

# The Fisher Effect, A Contradiction: Theory and Empirics<sup>1</sup>

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## Abstract

The Fisher Effect is presented to students of economics and finance as a stylized fact: i.e. “*The one-for-one relation between the inflation rate and the nominal interest rate is called the **Fisher Effect***” (Mankiw, 2012, p. 111). We prove in a contract-theoretic general equilibrium environment that this proportional relationship does not hold for private debt. Next, we take the theory to the data, employing cross-sectional observations for 74 countries and 24 years. The predictions of our theory – a contradiction of Fisher – are strongly supported empirically. These findings have potentially important implications for development finance theory and policy. In particular, inflation induced negative real interest rates may not signal financial market dysfunction.

**Keywords:** Inflation, Fisher Effect, Debt contract, Financial Markets

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## I. Introduction

The Fisher effect has been widely recognized since Irving Fisher's *The Theory of Interest* (1930) and is presented as a stylized fact to students of economics and finance. For example, Mankiw's *Macroeconomics* (2012) states, "According to the Fisher equation, a 1 percent increase in the rate of inflation in turn causes a 1 percent increase in the nominal interest rate. The one-for-one relation between the inflation rate and the nominal interest rate is called the **Fisher Effect**" (p. 111). A rigorous theoretical explanation of the Fisher effect is found in Wallace (2012) in the form of a no-arbitrage condition:

One way to do this is to (i) sell the date  $t$  good for money obtaining  $P_t$  units of money, (ii) lend the money at  $i_t$  thereby acquiring  $(1 + i_t)P_t$  units of money at date  $t + 1$ , and (iii) use that money to buy date  $t + 1$  good in the amount  $(1 + i_t)P_t/P_{t+1}$ . An alternative is to lend the date  $t$  good at the real rate  $r_t$ . This gives  $1 + r_t$  units of the date  $t + 1$  good. Equating the two amounts of date  $t + 1$  good, we get  $(1 + i_t) = (1 + r_t)(P_{t+1}/P_t)$ . This is often called the Fisher equation.

The Fisher equation continues to play an important role in finance and economics studies. A recent example is Allen, Carletti and Gale (2012), wherein both real and nominal contracting coexist. The difference between the two, and the fact that banks write nominal contracts, is central to their analysis.<sup>3</sup>

The point of the present study is to prove, in a general equilibrium framework, that the Fisher Effect does not hold for private debt. Theoretically, the interest rate on private debt is shown to increase with inflation, but less than the "one-for-one" relationship predicted by Fisher. Although the proof is messy, the intuition for this result is actually pretty simple. A standard private debt contract pays a total expected return that is *partly nominal*, (the interest return and contracted principle), and *partly real*, (recovery in bankruptcy states). In bankruptcy the lender

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<sup>3</sup> Our study was motivated by Boyd, Levine and Smith (2001) who reported finding negative real interest rates in many inflation-ridden developing economies. Here, we address the question as to why that is, and why it is not a surprising observation.

claims all the *real assets* of the firm. If the nominal component were to increase proportionally with inflation, debt holders would be over-compensated and debt issuers under-compensated.<sup>4</sup>

After proving the theorem we take the theory to the data, employing cross-section observations for 74 countries and 24 years. The predictions of our theory – a contradiction of Fisher – are strongly supported by the empirics. An important robustness check is to substitute government Treasury Bill rates for rates on private debt. With government debt we obtain a near one-for-one linear relationship exactly *a la* Fisher. This is consistent with the theory since our theorem only holds when default (bankruptcy) is possible. As briefly discussed in the conclusion, these findings have several implications for development finance and policy.

The rest of the paper proceeds as follows. Section II presents the theory and some numerical examples. It is shown, but not proved, that inflation reduces welfare. Section III. investigates the Fisher relationship and presents our theorem: interest rates on private debt increase less than proportionally with the rate of inflation. Section IV presents the empirics and Section V. concludes.<sup>5</sup>

## II. The Theory

### 2.1 Agents and Technologies

The economy lasts two periods and all agents are risk-neutral. There is a continuum of agents on  $[0, 1]$  who are identical. All agents value only consumption in period 1. Each agent is endowed in period 0 with  $E$  units of the single good and with access to a productive technology. The production technology converts the period 0 good into the period 1 good. The fixed cost of

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<sup>4</sup> A numerical example is provided in Section III.

<sup>5</sup> Our work is contract-theoretic and our results have nothing to do with the so-called Mundell-Tobin effect (1963, 1965).

activating this production technology is  $t$ , which is a deadweight cost. If this is done, we call the agent “an entrepreneur.” The production opportunity is constant returns to scale, risky, and requires an investment strictly greater than the endowment,  $E$ . To undertake the investment the entrepreneur must therefore borrow from another agent, who will thereafter be called “a lender”. The amount available for investment is  $2E - t$  and to keep things simple we assume that  $2E - t$  is the only scale possible. The real rate of return on investment  $r$  is a random variable and we assume that  $r$  is uniformly distributed on the interval,  $[L, U]$ , where  $L$  and  $U$  are exogenously given.

## 2.2 Costly State-verification

When returns are realized in period 1, they are freely observed by the entrepreneur. An investor can observe these return realizations only by paying a fixed monitoring cost  $M$  which is a dead-weight cost. This is a costly state verification environment and it is well known that in such an environment, the optimal contract is a “standard debt contract” – one that promises a constant payment to the lender in all states in which monitoring *does not* occur. Hereafter, we will call this promised constant return a “gross interest payment,”  $I$ . If this return is not paid, the contract calls for monitoring and the investor will receive all the assets of the entrepreneur net of monitoring costs. Under this piece-wise linear return structure, the entrepreneur becomes residual claimant in non-monitoring states and thus is essentially an equity holder. Monitoring states are associated with bankruptcy, and the investor has the rights of a debt holder.<sup>6</sup> We assume that agents have full ability to commit to contract terms in period zero, and further for

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<sup>6</sup> The optimality of this arrangement has been proved many times and will not be re-proved here (Gale and Hellwig, 1985; Townsend, 1978; etc).

technical reasons we assume that contracts cannot employ lotteries or other forms of extrinsic uncertainty.<sup>7</sup>

The nature of the game is that in period zero any agent may make a take-it-or-leave-it offer to any other agent. Since, all agents are identical in period 0, it must be true that in equilibrium the expected return of entrepreneurs and of investors is the same. Define  $\Delta$  = the expected real return of entrepreneurs, and  $\Omega$  = the expected real return of investors; then a property of equilibrium is that  $\Delta = \Omega$ , which is a form of participation constraint. The problem is to maximize the expected return of entrepreneurs (or investors) subject to the participation constraint. There is a single choice variable, the real interest payment,  $I$ .<sup>8</sup> The expected return of an entrepreneur is

$$\Delta = \int_{\frac{I}{2E-t}}^U ((2E - t)r\varphi(r) - I)dr, \quad (1)$$

which represents the residual return to the project in states in which interest is paid and no monitoring occurs.

The expected return of an investor is

$$\Omega = I \int_{\frac{I}{2E-t}}^U \varphi(r)dr + \int_L^{\frac{I}{2E-t}} ((2E - t)r\varphi(r) - M)dr, \quad (2)$$

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<sup>7</sup> See Boyd and Smith (1994) for an explanation of the technical reasons why we must prohibit “extrinsic uncertainty.” They further show that this theoretical restriction is totally irrelevant in actual applications of the model.

<sup>8</sup> It is more common to see this problem as one in which some agents are endowed with the good only (investors) and others are endowed with both the good and the production opportunity (entrepreneurs). Investors are also endowed with an alternative investment opportunity such as home production, which is inferior to entrepreneurial investment opportunity. With this alternative setup, the problem is to maximize the expected return of entrepreneurs, subject to participation by investors. We prefer our symmetric setup because all agents are identical in period zero. It is obvious that the problem we solve also solves a planning problem. None of our conclusions would be affected if we employed the more common model.

where the first term is expected real interest payments in non-monitoring states and the second term is the recover of investors in monitoring states, net of monitoring costs.

### **2.3 Inflation and Nominal Contracting**

We introduce inflation and nominal contracting in a stylistic way so as to avoid writing down and solving a monetary general equilibrium model.<sup>9</sup> For the problem at hand, we make two critical assumptions. First, agents are restricted to using nominal contracts. We know, based on experience in high inflation environments, that if inflation is sufficiently high, real contracts will emerge and will be increasingly employed if inflation persists. However, our model is not intended to deal with hyperinflation – only environments with moderate inflation where agents continue using nominal contracts. The second key assumption is that, as the rate of inflation increases, the future price level becomes less predictable. Existing empirical work strongly supports this second assumption (Barnes, 1999; Barnes, Boyd and Smith, 1999; Boyd, Levine and Smith, 2001).

As will become apparent, the combination of these two assumptions introduces uncertainty regarding future prices which complicates the contracting problem. Obviously, the problem could be eliminated by writing real contracts, a possibility that we prohibit by assumption.<sup>10</sup> We also ignore the question as to why the government would pursue policies that

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<sup>9</sup> That could be done, we believe, in the context of an overlapping-generations model with appropriate cash-in-advance constraints. For present purposes, however, that would add substantial complexity and little additional insight.

<sup>10</sup> In reality, writing real contracts is costly. The incentives to do so increase with the level and variability of inflation (Boyd, Levine and Smith, 2001).

result in inflation on average. There are many possible explanations an obvious one being seignorage.

In period 0, lenders and entrepreneurs enter into nominal contracts determining their payoffs in period 1. At the end of period 0, after such contracts are written, a price innovation determines the actual price level in period 1. The monetary authority chooses an expected rate of inflation but by assumption cannot precisely determine the actual future price level. Without loss of generality, we consider only policies of non-negative average inflation. In period 0, the decision period, the price level is  $P_0$ . The monetary authority chooses a mean price level for period 1 of  $P_m$  but the actual price level in period 1,  $P_1$ , is a random variable. A simple way to model this is to assume that the probability density function of  $P_1$  places all its mass on two points, a high price,  $P_h$ , and a low price,  $P_l$ . The probabilities of the high and low states are assumed to be equal to  $\frac{1}{2}$  and  $P_h \geq P_m \geq P_l$  and  $P_m = \frac{(P_h + P_l)}{2}$ .

In period 0, the entrepreneur and the investor enter into nominal debt contracts to be paid in period 1 after project returns are realized. The nominal debt payment (principal and interest) promised to investors is  $\Lambda$ . We assume that in period 0 the average future price level  $P_m$  and the distribution function of  $P_1$  are common knowledge. The expected real period 1 return to entrepreneurs can now be written in two parts,

$$\Delta = \frac{1}{2} \int_{\frac{\Lambda}{Pl(2E-t)}}^U \left( (2E-t)r - \frac{\Lambda}{Pl} \right) \varphi(r) dr + \frac{1}{2} \int_{\frac{\Lambda}{Ph(2E-t)}}^U \left( (2E-t)r - \frac{\Lambda}{Ph} \right) \varphi(r) dr, \quad (3)$$

reflecting the two future price levels. The expected real return to investors can similarly be written in two parts:

$$\Omega = \frac{1}{2} \left\{ \frac{\Lambda}{Pl} * \int_{\frac{\Lambda}{Pl(2E-t)}}^U \varphi(r) dr + \int_L^{\frac{\Lambda}{Pl(2E-t)}} ((2E-t)r - M) \varphi(r) dr \right\} + \frac{1}{2} \left\{ \frac{\Lambda}{Ph} * \int_{\frac{\Lambda}{Ph(2E-t)}}^U \varphi(r) dr + \int_L^{\frac{\Lambda}{Ph(2E-t)}} ((2E-t)r - M) \varphi(r) dr \right\}. \quad (4)$$

The agents' optimization problem (which, by inspection, is also the planning problem) can be written:

$$\begin{aligned} & \text{Max } \Delta, \quad \text{w. r. t. } \Lambda \\ & \text{Subject to: } \Omega = \Delta. \end{aligned} \quad (5)$$

The first-order condition has two solutions:<sup>11</sup>

$$\begin{aligned} \Lambda &= \frac{P_h P_l}{2(P_h^2 + P_l^2)} [(P_h + P_l)(2U(2E-t) - M) \\ &\pm \sqrt{(P_h + P_l)^2(2U(2E-t) - M)^2 - 4(P_h^2 + P_l^2)(2E-t)((L^2 + U^2)(2E-t) - 2LM)}], \end{aligned} \quad (6)$$

and the second order condition is:

$$\frac{(P_h^2 + P_l^2)(1-2\lambda)}{2P_h^2 P_l^2 (2E-t)(U-L)} \leq 0, \quad (7)$$

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<sup>11</sup> When solving numerically we discovered another constraint on problem (5). Realized bankruptcy costs must always be positive costs. With some parameters and inflation rates (extremely high inflation) this will not be the case. Obviously, such cases are outside the space of parameters/policies the model is intended to explain. In all numerical examples presented here, this constraint is satisfied.



where  $\lambda$  is the LaGrangian used to introduce the constraint  $\Delta = \Omega$ .<sup>12</sup>

## 2.4 Numerical Examples

In this model average inflation does no good and complicates the contracting problem. Some numerical solutions presented in Table 1 will help to explain this result. In a bit we shall prove our main theorem – that the private interest rate increases less than proportionally with the average rate of inflation.

In Table 1, the nominal gross debt payment  $\Lambda$  is the optimal value that solves problem (5). The second column shows solution values when  $P_m = 1$  and the mean rate of inflation is zero; in this case payoffs are the same as those under real contracting. Columns three and four show solutions when  $P_m = 1.1$  (1.2) and expected average inflation is ten percent (twenty percent).

The first thing to observe from Table 1 is that, as average inflation increases, expected equilibrium monitoring costs rise (shaded). Monitoring costs are a deadweight loss and therefore the expected consumption of all agents declines. In this example, as inflation increases from 0 to 20 percent, deadweight losses increase by 2.24 percent. This happens because inflation renders the contracting mechanism less precise and the effect worsens as inflation rises. It can be proved that a policy of average inflation causes a welfare loss. However, our proof is somewhat tedious and peripheral to the main point of this paper – so it is not reproduced here.

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<sup>12</sup> This condition for a maximum has been easily satisfied in all the numerical examples we have constructed.

We can, however, demonstrate how inflation reduces welfare by considering a special case: a two-state version of the model in which real returns can only be 1.0 and 1.5; both equiprobable. Assume that parameters are such that with no inflation it is optimal to monitor only in the low real return state. In that case monitoring would occur one half the time. Next, assume that there is inflation and that the period one price level can take on two values: 1.0 and 1.5, both equiprobable. There are now four possible outcomes in this simple environment all with probability 0.25: low return, low inflation; low return, high inflation; high return, low inflation; and high return, high inflation. To successfully monitor in all desired cases (those where real returns are low), requires monitoring three quarters of the time – a fifty percent increase in monitoring due to inflation.

### III. The Fisher Relationship

The next important result in Table 1. is that, as average inflation  $P_m$  increases, the nominal interest rate increases also – but less than proportionally. This is a contradiction of the Fisher Relationship. It will be proved in a moment, but the economic intuition is actually pretty simple. Consider the numerical example in Table 1 Column 4, in which the expected rate of inflation is 20%. Next, assume that the promised debt payment in that case is the Fisher payment: e.g. 20% higher than the no-inflation payment, which means that  $\Lambda_f = 124.71 * 1.2 = 149.65$ . Then, the expected return to entrepreneurs is  $\Delta = 106.86$ , and the expected return to savers is  $\Omega = 110.87$ . Why does this occur? A standard private debt contract pays a total expected return that is *partly nominal*, (the interest return and contracted principle), and *partly real*, (recovery in bankruptcy states). In bankruptcy, the lender claims all the *real assets* of the firm. If the nominal

component were to increase proportionally with inflation, investors would be over-compensated (and entrepreneurs under-compensated) because the assets claimed in bankruptcy are real assets, and are therefore automatically indexed.

**Theorem. The Nominal Contract Interest Rate on Private Debt Increases Less Than the Average Inflation Rate**

**Proof:**

The solution to the problem (5) gives, after some rearrangement, the optimal value of  $\Lambda$ , which we define as  $\Lambda^*$ .

$$\Lambda^* = \frac{P_h P_l (P_h + P_l)}{2(P_h^2 + P_l^2)} [(2U(2E - t) - M) \pm \sqrt{(2U(2E - t) - M)^2 - 4 \frac{(P_h^2 + P_l^2)}{(P_h + P_l)^2} (2E - t)((L^2 + U^2)(2E - t) - 2LM)}]. \quad (8)$$

Next, define the expected Fisher nominal interest payment as:  $\Lambda_f \equiv \Lambda_0^* \times P_m$ , where  $\Lambda_0$  is the real interest rate. Therefore,  $\Lambda_f$  is defined as an expected nominal interest payment that increases proportionally with the average rate of inflation.<sup>13</sup>

The proof proceeds in several steps. First, we employ equation (8) above with  $P_h = P_l = 1$  (which in our environment means no inflation) and obtain  $\Lambda_0$ .

$$\Lambda_0 = \frac{1}{2} \left[ (2U(2E - t) - M) \pm \sqrt{(2U(2E - t) - M)^2 - 2(2E - t)((L^2 + U^2)(2E - t) - 2LM)} \right].$$

<sup>13</sup> The Fisher equation specifies a contract interest rate that is proportional with inflation, but not an intercept. For that purpose we employ the optimal real rate of interest  $\Lambda_0$ , obtained by solving the maximization problem (5) with no inflation.

(9)

Next, we substitute  $\Lambda_0$  into the expression  $\Lambda_f \equiv \Lambda_0^* \times P_m$  to obtain the expected Fisher Payment  $\Lambda_f$ .

$$\Lambda_f = \frac{P_m}{2} [(2U(2E - t) - M) \pm \sqrt{(2U(2E - t) - M)^2 - 2(2E - t)((L^2 + U^2)(2E - t) - 2LM)}]. \quad (10)$$

Therefore, Equation (8) gives the equilibrium value of  $\Lambda^*$  and Equation (10) gives the expected Fisher payment  $\Lambda_f$ .

By inspection, there are just two pairs of terms that differentiate (8) and (10). The first pair of is  $\frac{P_h P_l (P_h + P_l)}{(P_h^2 + P_l^2)}$  from Equation (8) and  $P_m$  from Equation (10), which are the first terms in each equation. We shall prove that  $P_m > \frac{P_h P_l (P_h + P_l)}{(P_h^2 + P_l^2)}$ .

The second pair of terms is  $\frac{2(P_h^2 + P_l^2)}{(P_h + P_l)^2}$  from Equation (8) and 1.0 from Equation (10). Both these terms are inside the radical sign. We shall prove that  $\frac{2(P_h^2 + P_l^2)}{(P_h + P_l)^2} \geq 1.0$ . We continue to assume, without loss of generality, that  $P_l = 1.0$  and thus  $P_h = 2P_m - 1$ .

**Lemma 1.** Proof that  $P_m \geq \frac{P_h P_l (P_h + P_l)}{(P_h^2 + P_l^2)}$ .

Since,  $P_l = 1.0$  and  $P_h = 2P_m - 1$ , the right hand side of this inequality can be expressed as:

$$\frac{2(2P_m - 1)P_m}{(2P_m - 1)^2 + 1}. \quad (11)$$

If  $P_m = 1$ , then  $P_m = \frac{2(2P_m-1)P_m}{(2P_m-1)^2+1}$ .

If  $P_m > 1$ , then  $\frac{2(2P_m-1)}{(2P_m-1)^2+1} < 1$ , which implies  $P_m > \frac{2(2P_m-1)P_m}{(2P_m-1)^2+1}$ .

This follows from:  $(2P_m - 1)^2 + 1 - 2(2P_m - 1) = (2P_m - 2)^2 > 0$  if  $P_m > 1$ , therefore,  $(2P_m - 1)^2 + 1 > 2(2P_m - 1)$ , which means the numerator of the term  $\frac{2(2P_m-1)}{(2P_m-1)^2+1}$  is smaller than the denominator. Thus,  $\frac{2(2P_m-1)}{(2P_m-1)^2+1} < 1$ , if  $P_m > 1$ .

Next, we find the maximum value the term  $\frac{2(2P_m-1)P_m}{(2P_m-1)^2+1}$  in (11) can take. Taking the derivative of this term with respect to  $P_m$ :

$$\frac{d}{dP_m} \frac{2(2P_m - 1)P_m}{(2P_m - 1)^2 + 1} = \frac{-2P_m^2 + 4P_m - 1}{(2P_m^2 - 2P_m + 1)^2}$$

Setting the derivative equal to zero:

$$2P_m^2 - 4P_m + 1 = 0.$$

This is a quadratic with two possible solutions:

$$P_m = \frac{4 \pm \sqrt{16 - 4 * 2 * 1}}{2 * 2} = \frac{2 \pm \sqrt{2}}{2} = 1.7071, 0.2929$$

Next, we substitute the two solutions into (11) and obtain,

$$\left. \frac{2(2P_m - 1)P_m}{(2P_m - 1)^2 + 1} \right|_{P_m=1.7071} = 1.2071$$

$$\left. \frac{2(2P_m - 1)P_m}{(2P_m - 1)^2 + 1} \right|_{P_m=0.2929} = -0.2071$$

The expression in (11) must always be positive, otherwise, an important model restriction (limited liability for the investors) is violated. Therefore, the only acceptable solution is  $P_m = 1.7071$ .

We want to verify that this solution is indeed an interior maximum. Thus, we take the second derivative of the term  $\frac{2(2P_m-1)P_m}{(2P_m-1)^2+1}$  in (11),

$$\frac{d^2}{dP_m^2} \frac{2(2P_m-1)P_m}{(2P_m-1)^2+1} = \frac{4P_m(2P_m^2-6P_m+1)}{(2P_m^2-2P_m+1)^3}.$$

Then, we evaluate this expression for the second order condition at the two roots and obtain,

$$\left. \frac{4P_m(2P_m^2-6P_m+1)}{(2P_m^2-2P_m+1)^3} \right|_{P_m=1.7071} = -0.5858$$

This is a negative number, indicating  $P_m = 1.7071$  is an interior maximum of (11).  $\square$

**Lemma 2.** Proof that  $\frac{2(P_h^2+P_l^2)}{(P_h+P_l)^2} \geq 1.0$

After we substitute  $P_l = 1.0$  and  $P_h = 2P_m - 1$ , the expression on the left hand side in the statement of Lemma 2 above is equal to:

$$\frac{(2P_m-1)^2+1}{2P_m^2}. \tag{12}$$

By inspection, the left hand side of the expression is weakly greater than 1.0. The lower bound of the left hand term is 1.0, which occurs when expected inflation is zero,  $P_m = 1.0$ . For any larger value of  $P_m$ , the left hand side above is  $> 1.0$ .

Next, we show that the term  $\frac{(2P_m-1)^2+1}{2P_m^2}$  in (12) is increasing in  $P_m$ . Taking the derivative of this term with respect to  $P_m$ , we get

$$\frac{d}{dP_m} \frac{(2P_m-1)^2+1}{2P_m^2} = \frac{2(P_m-1)}{P_m^3}$$

For  $P_m > 1$ , it is always positive by inspection. This means that  $\frac{(2P_m-1)^2+1}{2P_m^2}$  is an increasing function of  $P_m$ .  $\square$

***Back to the main proof:***

Equation (8) describes two solutions to  $\Lambda^*$ . The mathematical expressions of the two solutions are similar, except in one the term under the radical is added and in the other it is subtracted.

Consider first the term under the radical in (8.), which is

$$\sqrt{(2U(2E-t) - M)^2 - 4 \frac{(P_h^2+P_l^2)}{(P_h+P_l)^2} (2E-t)((L^2+U^2)(2E-t) - 2LM)}.$$

We will show that this term is decreasing in inflation. Since imaginary solutions are not meaningful in the model this expression must be greater than or equal to zero. Inside the radical, the first term is  $(2U(2E-t) - M)^2$ , which is determined by parameters, and does not change with inflation. The second term is  $4 \frac{(P_h^2+P_l^2)}{(P_h+P_l)^2} (2E-t)((L^2+U^2)(2E-t) - 2LM)$ , which does change with inflation and is subtracted from the first term  $(2U(2E-t) - M)^2$ . There is a model restriction that  $(2E-t) > M$  because agents must have enough resources to monitor.  $U \geq 1$ , otherwise there are no gains to trade; and  $U > L \geq 0$ . Then,  $(2E-t)((L^2+U^2)(2E-t) - 2LM)$  is positive. By Lemma 2,  $\frac{2(P_h^2+P_l^2)}{(P_h+P_l)^2} \geq 1.0$  and is increasing in inflation. Therefore, the term under the radical is decreasing in inflation.

Next, we compare the two different solutions to  $\Lambda^*$  and  $\Lambda_f$  as described in (8) and (10) separately. We begin with the solutions in which the term under the radical sign is added.

### Case 1. Term under Radical is Added

Let's compare the terms in (8) and (10).

The first term  $\frac{P_m}{2} \geq \frac{P_h P_l (P_h + P_l)}{2(P_h^2 + P_l^2)}$ , by Lemma 1.

The second term  $(2U(2E - t) - M)$  is common to both expressions.

The third term involves the radical. As we have discussed above, the term

$$\sqrt{(2U(2E - t) - M)^2 - 4 \frac{(P_h^2 + P_l^2)}{(P_h + P_l)^2} (2E - t)((L^2 + U^2)(2E - t) - 2LM)}$$
 in Equation (8)

decreases in inflation. However, the term

$$\sqrt{(2U(2E - t) - M)^2 - 2(2E - t)((L^2 + U^2)(2E - t) - 2LM)}$$
 in Equation (10) does not depend on inflation. Therefore, for positive inflation,

$$\begin{aligned} & \sqrt{(2U(2E - t) - M)^2 - 2(2E - t)((L^2 + U^2)(2E - t) - 2LM)} \\ & > \sqrt{(2U(2E - t) - M)^2 - 4 \frac{(P_h^2 + P_l^2)}{(P_h + P_l)^2} (2E - t)((L^2 + U^2)(2E - t) - 2LM)} \end{aligned}$$

It is straightforward to see that the terms in Equation (8) are smaller than the terms in Equation (10) whenever inflation is positive. As inflation increases, the term inside the solid parenthesis in (8), which is

$$(2U(2E - t) - M) + \sqrt{(2U(2E - t) - M)^2 - 4 \frac{(P_h^2 + P_l^2)}{(P_h + P_l)^2} (2E - t)((L^2 + U^2)(2E - t) - 2LM)}$$

increases less than the comparable term in (10), which is

$$(2U(2E - t) - M) + \sqrt{(2U(2E - t) - M)^2 - 2(2E - t)((L^2 + U^2)(2E - t) - 2LM)}.$$

Furthermore, the term inside the solid parenthesis in (8) gets multiplied by a smaller term, which is  $\frac{P_h P_l (P_h + P_l)}{2(P_h^2 + P_l^2)}$  (by Lemma 1).

Therefore,  $\Lambda_f \geq \Lambda^*$  in Case 1.  $\square$

### Case 2. Term under the Radical is Subtracted



Again, comparing the terms in (8) and (10).

Our previous discussion regarding the individual terms in (8) and (10) holds here, except that the term under the radical is now subtracted from  $(2U(2E - t) - M)$ .

Since

$$\sqrt{(2U(2E - t) - M)^2 - 2(2E - t)((L^2 + U^2)(2E - t) - 2LM)} >$$

$$\sqrt{(2U(2E - t) - M)^2 - 4\frac{(P_h^2 + P_l^2)}{(P_h + P_l)^2}(2E - t)((L^2 + U^2)(2E - t) - 2LM)}$$

for positive inflation, the term inside the solid parenthesis in (8) is greater than the term inside the solid parenthesis in (10). This is because a bigger number is getting subtracted from  $(2U(2E - t) - M)$  in equation (10). Therefore, which one is larger, (8) or (10), depends on the magnitude of the terms multiplying the terms inside the solid parenthesis. Resultantly, we have been unable to prove this result analytically. However, it can be proved by examining a grid noting, importantly, that all functions are continuous and differentiable everywhere. This proof is presented in the Appendix.

As shown in the appendix, in this case also  $\Lambda_f - \Lambda^* \geq 0$ . That exhausts the possible cases and the theorem is proved.  $\square$

#### IV. Taking Theory to Data

In this section we test the theoretical prediction that, as inflation increases, the nominal interest rate on private debt increases less than proportionally. We employ two datasets from 74 countries over 24 years. “Lending Rate” comes from the World Development Indicators.<sup>14</sup> It is defined as the rate charged by banks on loans to prime corporate customers. There is one

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<sup>14</sup> <http://data.worldbank.org/indicator>

observation per country/year. Our second data source is an individual firm panel dataset from Compustat that includes 30,576 firms in the same countries and years.

With the firm-level data from Compustat we cannot measure the marginal cost of corporate borrowing directly, but must estimate it. All we have for each firm/year data point is total debt outstanding and total interest paid. The ratio of the two is an average interest rate on all of the firm's debt outstanding and is not the marginal object we wish to measure. Below, we explain how we estimate marginal debt costs with the Compustat firm data.

#### **4.1 Estimating Marginal Borrowing Costs with Individual Firm Data**

With the individual firm data we observe 4 objects in 2 years for each firm. Define  $I(t)$  = interest paid in year  $t$  and  $D(t)$  = total debt outstanding in year  $t$ . (For simplicity, firm subscripts are omitted.) If either  $D(t) < 0$  or  $I(t) < 0$ , we delete the observation as it is probably a reporting error. We also delete all cases with  $D(t) < D(t - 1)$  since this probably indicates that no new debt was issued. If no new debt was issued, there is no new marginal rate for us to record. This procedure deletes about one half of the sample but the individual firm dataset is quite large so this is not a problem.

Define  $r(t - 1)$  as the historic average interest rate that is observable and equals  $I(t - 1)/D(t - 1)$ . Define  $r(t)$  as the new marginal interest rate that we cannot observe. Now, we have to make some assumptions. If we assume that all old debt is short term and refinanced every period, the marginal rate is equal to the average rate and  $r(t) = I(t)/D(t)$ . If, on the other hand, we assume that all old debt is fixed rate and not refinanced,  $I(t) = I(t - 1) + r(t) *$

$[D(t) - D(t - 1)]$  and  $r(t) = [I(t) - I(t - 1)] / [D(t) - D(t - 1)]$ . In this case the marginal interest rate  $r(t)$  is the change in interest paid, divided by the change in total debt outstanding.

Any case between the two extremes is possible. However, in all our empirical tests the assumption of all short-term debt provided the best fit and highest values of statistical significance.<sup>15</sup> These are the results that will be present here.

## 4.2 Regression Results

In the regressions presented in Table 2, the dependent variable is one of the interest rate measures discussed above and the explanatory variable is annual inflation.<sup>16</sup> There are dummy variables for year, and clustering for country or firm. We delete all observations with average inflation exceeding 60%. The reason is that once long run inflation gets into such a high range agents will begin indexing many contracts including corporate borrowing rates. Such data are outside the range of economies the model is intended to describe. Finally, we delete all observations with negative interest rates or with interest rates exceeding 100%.<sup>17</sup>

In Table 2, column 1 we present results when the dependent variable is *Lending Rate*, from the World Development Indicators. Inflation enters in both the level and in a squared term ( $\text{Inflation}^2$ ). The first term is positive and highly significant and the second is negative and highly significant. The regression is plotted in Figure 1 with 90% confidence bands. Also

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<sup>15</sup> None of our conclusions are reversed if we assume that all debt is long-term and fixed rate. For brevity we do not reproduce those results here, but they are available for the interested reader.

<sup>16</sup> We have conducted robustness tests where inflation is averaged over 3 and 5 year periods, with commensurate decrease in sample size. For brevity, those results are not presented but they are completely consistent with the results in the paper.

<sup>17</sup> Our results are not particularly sensitive to the choice of these thresholds.

plotted for purposes of reference is a 45 degree line emanating from the function's intercept. It is clear that with these data, the nominal interest rate does not keep pace with the rate of inflation.

In Table 2, column 2 we present results with the individual firm data where the dependent variable is the estimated marginal corporate debt rate on debts of all maturities. The explanatory variables are as described previously. As seen in Figure 2, the coefficient of inflation is positive and highly significant and the coefficient of inflation squared is negative and highly significant. Again, the estimated marginal borrowing rate does not keep pace with the average rate of inflation. In Table 2, column 3 we present results with the individual firm data when the dependent variable is the marginal interest rate on long term debt. The regression results and plot are very similar to the others.

#### **4.3 A Robustness Test**

The theory predicts a non-proportional response of the nominal interest rate on private debt and we find support for that in the data. However, default-risk-free government debt should conform with the Fisher Relationship – at least if a government's debt is truly default risk free. This presents us with a natural robustness test, since we can estimate the same sort of regressions with government Treasury Bill rates (International Financial Statistics).<sup>18</sup> In column 4 of Table 2, we show the quadratic estimate for Treasury Bill. The level term is positive and highly significant but the squared term is insignificant. In column 5 we estimate the linear function and that function is plotted in Figure 4. The plotted function is very close to the 45 degree line, and

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<sup>18</sup> <http://www.imf.org/external/data.htm>

never near two standard deviations away. Thus, with interest rates on government securities, the data conform almost perfectly to the Fisher Effect.

## **V. Conclusion**

Our theoretical results are actually much more general than they might appear. Although we have employed a costly state verification environment, that is not necessary for our theorem to go through. All that is necessary is an environment in which, in bad states of the world, entrepreneurs get nothing and the investor gets the real assets of the firm. Other contracting environments such as Hart and Moore (1998) or Holmstrom and Tirole (1997) should give similar results.

This work also has some policy implications. If one takes nominal private interest rates from inflationary environments and adjusts for inflation, the real rate estimated in this way is very often negative. Boyd, Levine and Smith (2001) did this and concluded that inflation was interfering with financial markets; e.g. with negative real rates corporations arguably could not raise funds. Based on the results presented here, however, such an interpretation may be incorrect. For policy purposes, just estimating real rates of interest in the conventional way is not an adequate guide to the effects of inflation. This could be a significant new area of research for financial development scholars.

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**Table 1: Numerical Examples – Solutions to Problem (5)**

The nominal gross debt payment  $\Lambda$  is the optimal value that is the solution of problem (5). The second column shows solution values when  $P_m = 1$  and the mean rate of inflation is zero; in this case all payoffs are the same as those under real contracting. Columns three and four show solutions when  $P_m = 1.1$  (1.2) and expected average inflation is ten percent (twenty percent). The parameter values are  $U = 21$ ,  $L = 1$ ,  $E = 10$ ,  $t = 0.01$ ,  $M = 8$ .

|                         | <b>Zero Inflation</b> | <b>10% Inflation</b> | <b>20% Inflation</b> |
|-------------------------|-----------------------|----------------------|----------------------|
| $\Lambda$               | 124.706               | 136.285              | 146.368              |
| $\Lambda/P_m$           | 124.706               | 123.895              | 121.973              |
| Average Bankruptcy Cost | 2.09538               | 2.099815             | 2.110425             |
| SOC                     | -0.0000344            | -0.0000292           | -0.0000263           |
| Second Solution(*)      | 706.874               | 763.457              | 797.587              |
| SOC of Second Solution  | 0.0000344             | 0.0000292            | 0.0000263            |

(\*) Recall that we have two sets of first order conditions. The “second solution” can be ignored because it is a minimum in the examples presented here.



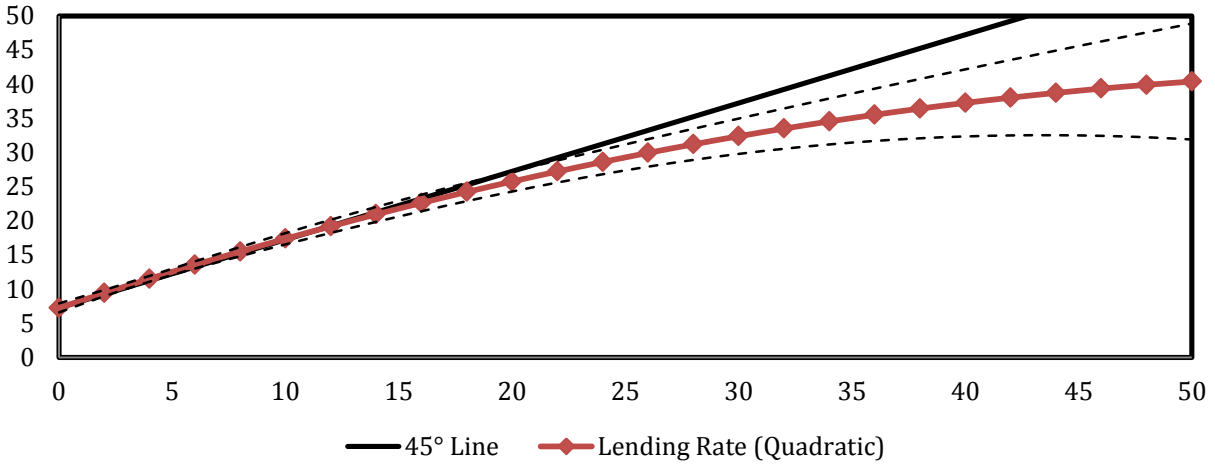
**Table 2. Regressing Interest Rates on Inflation**

Dependent Variables are defined at the top of the columns. Lending Rate is the rate charged by banks on loans to prime customers. M Total Debt is the marginal rate of return on the total debt (long-term + short-term) of firms. M Long Debt is the average marginal rate of return on the long-term debt of firms. Govt Tbill is the short-term rates on government securities. Inflation is the annual CPI inflation. Inflation<sup>2</sup> is the squared value of Inflation. All regressions include year dummy variables and firm or country clustering.

|                        | (1)                     | (2)                    | (3)                    | (4)                   | (5)                   |
|------------------------|-------------------------|------------------------|------------------------|-----------------------|-----------------------|
|                        | Lending Rate            | M Total Debt           | M Long Debt            | Govt TBill            | Govt Tbill            |
| Inflation              | 1.1003<br>(0.0866)***   | 0.4472<br>(0.0899)***  | 0.7564<br>(0.0291)***  | 1.0716<br>(0.1222)*** | 0.9654<br>(0.0706)*** |
| Inflation <sup>2</sup> | -0.0087<br>(29.2962)*** | -0.0004<br>(0.9079)*** | -0.0007<br>(0.2652)*** | -0.0037<br>(51.7371)  |                       |
| Constant               | 7.2437<br>(0.3371)***   | 4.6558<br>(0.3781)***  | 4.1972<br>(0.1093)***  | 2.5506<br>(0.3557)*** | 2.8885<br>(0.3314)*** |
| Observations           | 1198                    | 51477                  | 35409                  | 857                   | 857                   |
| Adj. R-squared         | 0.37                    | 0.15                   | 0.21                   | 0.59                  | 0.58                  |

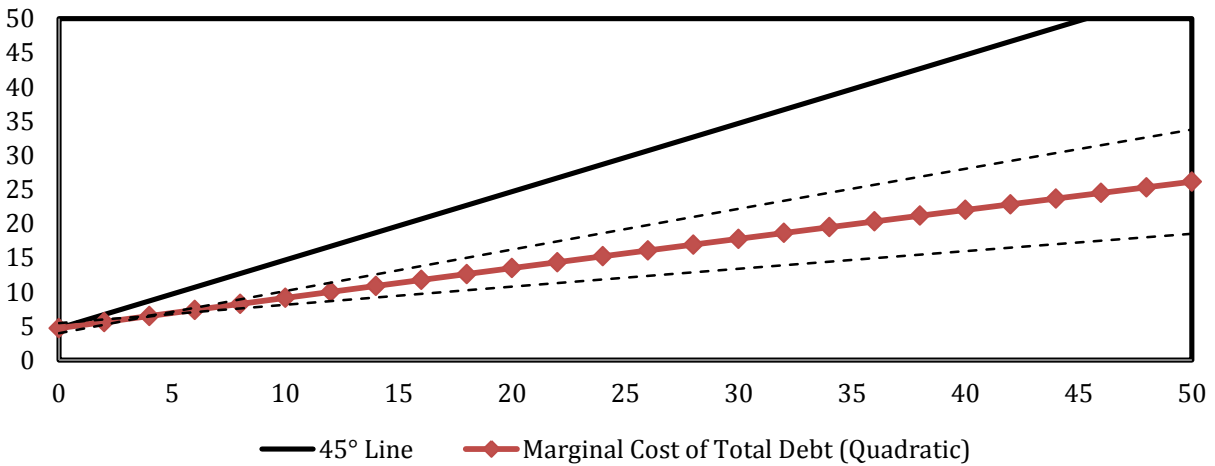
**Figure 1: The Lending Rate and Inflation**

In the horizontal axis, we plot the Inflation rate, while on the vertical axis we plot the predicted Lending Rate from equation (1) in Table 2. The solid black line represents the 45 degree line through the function's intercept. The dotted lines represent the 90% confidence band.



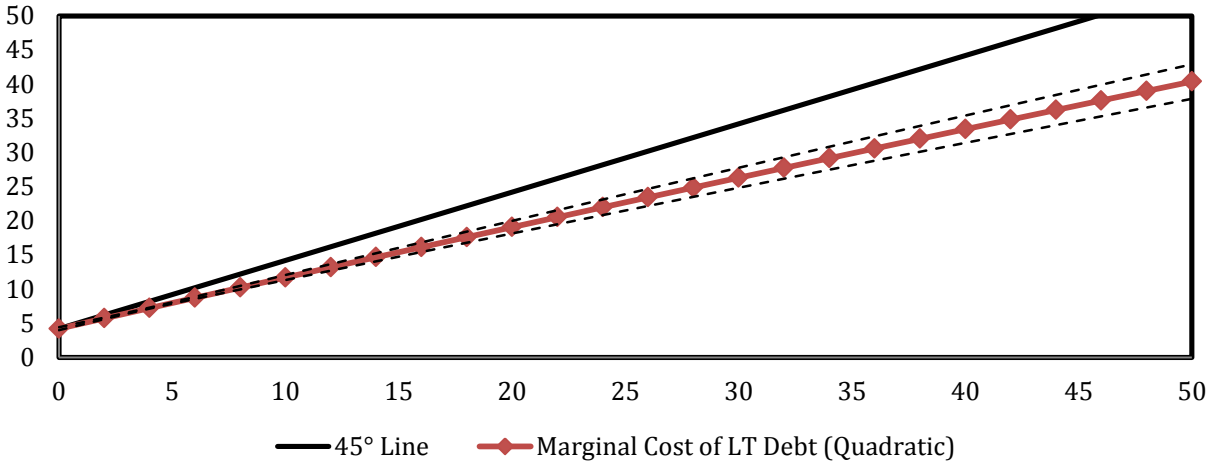
**Figure 2: Marginal Corporate Borrowing Rate, All Maturities, and Inflation**

In the horizontal axis, we plot the Inflation rate, while on the vertical axis we plot the predicted Marginal Cost of Total Debt from equation (2) in Table 2. The solid black line represents the 45 degree line through the function's intercept. The dotted lines represent the 90% confidence band.



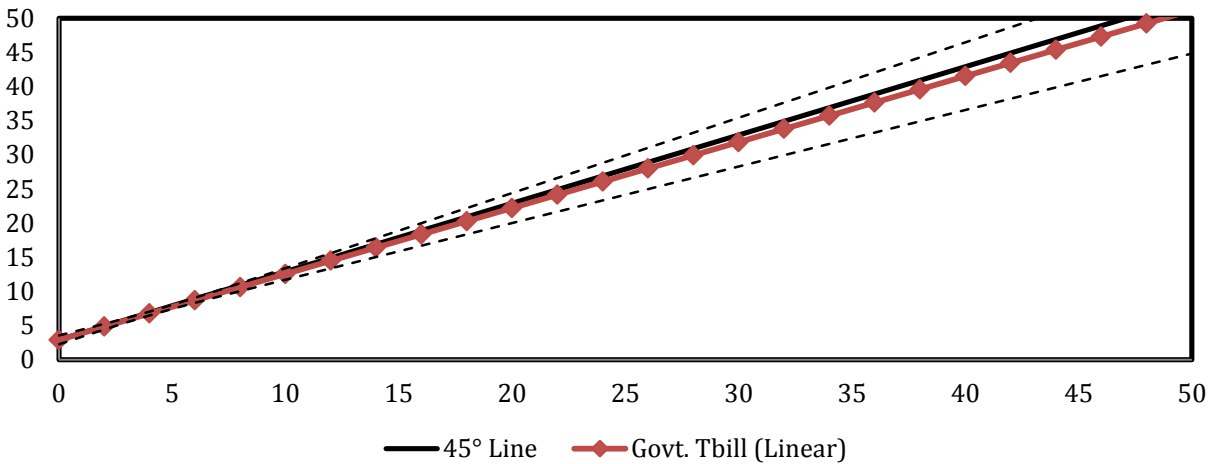
### Figure 3: Marginal Corporate Borrowing Rate, Long-term Debt and Inflation

In the horizontal axis, we plot the Inflation rate, while on the vertical axis we plot the predicted Marginal Cost of Long-term Debt from equation (3) in Table 2. The solid black line represents the 45 degree line through the function's intercept. The dotted lines represent the 90% confidence band.



### Figure 4: Government Tbill Rate, Linear Regression

In the horizontal axis, we plot the Inflation rate, while on the vertical axis we plot the predicted Govt Tbill Rate from equation (5) in Table 2. The solid black line represents the 45 degree line through the function's intercept. The dotted lines represent the 90% confidence band.



**Appendix A: Grid Search to Prove that  $\Lambda_f - \Lambda^* \geq 0$  in the Case where:**

$$\Lambda^* = \frac{P_h P_l (P_h + P_l)}{2(P_h^2 + P_l^2)} [(2U(2E - t) - M) - \sqrt{(2U(2E - t) - M)^2 - 4 \frac{(P_h^2 + P_l^2)}{(P_h + P_l)^2} (2E - t)((L^2 + U^2)(2E - t) - 2LM)}].$$

The parameters that can change are  $t, P_m, E, M, L$  and  $U$ . However, the grid search process is greatly simplified by the existence of numerous bounds.

1.  $(2E - t) > M$ . Endowed resources must be large enough to pay for monitoring costs and the cost of activating the entrepreneurial investment. For present purposes,  $t$  is a nuisance parameter and can be set equal to zero. Further,  $M$  is bounded above by  $E$  and we shall use this feature.
2.  $U > L \geq 0$ . All returns must be non-negative and the upper bound must be strictly greater than the lower bound. Further, it must always be possible to pay the monitoring cost  $M$  and activation cost  $t$  so that  $M + t \leq L$ . It follows that  $M$  is bounded both above and below. Therefore, in our grid we only examine  $M$  relative to  $E$ .
3.  $U \geq 1$ . Otherwise there are no gains to trade.
4.  $L < \frac{\Lambda}{2E-t} < U$ . If this bound is violated, the cost of bankruptcy in low inflation state becomes negative. That is, there is a positive return to going bankrupt. This violates the spirit of the model and such cases are not allowed.
5.  $L < \frac{\Lambda}{(2P_m-1)(2E-t)} < U$ . If this bound is violated, the cost of bankruptcy in high inflation state becomes negative. That is, there is a positive return to going bankrupt. This violates the spirit of the model.

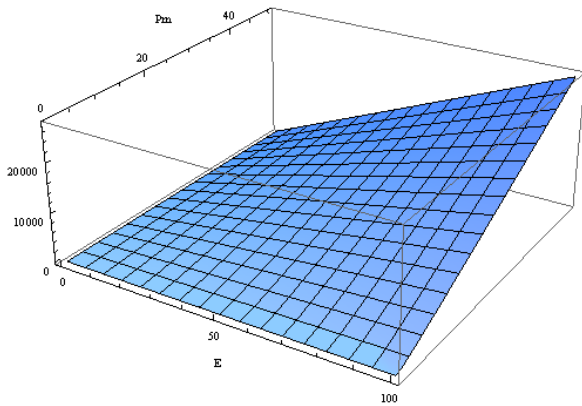
Now, to conduct our grid search, we consider the value of  $\Lambda_f - \Lambda^*$ . First, we fix the values of  $U = 10$ , and  $L = 0$ . Then, we plot the relationship with  $\Lambda_f - \Lambda^*$ . In these graphs, we vary inflation  $P_m$  from 1 to 50 (4900%) and the endowment  $E$  from 0 to 100. The monitoring cost  $M$  enters in relation to the endowment  $E$ . In the first plot, we present the case where the monitoring

cost is zero. In the second, third and fourth plots, the monitoring cost takes on higher and higher values.<sup>19</sup>

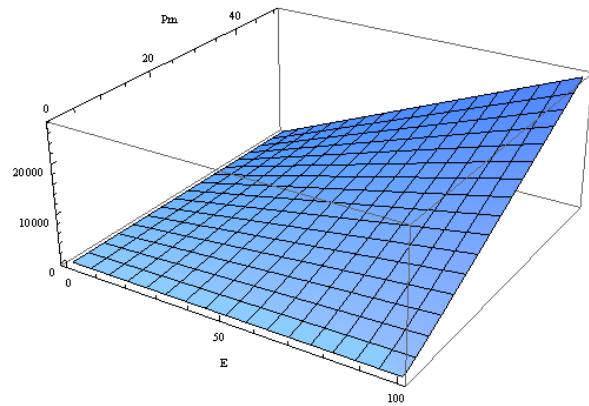
Three conclusions are clear from the graphs. First, as inflation increases,  $\Lambda_f - \Lambda^*$  stays positive and keeps growing larger as inflation increases. Second, as the monitoring cost becomes larger, the possible numerical solutions of  $\Lambda^*$  become a smaller set. Third, when  $Pm$  and  $U$  become very large,  $\Lambda_f - \Lambda^*$  continues growing – there is no evidence of an upper asymptote.

### Plots

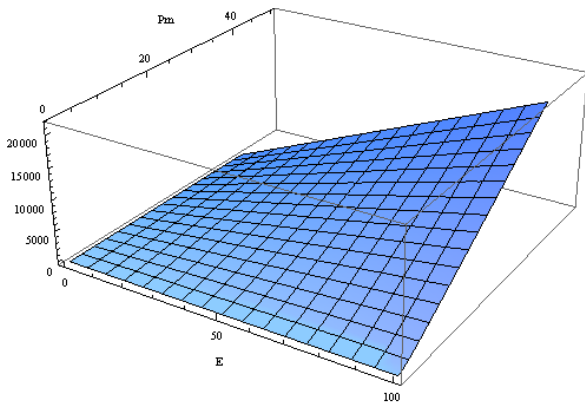
$M = 0.0001$



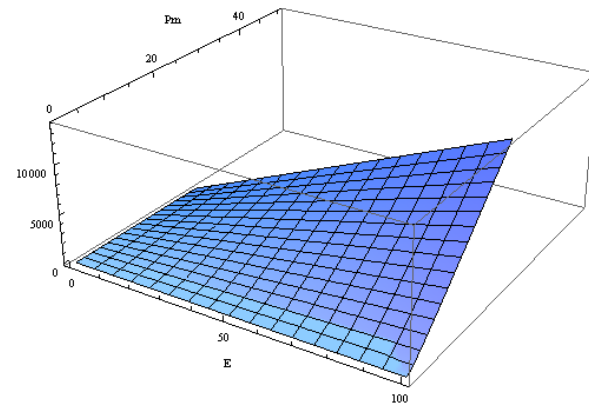
$M = 0.25 * E$



$M = 0.50 * E$



$M = 0.75 * E$



<sup>19</sup> Examining a much finer grid for  $M$  produces the same conclusions.

## **Appendix B – Definitions**

$A$  = nominal debt payment promised to investors

$\Delta$  = the expected real return of entrepreneurs.

$D$  = total payment promised the lender in non-monitor states.

$E$  = initial endowment of good to all agents.

$I$  = real interest payment on debt.

$M$  = monitoring cost. Exogenously fixed, a deadweight cost.

$L$  = lower bound of the real return distribution.

$\Omega$  = the expected real return of investors.

$s$  = parameter determining price level variability,  $s \geq 1$ .

$T$  = total return on entrepreneurial project before any monitoring costs.

$t$  = cost (fixed and deadweight) of activating the entrepreneurial investment.

$U$  = upper bound of the real return distribution (exogenously fixed).

$I(t)$  = interest paid in year  $t$

$D(t)$  = total debt outstanding in year  $t$

$r(t)$  = average interest rate in year  $t$