at a pre-specified threshold $x^C$.

**Proposition IA.3.** *The value of firm $i$ when both firms invest at a given threshold $x^C$ is*

$$V_i^C(x; x^C) = \begin{cases} \left(\frac{x}{x^C}\right)^{\phi_0} \left[\frac{h_i x^C}{h_i+h_j+\delta} - K\right] & \text{if } x < x^C \\ \frac{h_i x}{h_i+h_j+\delta} - K & \text{if } x \geq x^C \end{cases}, \quad i \neq j, \tag{IA.9}$$

*where*

$$\phi_0 = \frac{1}{2} - \frac{r-\delta}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{r-\delta}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} > 1. \tag{IA.10}$$

The optimal joint threshold $x_i^C$ from each firm's individual perspective is obtained by maximizing (IA.9) with respect to $x^C$, yielding

$$x_i^C = \frac{\phi_0}{\phi_0 - 1} \frac{h_i + h_j + \delta}{h_i} K, \quad i, j = 1, 2. \tag{IA.11}$$

Notice that if $h_i \neq h_j$, the two firms disagree on the optimal joint threshold. For example, if $h_1 > h_2$, then, from (IA.11), $x_1^C < x_2^C$. This implies that in an equilibrium with simultaneous investment and different levels of technological innovativeness, one of the two firms adopts a strategy that is not value maximizing, although, as discussed in Proposition IA.4, no profitable deviations exist.

## IA.2 Equilibrium investment strategies

Given the value of investing as the leader, the laggard, and simultaneously, we can now characterize the set of Markov perfect equilibria of the innovation game. Proposition IA.4 below shows that there are three types of equilibria: (i) sequential, (ii) preemptive, and (iii) simultaneous. To understand the structure of each type of equilibrium, let us assume, without loss of generality, that $h_1 > h_2$. From Lemma IA.3, this implies that $V_1^L(x) > V_2^L(x)$ and $V_1^G(x) > V_2^G(x)$, for all $x$.

When firm 2 has no incentive to become the leader, $V_2^G(x) > V_2^L(x)$, for all $x < x_2^G$, then the equilibrium is of the *sequential* type. Firm 1 acts as a "designated leader," investing at the threshold $x_1^D$, which it would have chosen if it had had the exclusive right to invest first (see equation (IA.36) in Section IA.6). Firm 2 optimally chooses the investment threshold $x_2^G$.[7]

When both firms have an incentive to become leaders, $V_i^L(x) > V_i^G(x)$, $i = 1, 2$, for some $x$, then the equilibrium is of the *preemptive* type, with firm 1 acting as the leader and firm 2 as the laggard. The preemptive threat of firm 2 (the less innovative) induces firm 1 to "retaliate" by investing at a lower threshold. The preemptive threat of firm 2 vanishes when firm 2 is indifferent about being the leader or being the laggard, which happens at the value $x_2^P$, defined as

$$x_2^P = \inf\{x : V_2^L(x) = V_2^G(x)\}. \tag{IA.12}$$

[7]Notice that, as shown in Lemma IA.4 in Section IA.6, if $h_1 > h_2$, there always exists a unique $\hat{x}$ such that $V_1^L(x) > V_1^G(x)$, for all $x \in [\hat{x}, x_2^G]$, while it is possible that $V_2^L(x) < V_2^G(x)$ for all $x$. This implies that the more innovative firm always has the incentive to be the leader and rules out the case in which neither firm has incentive to lead.

Proposition IA.4 below shows that at $x_2^P$, $V_1^L(x_2^P) > V_1^G(x_2^P)$, and therefore firm 1 invests at the threshold equal to the minimum between $x_2^P$ and the investment threshold $x_1^D$. In both of these cases, in a preemptive equilibrium, firm 2 optimally invests at the threshold $x_2^G$.

Finally, a *simultaneous* equilibrium can be sustained if $V_i^L(x) < V_i^C(x; x^C)$, for all $x$, $i = 1, 2$. These conditions ensure that there is no unilateral incentive to deviate from the strategy to invest simultaneously at $x^C$, and therefore an equilibrium involving a joint investment threshold can be sustained. There are infinitely many of these equilibria, depending on the pre-specified threshold $x^C$. As in Fudenberg and Tirole (1985) and Weeds (2002), we reduce the multiplicity of these equilibria by focusing on the Pareto-dominating one, which, as Proposition IA.4 below shows, is the equilibrium with the optimal joint investment threshold $x_1^C$ for firm 1, derived in equation (IA.11). In fact, if $h_1 > h_2$, then the only sustainable joint investment threshold is $x_1^C < x_2^C$, because, under the condition (IA.2) for $x(0)$, firm 1 would always have an incentive to deviate from the alternative joint threshold $x_2^C$ which maximizes firm 2's value. Importantly, the existence of simultaneous equilibria does not rule out the existence of both preemptive and sequential equilibria, and hence, in principle, there can be multiple equilibria when $V_i^L(x) < V_i^C(x; x^C)$, for all $x$, $i = 1, 2$.

The following proposition describes the regions of technological efficiencies $h_1$ and $h_2$, for which each of the three types of equilibria described above occurs.

**Proposition IA.4.** *Assume $x(0)$ satisfies condition (IA.2), and let $h_1 > h_2$. Let $x_2^G$, $x_2^P$ be defined, respectively, by (IA.4) and (IA.12) ,and let $x_1^D$ be the threshold defined implicitly by condition (IA.36) of Lemma IA.5 of Section IA.6. Then, for every $h_1$, there exists two thresholds for $h_2$, $J(h_1)$ and $S(h_1)$, such that the Markov perfect equilibrium is:*

1. *Sequential, if $h_2 < S(h_1)$, with firm 1 investing at the threshold $x_1^D$ and firm 2 investing at the threshold $x_2^G > x_1^D$.*

2. *Preemptive, if $h_2 > S(h_1)$, with firm 1 investing at the threshold $x_1^P = \min\{x_1^D, x_2^P\}$ and firm 2 investing at the threshold $x_2^G > x_1^P$.*

3. *Simultaneous, if $h_2 > J(h_1)$, with both firms investing at the threshold $x_1^C$.*

We refer to equilibria of the preemptive and sequential types as *leader-laggard equilibria.* Figure IA.2 depicts the regions of different types of equilibria from Proposition IA.4 in the

$(h_1, h_2)$ plane. The solid line is the threshold $J(h_1)$, and the dash-dotted line is the threshold $S(h_1)$. Since we assume $h_1 > h_2$, the relevant region in Figure IA.2 is the shaded area below the 45-degree line.

Figure IA.2 Panel A (Panel B) depicts the case when the volatility of the process $x$ is low (high). Simultaneous equilibria are more likely to occur when volatility is high. Intuitively, the higher the volatility, the more valuable the option to wait is, and the less incentive the leader has to preempt by investing early. For sufficiently low levels of volatility (Panel A), the threshold $J(h_1)$ is always above the 45-degree line, and simultaneous equilibria do not occur. From the threshold $S(h_1)$, we infer that when $h_2$ is sufficiently smaller than $h_1$, firm 2 has no interest in becoming the leader, and hence the equilibria are sequential. As $h_2$ increases and crosses the threshold $S(h_1)$, firm 2 has an incentive to become the leader, and preemptive equilibria ensue. Finally, as discussed in Weeds (2002), note that on the 45-degree line, when $h_1 = h_2$, there are only preemptive or simultaneous equilibria.

## IA.3 Equilibrium firm values

Given the characterization of the equilibria in Proposition IA.4, we derive firm values and betas for each equilibrium type.

**Proposition IA.5.** *Let $V_i^G(x)$ and $V_i^L(x)$, $i = 1, 2$ be given by Propositions IA.1 and IA.2, respectively, and let $h_1 > h_2$.*

1. *In a leader-laggard (sequential or preemptive) equilibrium, the value of the leader, $V_1^{\text{PS}}$, and of the laggard, $V_2^{\text{PS}}$, are:*

   (a) *If $x < x_1^P$,*

$$V_1^{\text{PS}}(x) = \left(\frac{x}{x_1^P}\right)^{\phi_0} V_1^L(x_1^P) \quad \text{and} \quad V_2^{\text{PS}}(x) = \left(\frac{x}{x_1^P}\right)^{\phi_0} V_2^G(x_1^P), \tag{IA.13}$$

   *where $\phi_0$ is given in (IA.10).*

*(b) If* $x_1^P < x < x_2^G$,

$$V_1^{\mathrm{PS}}(x) = \underbrace{\frac{h_1 x}{h_1+\delta}}_{\equiv a(x)} - \underbrace{\left(\frac{x}{x_2^G}\right)^{\phi_1}\left[\frac{h_1 x_2^G}{h_1+\delta} - \frac{h_1 x_2^G}{h_1+h_2+\delta}\right]}_{\equiv b(x)}, \tag{IA.14}$$

$$V_2^{\mathrm{PS}}(x) = \left(\frac{x}{x_2^G}\right)^{\phi_1}\left[\frac{h_2 x_2^G}{h_1+h_2+\delta} - K\right]. \tag{IA.15}$$

*(c) If* $x > x_2^G$

$$V_1^{\mathrm{PS}}(x) = \frac{h_1 x}{h_1+h_2+\delta} \quad \text{and} \quad V_2^{\mathrm{PS}}(x) = \frac{h_2 x}{h_1+h_2+\delta}, \tag{IA.16}$$

*where* $\phi_1$ *is given in (IA.5),* $x_1^P = \min\{x_1^D, x_2^P\}$ *in a preemptive equilibrium, and* $x_1^P = x_1^D$ *in a sequential equilibrium, with* $x_2^P$ *given by (IA.12) and* $x_1^D$ *given implicitly by condition (IA.36) in Lemma IA.5 of Section IA.6.*

2. *In a simultaneous equilibrium, the value of each firm,* $V_i^{\mathrm{S}}(x)$, *is*

$$V_i^{\mathrm{S}}(x) = \begin{cases} \left(\frac{x}{x_1^C}\right)^{\phi_0}\left[\frac{h_i x_1^C}{h_1+h_2+\delta} - K\right] & \text{if } x < x_1^C \\ \frac{h_i x}{h_1+h_2+\delta} & \text{if } x > x_1^C \end{cases}, \quad i = 1, 2, \tag{IA.17}$$

*where* $x_1^C$ *is defined in (IA.11).*

Note that when the investment takes place, the values of both firms increase discontinuously. This happens because at the time of investing, the option to invest is converted into assets in place, and we assume that the investment cost $K$ is financed through an influx of new equity capital.[8]

## IA.4 Innovation arrival rates and firms' betas

In this subsection, we analyze the effect of a change in the innovation hazard rates, $h_1$ and $h_2$, on firms' betas. Figure IA.3 shows the betas for the leader and the laggard in leader-laggard equilibria, derived in Proposition 1. Panel A analyzes the case of the laggard "catching up," i.e., the hazard rate of the leader is set to $h_1 = 0.1$, and we consider three levels of $h_2 = \{0.06, 0.08, 0.09\}$. Panel B analyzes the case of the leader "pulling ahead," i.e., the hazard

[8]If, for example, $K$ were financed through debt, the firm values would need to be adjusted to incorporate the present value of the liability cash flows.

rate of the laggard is set to $h_2 = 0.1$, and we consider three levels of $h_1 = \{0.11, 0.13, 0.15\}$. In both panels, the bottom part of the graph plots the beta of the leader, and the top part plots the beta of the laggard (equation (5) in Proposition 1 of the main text). Figure IA.3 highlights that as $h_1$ and $h_2$ change, the investment thresholds change. Specifically, both $x_1^P$ and $x_2^G$ (i) decrease with $h_2$, for a given level of $h_1$, and (ii) increase with $h_1$, for a given level of $h_2$.

Figure IA.3 Panel A shows that, as $h_2$ increases, the upper bound of the beta of the laggard is unaffected, while the lower bound of the leader's beta decreases. To understand the behavior of the leader's beta in Panel A, for $x \in [x_1^P, x_2^G]$, note that, from equation (IA.14) of the main text, the leader's value $V_1^{\text{PS}}(x)$ is a portfolio of assets in place, $a(x)$, and a short position in the option to innovate, $b(x)$. From equation (5), the leader's beta for $x \in [x_1^P, x_2^G]$ is given by $1 - \omega(x)(\phi_1 - 1)$, where $\omega(x) = b(x)/(a(x) - b(x)) > 0$ stands for the fraction of the value $V_1^{\text{PS}}$ represented by the innovation option. As $h_2$ increases, the laggard's innovation option $b(x)$ becomes more valuable, and, because $a(x)$ does not depend on $h_2$, $\omega(x)$ increases. This, and the fact that a change in $h_2$ does not affect $\phi_1$, imply that an increase in $h_2$ reduces the beta of the leader.

Figure IA.3 Panel B shows that, as $h_1$ increases, the upper bound of the laggard's beta increases. According to Proposition 1, in leader-laggard equilibria, the beta of the laggard increases with the innovativeness of the leader, because this makes the laggard's option to innovate more sensitive to the underlying process $x$. This effect can be seen by inspecting the expression for the laggard's beta $\beta_2^{\text{PS}}$ in equation (5). From equation (IA.5), we see that $\phi_1$ increases with the leader's hazard rate $h_1$, and hence $\beta_2^{\text{PS}}$ increases with $h_1$, as well.

The implications of a change in $h_1$ on the beta of the leader in Panel B are more subtle, because a change in $h_1$ has both direct and indirect effects. An increase in $h_1$, keeping $\omega(x)$ in equation (5) fixed, implies an increase in $\phi_1$ and hence a decline in $\beta_1^{\text{PS}}$ (direct effect). However, as $h_1$ increases, $\omega(x)$ changes as well. In particular, a higher $h_1$ reduces the value of the innovation option $b(x)$ held by the laggard and increases the value $a(x)$ of the leader's expected discounted profits from the patent, causing $a(x) - b(x)$ to increase and $\omega(x)$ to decrease. A decrease in $\omega(x)$ causes an increase in $\beta_1^{\text{PS}}$ (indirect effect). It is not clear, a priori, which of the two effects prevails.

In summary, because changes in innovation efficiencies affect equilibrium investment thresholds and have mixed implications for the leader's beta, comparative statics on innovation efficiencies do not lead directly to testable predictions.

## IA.5 Betas in a sequential equilibrium of an $N$-firm race

The construction of the full set of equilibria in an $N$-firm game provides little guidance for our empirical analysis, because the solution involves identifying all the possible subsets of firms investing in either simultaneous or preemptive/sequential equilibria.[9] Note, however, that in every possible equilibrium configuration, the mechanism described above for the two-firm case still operates: In every equilibria in which at least two firms invest at different dates, the beta of the first firm to invest is weakly lower than that of the firm to invest next. Since firms' betas are the same in simultaneous equilibria, we find that under no equilibrium configurations in an $N$-firm game are the predictions of Proposition 1 overturned. Proposition IA.6 shows the patterns of beta in an $N$-firm game in which firms invest sequentially.

**Proposition IA.6.** *Suppose $N$ firms have hazard rates $h_1 > h_2 > \ldots > h_N$, and $x_1 < x_2 < \ldots < x_N$ are the investment thresholds in a leader-laggard equilibrium of an $N$-firm game. The beta of firm $m = 1, \ldots, N$ is*

$$\beta_m^{\text{PS}}(x) = \begin{cases} \phi_0 & \text{if } x < x_1, \\ \begin{cases} \phi_n & \text{for } n < m \\ 1 - \omega_{m,n}(x)(\phi_n - 1) & \text{for } n \geq m \end{cases} & \text{if } x_{n-1} < x < x_n,\ n = 2, \ldots, N-1, \\ 1 & \text{if } x > x_N, \end{cases} \tag{IA.18}$$

*where $\omega_{m,n}(x) > 0$ is defined in (IA.56), $\phi_0$ is defined in (IA.10), and*

$$\phi_n = \frac{1}{2} - \frac{r-\delta}{\sigma^2}\sqrt{\left(\frac{1}{2} - \frac{r-\delta}{\sigma^2}\right)^2 + \frac{2\left(r + \sum_{i=1}^{n} h_i\right)}{\sigma^2}} > 1,\ n = 1, \ldots, N-1. \tag{IA.19}$$

Equation (IA.18) is a generalization of equation (5) in the main text. The proposition implies that, for any pair $(i, j)$ of firms in leader-laggard equilibria with $h_i > h_j$, (i) the two firms have the same beta, $\beta_i^{\text{PS}} = \beta_j^{\text{PS}}$, or (ii) the more innovative firm has a strictly lower beta $\beta_i^{\text{PS}} < \beta_j^{\text{PS}}$.

[9] For example, Bouis, Huisman, and Kort (2009) characterize equilibria investment thresholds in an oligopoly product market game without technological uncertainty.

This holds for any given value of $x$. Furthermore, from equation (IA.19), the maximum beta across all possible realizations of $x$ of a firm with innovativeness $h_n$ is increasing in $n$.

## IA.6 Proofs

The following lemma contains two preliminary results that will be used extensively in the sequel.

**Lemma IA.1.** *Let $x(t)$ be the stochastic process in (3) with $\mu < r$ and $\tau = \inf\{t > 0 : x(t) > x^*\}$, $x^* > x(0)$. Then*

$$E\left[e^{-r\tau}\right] = \left(\frac{x(0)}{x^*}\right)^{\phi} \text{ and} \tag{IA.20}$$

$$E\left[\int_0^{\tau} e^{-rt} x(t) dt\right] = \frac{x(0)}{r-\mu}\left[1 - \left(\frac{x(0)}{x^*}\right)^{\phi-1}\right], \tag{IA.21}$$

*where $\phi = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} > 1$ is the positive root of the quadratic equation*

$$\frac{1}{2}\sigma^2\phi(\phi-1) + \mu\phi - r = 0. \tag{IA.22}$$

**Proof:** The proof is standard and can be found, for example, in Harrison (1985), Chapter 3, or Dixit and Pindyck (1994), pp. 315–316. ∎

### Proof of Proposition IA.1

By the law of iterated expectations, we can express (IA.3) for $x < x_i^G$ as

$$\begin{aligned} V_i^G(x) &= \sup_{\tau_i^G} E\left[e^{-(r+h_j)\tau_i^G} E_{\tau_i^G}\left[\int_{\tau_i^G}^{\infty} e^{-(r+h_i+h_j)(t-\tau_i^G)} h_i x(t) dt - K\right]\right],\ x(0) = x, \\ &= \sup_{x_i^G} \left(\frac{x}{x_i^G}\right)^{\phi_j} \left(\frac{h_i x_i^G}{h_i + h_j + \delta} - K\right), \end{aligned} \tag{IA.23}$$

where the last equality follows from (IA.20) in Lemma IA.1. Maximizing with respect to $x_i^G$ yields (IA.4) and (IA.6). ∎

## Proof of Proposition IA.2

From (IA.21) in Lemma IA.1, the leader's value (IA.7) for $x < x_i^G$ can be written as

$$V_j^L(x) = \frac{h_j x}{h_j + \delta}\left[1 - \left(\frac{x}{x_i^G}\right)^{\phi_j - 1}\right] + \left(\frac{x}{x_i^G}\right)^{\phi_j} \frac{h_j x_i^G}{h_i + h_j + \delta} - K, \qquad \text{(IA.24)}$$

from which (IA.8) follows. ∎

In the following lemma we collect useful properties of the investment thresholds and value functions.

**Lemma IA.2.**[10] *Let $x_i^G$ be defined as in (IA.4), $i = 1, 2$. If $h_1 > h_2$ then $x_1^G < x_2^G$.*

**Proof:** Let us set $h_2 = h > 0$ and $h_1 = (1+\alpha)h$, $\alpha \geq 0$. If $\alpha = 0$, $x_1^G = x_2^G$. Hence, to prove the lemma we need to show that $x_1^G \leq x_2^G$ for $\alpha \geq 0$.

From (IA.5) we define the function

$$\phi(\alpha) = \gamma + \sqrt{\gamma^2 + k(\alpha)} > 1, \qquad \text{(IA.25)}$$

where $\gamma \equiv \frac{1}{2} - \frac{r-\delta}{\sigma^2}$, $k(\alpha) = 2\xi(r + h(1+\alpha))$, and $\xi \equiv \frac{1}{\sigma^2}$. By definition, $\phi(\alpha)$ is the positive solution of the quadratic equation

$$\frac{1}{2}\sigma^2\phi(\phi - 1) + (r - \delta)\phi - (r + h(1+\alpha)) = 0. \qquad \text{(IA.26)}$$

The thresholds $x_1^G$ and $x_2^G$ in (IA.4) can then be expressed as follows:

$$x_1^G = \frac{\phi(0)}{\phi(0) - 1}\frac{h(2+\alpha) + \delta}{h(1+\alpha)} \quad \text{and} \quad x_2^G = \frac{\phi(\alpha)}{\phi(\alpha) - 1}\frac{h(2+\alpha) + \delta}{h}. \qquad \text{(IA.27)}$$

Therefore, to show that $x_1^G \leq x_2^G$, it is therefore sufficient to show that the function

$$f(\alpha) = \frac{\phi(\alpha)}{\phi(\alpha) - 1}(1+\alpha), \quad \alpha \geq 0 \qquad \text{(IA.28)}$$

is increasing in $\alpha$. Taking the first derivative with respect to $\alpha$, we get

$$f'(\alpha) = \frac{-\phi'(\alpha)(1+\alpha) + \phi(\alpha)(\phi(\alpha) - 1)}{(\phi(\alpha) - 1)^2}. \qquad \text{(IA.29)}$$

[10] We thank Alberto Romero for help with the proof of this lemma.

Because $\phi(\alpha)$ is the positive root of quadratic equation (IA.22), with $\mu$ replaced by $r-\delta$ and $r$ replaced by $r+h(1+\alpha)$, we can write $\phi(\alpha)(\phi(\alpha)-1) = k(\alpha)+2(\gamma-1)$. Moreover, $\phi'(\alpha) = \frac{\xi h}{\phi(\alpha)-\gamma}$. Hence, $f'(\alpha) \geq 0$ if and only if

$$\xi h(1+\alpha) \leq UB(\gamma), \tag{IA.30}$$

where $UB(\gamma) \equiv (k(\alpha)+(2\gamma-1)(\gamma+\sqrt{\gamma^2+k(\alpha)}))\sqrt{\gamma^2+k(\alpha)}$. We now show that $\inf_\gamma UB(\gamma) \geq \xi h(1+\alpha)$, thus proving that $x_1^G \leq x_2^G$. Simple algebraic manipulation allows us to rewrite $UB(\gamma)$ as

$$UB(\gamma) = \left(\sqrt{\gamma^2+k(\alpha)}+\gamma-1\right)\left(\sqrt{\gamma^2+k(\alpha)}\right)\left(\sqrt{\gamma^2+k(\alpha)}+\gamma\right) > 0. \tag{IA.31}$$

Taking the derivative of $UB(\gamma)$ with respect to $\gamma$ we obtain

$$UB'(\gamma) = \left(\gamma+\sqrt{\gamma^2+k(\alpha)}\right)^2 + \frac{UB(\gamma)}{\gamma^2+k(\alpha)}\left(\gamma+\sqrt{\gamma^2+k(\alpha)}\right) > 0. \tag{IA.32}$$

Because $\gamma \equiv \frac{1}{2} - \frac{r-\delta}{\sigma^2}$ and $\delta \geq r$, for any $r$, $\gamma$ is minimum for $\delta = 0$. Let $\hat{\gamma} = \inf_\delta \gamma = \frac{1}{2} - \frac{r}{\sigma^2}$. Hence, for every $0 \leq \delta \leq r$, $UB(\hat{\gamma}) < UB(\gamma)$. From (IA.31), using the definition of $\hat{\gamma}$ we obtain

$$UB(\hat{\gamma}) = \left(\sqrt{(1-\hat{\gamma})^2+c(\alpha)}+\hat{\gamma}-1\right)\left(\sqrt{(1-\hat{\gamma})^2+c(\alpha)}\right)\left(\sqrt{(1-\hat{\gamma})^2+c(\alpha)}+\hat{\gamma}\right), \tag{IA.33}$$

where $c(\alpha) \equiv 2\xi(1+\alpha)$. From (IA.32), $UB'(\hat{\gamma}) > 0$, and, from the definition of $\hat{\gamma} = \frac{1}{2} - \frac{r}{\sigma^2}$, we obtain $\frac{\partial UB(\hat{\gamma})}{\partial r} = \frac{\partial UB(\hat{\gamma})}{\partial \hat{\gamma}} \frac{\partial \hat{\gamma}}{\partial r} < 0$. The lowest bound of $UB(\hat{\gamma})$ is thus obtained for $r \to \infty$. Direct computation shows that $\lim_{\hat{\gamma}\to\infty} = \frac{c(\alpha)}{2} = \xi h(1+\alpha)$, thus verifying condition (IA.30). ∎

**Lemma IA.3.** *Let $V_i^G(x)$ and $V_i^L(x)$, $i = 1, 2$, be defined as in (IA.6) and (IA.7), respectively. Then, if $h_1 > h_2$: (i) $V_1^G(x) > V_2^G(x)$, and (ii) $V_1^L(x) > V_2^L(x)$ for all $x$.*

**Proof:** Part (i) follows immediately from the convexity of $V_i^G(x)$, $i = 1, 2$, and Lemma IA.2. To prove (ii), from the definition of $V_i^L(x)$, $i = 1, 2$, in (IA.7), and the fact that $x_1^G > x_2^G$ (Lemma IA.2), we see that $V_1^L(x) > V_2^L(x)$ for $x > x_2^G$. For $x \in [x_1^G, x_2^G]$ we show that the difference

$$\begin{aligned} V_1^L(x) - V_2^L(x) &= x\left(\frac{h_1}{h_1+\delta} - \frac{h_2}{h_1+h_2+\delta}\right) - \left(\frac{x}{x_2^G}\right)^{\phi_1} h_1 x_2^G \left(\frac{1}{h_1+\delta} - \frac{1}{h_1+h_2+\delta}\right) \\ &> h_1 x\left(\frac{1}{h_1+\delta} - \frac{1}{h_1+h_2+\delta}\right) - \left(\frac{x}{x_2^G}\right)^{\phi_1} h_1 x_2^G \left(\frac{1}{h_1+\delta} - \frac{1}{h_1+h_2+\delta}\right) \\ &= \left(\frac{1}{h_1+\delta} - \frac{1}{h_1+h_2+\delta}\right)\left(\frac{x\,(x_2^G)^{\phi_1} - x^{\phi_1} x_2^G}{(x_2^G)^{\phi_1}}\right) > 0, \end{aligned} \tag{IA.34}$$

where the first inequality follows from $h_1 > h_2$, and the last inequality follows from $x < x_2^G$ and $\phi_1 > 1$. For $x \in [0, x_1^G]$ we note that: (a) $V_1^L(x_1^G) > V_2^L(x_1^G)$ by (IA.34); (b) $\frac{\partial V_1^L(x)}{\partial x}\Big|_{x=0} = \frac{h_1}{h1+h_2+\delta}$ and $\frac{\partial V_2^L(x)}{\partial x}\Big|_{x=0} = \frac{h_2}{h1+h_2+\delta}$, thus $\frac{\partial V_1^L(x)}{\partial x}\Big|_{x=0} > \frac{\partial V_2^L(x)}{\partial x}\Big|_{x=0}$; and (c) $V_1^L(x)$ and $V_2^G(x)$ are strictly concave in $x \in [0, x_1^G]$. These facts imply that $V_1^L(x)$ and $V_2^L(x)$ cannot cross for any $x < x_1^G$, and therefore $V_1^L(x) > V_2^L(x)$. ∎

**Lemma IA.4.** *If $h_1 > h_2$, then: (i) there always exists a unique $\hat{x}$ such that $V_1^L(x) > V_1^G(x)$ for all $x \in [\hat{x}, x_2^G]$, and (ii) there exists an $h_2$ such that $V_2^L(x) < V_2^G(x)$ for all $x \in [0, x_2^G]$.*

**Proof:** (i) Let $D_1(x) = V_1^L(x) - V_1^G(x)$, $x \in [0, x_2^G]$. Note that $D_1(0) = -K$, $D_1(x_2^G) = 0$, $\frac{\partial V_1^L(x)}{\partial x}\Big|_{x=x_2^G} = \left(\frac{1}{h_1+\delta} - \frac{1}{h_1+h_2+\delta}\right) h_1(1-\phi_1) < 0$. Hence, $D_1(x)$ has a unique root $\hat{x} \in [0, x_2^G]$. (ii) Let $D_2(x) = V_2^L(x) - V_2^G(x)$, $x \in [0, x_1^G]$. Note that if $h_1 = h_2$, then $x_1^G = x_2^G$, and, by (i), there exists an $\hat{x} \in [0, x_1^G]$ such that $D_2(x) > 0$ for $x \in [\hat{x}, x_1^G]$. Moreover, $\lim_{h_2 \to 0} = -K$ for $x < x_2^G$. Hence, there exists an $h_2 > 0$ such that $V_2^L(x) < V_2^G(x)$ for all $x \in [0, x_2^G]$. ∎

**Lemma IA.5.** *Suppose the roles of the firms are preassigned and firm $j$ is the designated leader who cannot be preempted by firm $i$. The value of firm $j$ is*

$$V_j^D(x) = \begin{cases} \left(\frac{x}{x_j^D}\right)^{\phi_0} V_j^L(x_j^D) & \text{if } x < x_j^D \\ V_j^L(x) & \text{if } x \geq x_j^D \end{cases}, \tag{IA.35}$$

*where $V_j^L(x)$ is defined in Proposition IA.2, and $x_j^D$ is implicitly determined by the smooth pasting condition*

$$(\phi_j - \phi_0)\frac{h_j}{h_j+\delta}\frac{h_i x_i^G}{h_i+h_j+\delta}\left(\frac{x_j^D}{x_i^G}\right)^{\phi_j} + (\phi_0 - 1)\frac{h_j x_j^D}{h_j+\delta} - \phi_0 K = 0, \tag{IA.36}$$

*with $\phi_0$ given in equation (IA.10), $\phi_j$ given in equation (IA.5), and $x_i^G$ given in equation (IA.4).*

**Proof:** The optimal value of firm $j$ as the designated leader $V_j^D(x)$ is given by

$$V_j^D(x) = \sup_{\tau_j^D} E\left[e^{-r\tau_j^D} V_j^L(x_j^D)\right], \tag{IA.37}$$

where $\tau_j^D = \inf\{t > 0 : x(t) \geq x_j^D\}$. From Lemma IA.1, the value of the designated leader (IA.37) for $x < x_i^D$ can be written as

$$V_j^D(x) = \sup_{x_i^D} \left[\left(\frac{x}{x_j^D}\right)^{\phi_0} V_j^L(x_j^D)\right]. \tag{IA.38}$$

Maximizing with respect to $x_i^D$ yields (IA.37), where $x_i^D$ is implicitly defined by (IA.36). Because $V_i^L(x)$ is increasing and concave for $x \in [0, x_j^G]$, it follows that $x_i^D <_j^G$. ∎

## Proof of Proposition IA.3

The value of firm $i$ when both firms invest at a pre-specified threshold $x^C$ is given by

$$V_i(x; x^C) = E\left[e^{-r\tau^C}\left(\int_{\tau^C}^{\infty} e^{-(r+h_i+h_j)(t-\tau^C)} h_i x(t) dt - K\right)\right], \quad i = 1, 2, \tag{IA.39}$$

where $\tau^C = \inf\{t > 0 : x(t) \geq x^C\}$. The proposition follows immediately from the law of iterated expectations and Lemma IA.1. ∎

## Proof of Proposition IA.4

From Lemma IA.3, $h_1 \geq h_2$ implies $V_1^L(x) \geq V_2^L(x)$, $V_1^C(x; x^C) > V_2^C(x; x^C)$, and $x_1^C \leq x_2^C$. A simultaneous equilibrium can only occur when $V_1^C(x; x_1^C) > V_1^L(x)$ and $V_2^C(x; x_1^C) > V_2^L(x)$ for all $x$. The only sustainable joint investment threshold is $x_1^C$, because, given that $x(0) < x_1^C$, firm 1 will always have incentive to deviate from the alternative joint threshold $x_2^C$ that maximizes firm 2's joint value. Because $V_1^L(x)$ is concave and decreasing in $h_2$ in $x \in [0, x_2^G]$, and $V_1^C(x)$ is convex, for every $h_1$ there exists a pair $(x^*, h_2^*)$ such that $V_1^L(x^*) = V_1^C(x^*; x_1^C)$ and $\frac{\partial V_1^L(x)}{\partial x} = \frac{\partial V_1^C(x; x_1^C)}{\partial x}|_{x=x^*}$. Let $J(h_1) = h_2^*$. For $h_2 > J(h_1)$, $V_1^L(x^*) < V_1^C(x^*; x_1^C)$ and a simultaneous equilibrium is possible. For $h_2 < J(h_1)$, $V_1^L(x^*) > V_1^C(x^*; x_1^C)$ and no simultaneous equilibria are possible. If $J(h_1) > h_1$ then no simultaneous equilibria are possible when $h_1 \geq h_2$.

Similarly, for every $h_1$ there exists a pair $(x^{**}, h_2^{**})$ such that $V_2^L(x^{**}) = V_2^C(x^{**}; x_1^C)$ and $\frac{\partial V_2^L(x)}{\partial x} = \frac{\partial V_2^C(x;x_1^C)}{\partial x}|_{x=x^{**}}$. Let $\widehat{J}(h_1) = h_2^{**}$. For $h_2 > \widehat{J}(h_1)$, $V_2^L(x) > V_2^C(x; x_1^C)$ for some $x$ and a simultaneous equilibrium is not sustainable. For $h_2 < \widehat{J}(h_1)$, $V_2^L(x) < V_2^C(x; x_1^C)$ for all $x$ and simultaneous equilibria are possible. Furthermore, if $J(h_1) < h_1$, then there exists a unique $\tilde{h}_1$ such that $\widehat{J}(h_1) > h_1 > J(h_1)$ for all $h_1 < \tilde{h}_1$ and $J(\tilde{h}_1) = \widehat{J}(\tilde{h}_1) = \tilde{h}_1$. Hence, if $h_1 > h_2$, a simultaneous equilibrium can emerge only if $h_2 > J(h_1)$ and $h_1 \leq \tilde{h}_1$.

A sequential equilibrium emerges if $V_1^C(x; x_1^C) < V_1^L(x)$ for some $x$ and $V_2^L(x) < V_2^G(x)$ for all $x$. Because $V_2^L(x)$ is concave and increasing in $h_2$, and $V_2^G(x)$ is convex in $x \in [0, x_1^G]$, for every $h_1$ there exists a pair $(x', h_2')$ such that $V_2^L(x') = V_2^G(x')$ and $\frac{\partial V_2^L(x)}{\partial x} = \frac{\partial V_2^G(x)}{\partial x}|_{x=x'}$. Let $S(h_1) = h_2'$. For $h_2 < S(h_1)$, $V_2^L(x) < V_2^G(x)$ for all $x$, and the equilibrium is of the sequential type. For $h_2 > S(h_1)$, $V_2^L(x') > V_2^G(x')$ and so no sequential equilibrium is possible. In the last case, firm 2 will attempt to preempt firm 1, as long as $V_2^L(x) = V_2^G(x)$. Let $x_2^P = \inf\{x \in [0, x_1^G] : V_2^L(x) = V_2^G(x)\}$. Then, if $h_2 > S(h_1)$, firm 2 will try to preempt firm 1 until $x \geq x_2^P$. Because $h_1 > h_2$, by Lemma IA.3, $V_1^L(x_2^P) - V_1^G(x_2^P) > 0$. Hence, the optimal response of firm 1 is to "$\epsilon$-preempt" firm 2 and invest at $x_1^P = \min\{x_2^P - \epsilon, x_1^D\}$, where $\epsilon > 0$ and $x_1^D$ is the optimal investment threshold of firm 1 as a designated leader, defined in (IA.36) of Lemma IA.5.

The two thresholds $J(h_1) < h_1$ and $S(h_1) < h_1$ partition the space $(h_1, h_2)$, $h_1 > h_2$, into four regions, each characterized by a different equilibrium.

1. *Region 1:* $h_2 > J(h_1)$ and $h_2 > S(h_1)$. Two types of equilibria: Simultaneous and preemptive.
2. *Region 2:* $h_2 > J(h_1)$ and $h_2 < S(h_1)$. Two types of equilibria: Simultaneous and sequential.
3. *Region 3:* $h_2 < J(h_1)$ and $h_2 > S(h_1)$. Unique preemptive equilibrium.
4. *Region 4:* $h_2 < J(h_1)$ and $h_2 < S(h_1)$. Unique sequential equilibrium. ∎

## Proof of Proposition IA.5

The firms' value in the case of preemptive or sequential equilibrium follows directly from Propositions IA.1 and IA.2, while the firms' value in the case of simultaneous equilibria follows from Proposition IA.3. ∎

**Proof of Proposition IA.6**

The proof is by induction. We consider the case of three firms first and then generalize to the case of $N$ firms. Let $\tau_1 < \tau_2 < \tau_3$ be the investment times of firms 1, 2, and 3, respectively, corresponding to the thresholds $x_1 < x_2 < x_3$. Following the derivation of leader's and laggard's payoff in Section IA.1, the value of firm 1 at threshold $x_1$ is given by

$$\begin{aligned} V_1(x_1) &= E\left[\int_0^{\tau_2} e^{-(r+h_1)t} h_1 x(t)dt + e^{-(r+h_1)\tau_2}\left(\int_{\tau_2}^{\tau_3} e^{-(r+h_1+h_2)(t-\tau_2)} h_1 x(t)dt + \right.\right. \\ &\qquad \left.\left. e^{-(r+h_1+h_2)(\tau_3-\tau_2)} \int_{\tau_3}^{\infty} e^{-(r+h_1+h_2+h_3)(t-\tau_3)} h_1 x(t)dt\right)\right] - K \\ &= \frac{h_1 x_1}{Y_1+\delta} - K - \left(\frac{x_1}{x_2}\right)^{\phi_1} h_1 \left(x_2\Delta_2 + \left(\frac{x_2}{x_3}\right)^{\phi_2} h_1 x_3 \Delta_3\right), \end{aligned} \tag{IA.40}$$

where $Y_k \equiv \sum_{i=1}^k h_i$, $\Delta_k \equiv \frac{1}{Y_{k-1}+\delta} - \frac{1}{Y_2k+\delta}$, $k \geq 2$, and $\phi_k$ defined as in (IA.19). Following similar construction we obtain

$$V_1(x_2) = \frac{h_1 x_2}{Y_2+\delta} - \left(\frac{x_2}{x_3}\right)^{\phi_2} h_1 x_3 \Delta_3 \tag{IA.41}$$

$$V_1(x_3) = \frac{h_1 x_3}{Y_3+\delta} \tag{IA.42}$$

$$V_2(x_1) = \left(\frac{x_1}{x_2}\right)^{\phi_1}\left(\frac{h_2 x_2}{Y_2+\delta} - K - \left(\frac{x_2}{x_3}\right)^{\phi_2} h_2 x_3 \Delta_3\right) \tag{IA.43}$$

$$V_2(x_2) = \frac{h_2 x_2}{Y_2+\delta} - K - \left(\frac{x_2}{x_3}\right)^{\phi_2} h_2 x_3 \Delta_3 \tag{IA.44}$$

$$V_2(x_3) = \frac{h_2 x_3}{Y_3+\delta} \tag{IA.45}$$

$$V_3(x_1) = \left(\frac{x_1}{x_2}\right)^{\phi_1}\left(\frac{x_2}{x_3}\right)^{\phi_2}\left(\frac{h_3 x_3}{Y_3+\delta} - K\right) \tag{IA.46}$$

$$V_3(x_2) = \left(\frac{x_2}{x_3}\right)^{\phi_2}\left(\frac{h_3 x_3}{Y_3+\delta} - K\right) \tag{IA.47}$$

$$V_3(x_3) = \frac{h_3 x_3}{Y_3+\delta} - K. \tag{IA.48}$$

These quantities can be used to derive firm values in leader-laggard equilibria for all $x$ as follows:

$$V_1^{\mathrm{PS}}(x) = \begin{cases} \left(\frac{x}{x_1}\right)^{\phi_0} V_1(x_1) & \text{if } x < x_1 \\ \frac{h_1 x}{Y_1+\delta} - \left(\frac{x}{x_2}\right)^{\phi_1} h_1 \left(x_2\Delta_2 + \left(\frac{x_2}{x_3}\right)^{\phi_2} h_1 x_3 \Delta_3\right) & \text{if } x_1 < x < x_2 \\ \frac{h_1 x}{Y_2+\delta} - \left(\frac{x}{x_3}\right)^{\phi_2} h_1 x_3 \Delta_3 & \text{if } x_2 < x < x_3 \\ \frac{h_1 x}{Y_3+\delta} & \text{if } x > x_3 \end{cases} \tag{IA.49}$$

$$V_2^{\mathrm{PS}}(x) = \begin{cases} \left(\frac{x}{x_1}\right)^{\phi_0} V_2(x_1) & \text{if } x < x_1 \\ \left(\frac{x}{x_2}\right)^{\phi_1} V_2(x_2) & \text{if } x_1 < x < x_2 \\ \frac{h_2 x}{Y_2+\delta} - \left(\frac{x}{x_3}\right)^{\phi_2} h_2 x_3 \Delta_3 & \text{if } x_2 < x < x_3 \\ \frac{h_2 x}{Y_3+\delta} & \text{if } x > x_3 \end{cases} \tag{IA.50}$$

$$V_3^{\mathrm{PS}}(x) = \begin{cases} \left(\frac{x}{x_1}\right)^{\phi_0} V_3(x_1) & \text{if } x < x_1 \\ \left(\frac{x}{x_2}\right)^{\phi_1} V_3(x_2) & \text{if } x_1 < x < x_2 \\ \left(\frac{x}{x_3}\right)^{\phi_2} V_3(x_3) & \text{if } x_2 < x < x_3 \\ \frac{h_3 x}{Y_3+\delta} & \text{if } x > x_3 \end{cases}. \tag{IA.51}$$

Generalizing to the case of $N$ firms, we obtain

$$V_m^{\mathrm{PS}}(x) = \begin{cases} \left(\frac{x}{x_1}\right)^{\phi_0} V_m(x_1) & & \text{if } x < x_1 \\ \left(\frac{x}{x_{n+1}}\right)^{\phi_n} V_m(x_{n+1}) & \text{for } n < m & \multirow{2}{*}{\text{if } x_n < x < x_{n+1},\ n = 1, \ldots, N-1} \\ \frac{h_m x}{Y_n+\delta} - \left(\frac{x}{x_{n+1}}\right)^{\phi_n} h_m \Gamma_{m,n} & \text{for } n \geq m & \\ \frac{h_m x}{Y_N+\delta} & & \text{if } x > x_N \end{cases}, \tag{IA.52}$$

with

$$\Gamma_{m,n} = \begin{cases} x_{n+1}\Delta_{n+1} + \prod_{k=n+1}^{N-1} \left(\frac{x_k}{x_{k+1}}\right)^{\phi_k} x_{k+1}\Delta_{k+1} & \text{if } m \leq n \\ 1 & \text{if } m > n \end{cases}, \tag{IA.53}$$

$\Delta_k = \frac{1}{Y_{k-1}+\delta} - \frac{1}{Y_k+\delta} > 0$, and, for $n = 1, \ldots, N-1$,

$$V_m(x_n) = \begin{cases} \frac{h_m x_n}{Y_n+\delta} - \left(\frac{x_n}{x_{n+1}}\right)^{\phi_n} h_m \Gamma_{m,n} & \text{if } m < n \\ \frac{h_m x_n}{Y_n+\delta} - K - \left(\frac{x_n}{x_{n+1}}\right)^{\phi_n} h_m \Gamma_{m,n} & \text{if } m = n \\ \left(\frac{x_n}{x_{n+1}}\right)^{\phi_n} V_m(x_{n+1}) & \text{if } m > n \end{cases}, \tag{IA.54}$$

and

$$V_m(x_N) = \begin{cases} \frac{h_m x_N}{Y_N+\delta} & \text{if } m < n \\ \frac{h_N x_N}{Y_N+\delta} - K & \text{if } m = N \end{cases}. \tag{IA.55}$$

Using the definition of beta in (4) and the equilibrium firm values in (IA.52), we obtain (IA.18) where

$$\omega_{m,n}(x) = \frac{a_{m,n}(x)}{a_{m,n}(x) - b_{m,n}(x)} > 0, \tag{IA.56}$$

with $a_{m,n}(x) = \frac{h_m x}{Y_n+\delta}$ and $b_{m,n}(x) = \left(\frac{x}{x_{n+1}}\right)^{\phi_n} h_m \Gamma_{m,n}$. ∎

## Proof of Corollary 2

To prove the first statement in the corollary, let us consider the ex-ante innovation probability that ignores each firm's investment strategy, $h_i/(h_1 + h_2)$, $i = 1, 2$. From (7) we see that $Y_1 > h_1/(h_1 + h_2)$ and $Y_2 < h_2/(h_1 + h_2)$. This means that, when the leader invests, its probability of innovating raises above the ex-ante value, while that of the laggard drops below the ex-ante value. When the laggard also invests, i.e., when $\tau_2^G = 0$ in (7), then the the leader's innovation probability *decreases*, while the laggard's probability *increases* to their ex-ante values. Because, by Proposition 1, a firm's investment is associated with an increase in the beta of its rival, the first statement of the corollary follows.

To prove the second statement in the corollary, consider first the case in which the leader invests. As Proposition 1 shows, the laggard's beta increases from $\phi_0$ to $\phi_1$, i.e., $\Delta\beta_2 = \phi_1 - \phi_0$. From (IA.5) and (7) we see that both $\phi_1$ and $Y_1$ are increasing functions of $h_1$. Hence, a large increase in the laggard's beta ($\Delta\beta_2$) is associated with a large increase in the leader's innovation output ($\Delta Y_1$). Similarly, let us consider the case in which the laggard invests. From (5) in Proposition 1, we see that when the laggard invests the Leader's beta changes from $1-\omega(x_2^G)(\phi_1-1)$ to 1, i.e., using the expression of $\omega(x)$,

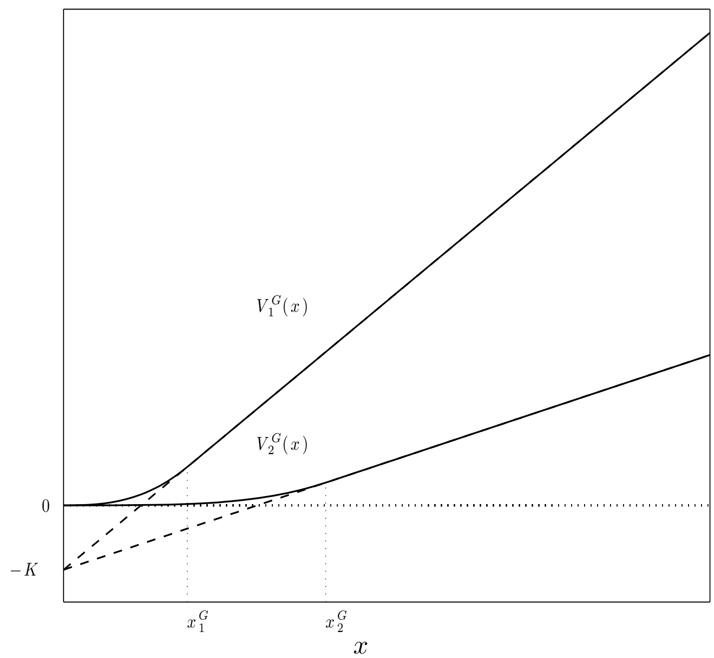
$$\Delta\beta_1 = \frac{h_2}{h_1 + \delta}(\phi_1 - 1). \tag{IA.57}$$

From (IA.57) and (7) we see that both $\Delta\beta_1$ and $Y_2$ are increasing functions of $h_2$. Hence, the second statement of the corollary follows. ∎

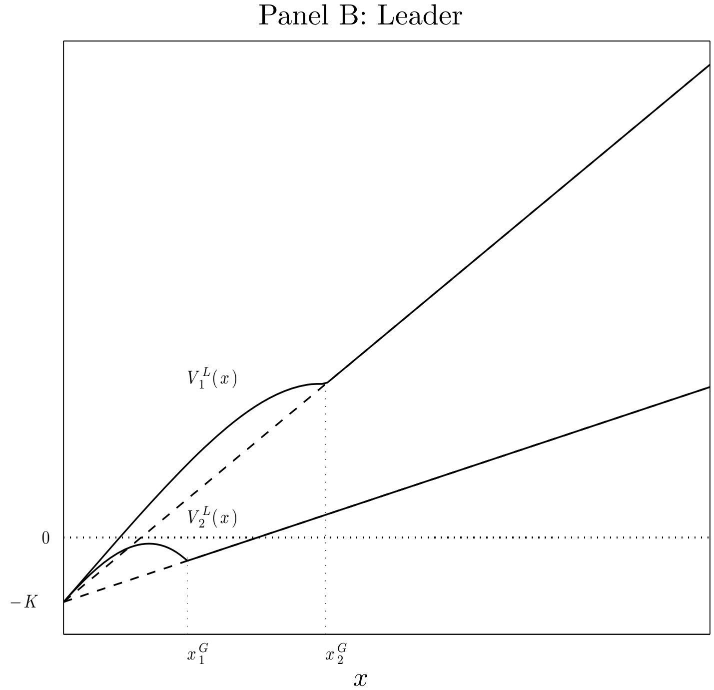
**Figure IA.1: Values of the laggard and of the leader**

The figure reports the value of firm $i = 1, 2$ when it invests as the laggard (Panel A) and when it is the leader (Panel B), assuming that $h_1 > h_2$. The values in Panel A (Panel B) illustrate the results in Proposition IA.1 (Proposition IA.2). The optimal investment thresholds for the laggard are $x_1^G$ and $x_2^G$, derived in Proposition IA.1. Parameter values: $h_1 = 0.5$, $h_2 = 0.2$, $\delta = 2\%$, $\sigma = 75\%$, and $K = 1$.

Panel A: Laggard




Panel B: Leader



## Figure IA.2: Equilibrium regions

The figure reports the thresholds $J(h_1)$ (solid line) and $S(h_1)$ (dash-dotted line) and the corresponding Markov perfect equilibrium regions derived in Proposition IA.4. Parameter values: $\delta = 2\%$ and $K = 1$.

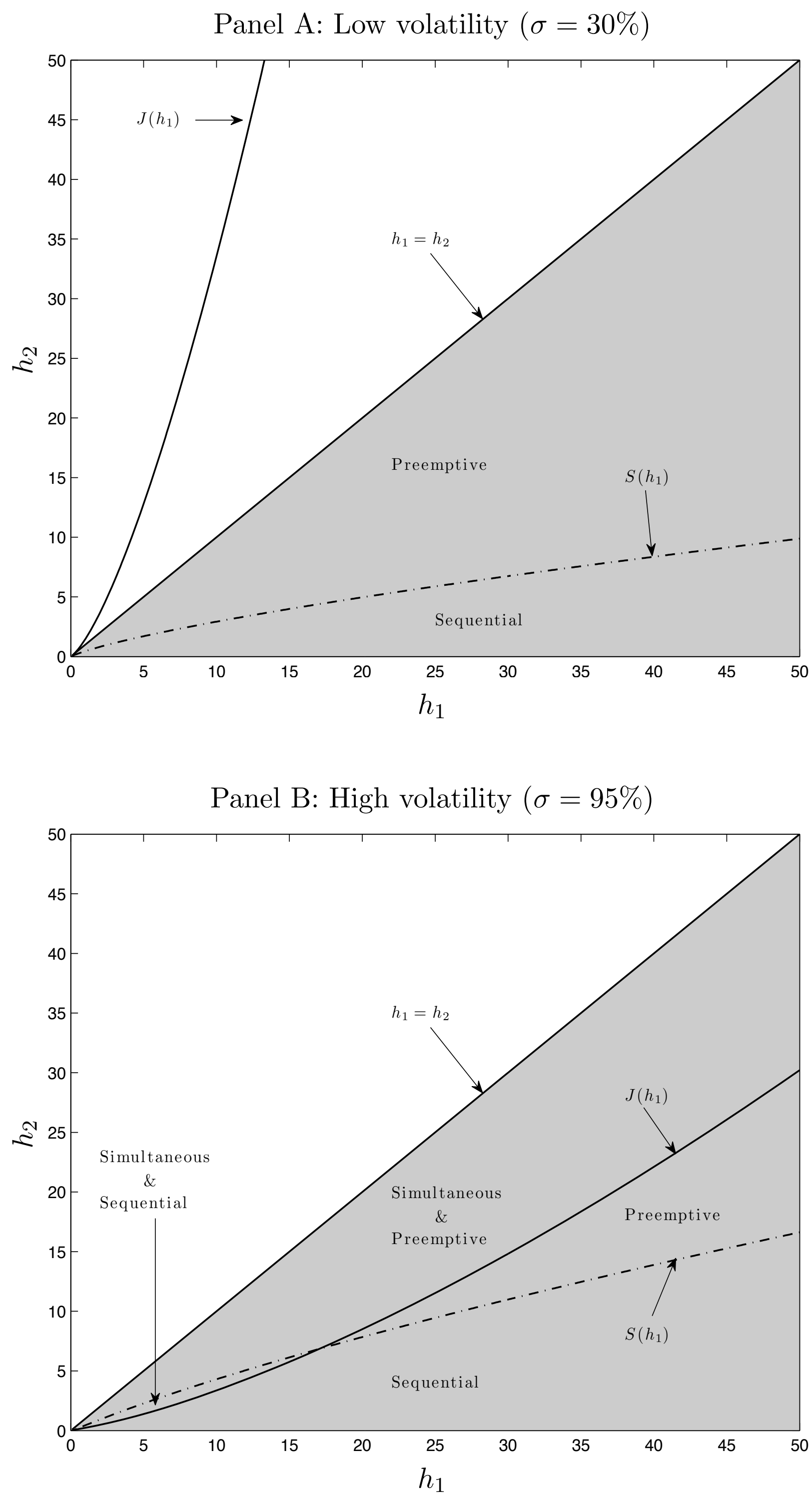
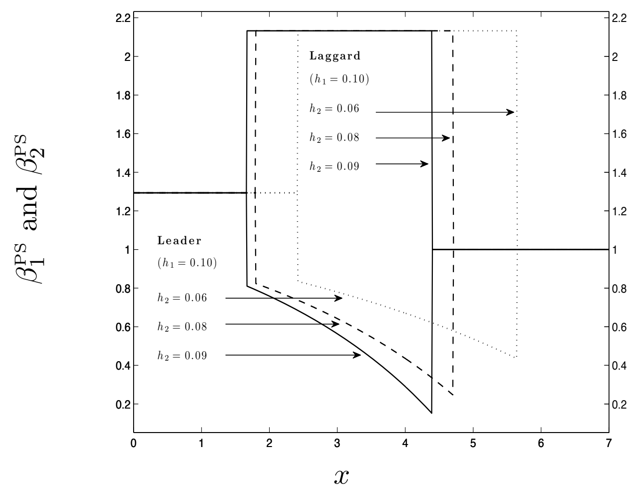


**Figure IA.3: Leader's and laggard's betas**

The figure reports the beta of the leader, $\beta_1^{\text{PS}}$, and the laggard, $\beta_2^{\text{PS}}$, in leader-laggard equilibria, derived in Proposition 1. Parameter values: $\delta = 2\%$, $K = 1$, and $\sigma = 30\%$.

Panel A: $h_2$ varies, $h_1 = 0.1$




Panel B: $h_1$ varies, $h_2 = 0.1$