Corporate Bond VIX*

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April 19, 2019

Abstract

We synthetically create option contracts on a corporate bond index using CDX swaptions, overcoming the limitations that originate from the lack of traded corporate bond options. Our approach allows us to estimate various risk-neutral quantities concerning the corporate bond market in a model-free manner. Specifically, we construct a forward-looking volatility measure, the corporate bond VIX, as well as the variance risk premium, and examine their properties to understand the role of volatility risk in this market. Coupled with the results on higher moments, these analyses reveal the usefulness of our synthetic options in characterizing the aggregate behavior of corporate bonds.

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1 Introduction

The introduction of the CBOE Volatility Index (in short, VIX) brought about a significant breakthrough in understanding volatility risk, which has been one of the central issues in modern finance for academics and practitioners alike. This index, which represents the risk-neutral expectation of future 1-month stock market volatility, is calculated in a model-free manner using the prices of out-of-the-money calls and puts on the S&P 500. Not only is the VIX an important tool for research, but it also provides the foundation for volatility products that are actively traded, such as VIX futures and options.

The VIX pertains to the aggregate stock market, but this is just one facet of the entire capital market. In fact, an even larger portion of the capital market is occupied by the bond market. Given that volatility risk is a universal subject regardless of markets, one would naturally expect similar volatility indices for the bond market. Not surprisingly, the CBOE has recently launched a new volatility index called the Treasury VIX, or TYVIX, which is a forward-looking volatility measure similar to the VIX, but based on Treasury securities and their options.[1]

Yet, what is still missing is an index that captures volatility risk in the corporate bond market whose market capitalization is as large as the Treasury market. The innate volatility risk of corporate bonds is very different from that of default-free government bonds: while the latter is entirely driven by interest rate volatility, the former is additionally affected by credit risk volatility. As a matter of fact, the prices of corporate bonds with floating rate coupons (namely, floating rate notes or FRNs) are insensitive to interest rate risk and, therefore, are completely driven by credit risk.

In this paper, we construct a volatility index for the corporate bond market, which we call the corporate bond VIX (or simply, CBVIX), from March 2012 to September 2018. This index measures the risk-neutral expectation of future 1-month volatility on an aggregate

[1] Specifically, the TYVIX represents the risk-neutral expectation of future 1-month volatility of 10-year maturity Treasury note futures.

corporate bond price index that consists of 5-year FRNs issued by various investment grade firms. To use a similar model-free approach adopted by the VIX and TYVIX, it is essential to acquire a cross section of options on the corporate bond index. However, such data do not exist: neither options written on aggregate bond indices nor ones on corporate bond ETFs are actively traded with meaningful cross sections.

We tackle this issue by using credit derivatives. First, we exploit the fact that a defaultable FRN issued by a certain firm can be replicated by the portfolio of (i) the corresponding risk-free FRN and (ii) the credit default swap (CDS) contract that is exposed to the firm's credit risk. The price of the risk-free FRN is essentially par because floating rate coupons bear little interest risk. This implies that, under no arbitrage, the price of the firm's synthetic FRN with a 1% quoted margin should equal the face value of the bond minus the quoted upfront fee for the firm's CDS contract.

Then, we create our synthetic corporate bond index as the average price of synthetic FRNs issued by a large number of investment grade firms. To capture the volatility of the aggregate corporate bond market, we choose the pool of firms in the CDX North American Investment Grade index (in short, CDX). Since the CDX represents 125 equally-weighted investment grade single-name CDS contracts across five industrial sectors, it can serve as a good proxy for the average behavior of the corporate bond market. Consequently, we show that the future level of the synthetic corporate bond index can be expressed in terms of the future upfront fee for the CDX and the accumulative loss due to defaults in the CDX.

Lastly and most importantly, we use the data on CDX swaptions. An important advantage of using the CDX for our analyses is that its "calls" and "puts" are actively traded. These options are called credit swaptions because they grant investors the right (but not the obligation) to enter into a 5-year CDX contract. A receiver swaption allows the holder to enter into a contract as the protection seller while a payer swaption allows the holder to do so as the protection buyer. We show that using these CDX swaptions, it is possible to replicate call and put options on the 5-year synthetic corporate bond index, which enable us to calculate various risk-neutral quantities in a model-free fashion. The CBVIX is one such example.

The resulting CBVIX shows significant time series variations with a sample mean of 1.76%. The CBVIX is high when the synthetic corporate bond index is low, implying asymmetric volatility in the corporate bond market. While its level is much lower compared to the equity VIX, they fluctuate in a consistent manner with a correlation of 0.71. Although both are based on bonds, the CBVIX and the TYVIX have a weaker correlation of 0.43, showing similar yet distinct patterns.

Based on the CBVIX, we examine the role of variance risk in the corporate bond market by constructing the monthly time series of the variance risk premium, which is defined as the difference between the risk-neutral and physical expectations of future corporate bond index variance. We find that most of the time, the corporate bond variance risk premium is positive and shows substantial time variations. Furthermore, while the corporate bond variance risk premium has negative contemporaneous correlations with bond and equity returns, it positively predicts future bond and equity returns. The predictability remains significant even when we control for the equity variance risk premium. These robust empirical results suggest that the variance risk premium in the corporate bond market captures an important source of systematic risk shared by both bond and equity markets.

Motivated by Martin (2017), we construct an additional model-free volatility index called the simple CBVIX, which measures the 1-month risk-neutral volatility of the price relative of the corporate bond index (or equivalently, that of the simple return on the corporate bond index), scaled by the gross risk-free rate. While the time series of the CBVIX and the simple CBVIX closely resemble one another, we find that the CBVIX is always larger than the simple CBVIX. As Martin (2017) points out, this finding indicates a rejection of the assumption that the corporate bond index return and the pricing kernel are conditionally log-normal at the 1-month horizon.

Our model-free option-based estimation is not limited to the second moment: we further estimate higher-order moments of the corporate bond index and find large magnitudes of (negative) skewness and kurtosis, especially during the eurozone debt crisis. In addition, we nonparametrically estimate the distributions of the price relative under the risk-neutral and physical measures and discover that they imply a non-monotonic U-shaped pricing kernel.

In sum, this paper highlights the usefulness of CDX swaptions in studying the corporate bond market. Normally, corporate bonds are issued with various maturities, coupons, and embedded options, which makes it difficult to create an aggregate price index and trade option contracts written on it. We circumvent this problem by replicating option contracts on our synthetic corporate bond index using CDX swaptions. Equipped with these synthetic options, one can take advantage of a vast array of option-based tools that the literature has developed for the stock market and gain more insight into the corporate bond market.

Our paper builds on prior studies that propose a model-free approach to estimate the risk-neutral moments of future returns based on options. According to Bakshi and Madan (2000), Carr and Madan (2001), and Bakshi, Kapadia, and Madan (2003), we can span the risk-neutral expectation of a twice-differentiable payoff function by using prices of out-of-the money European calls and puts. Generally, these papers apply this model-free approach to equity options in order to study the properties of the equity market. While we adopt the same techniques, we instead apply them to synthetic bond index options to examine the properties of the corporate bond market.

With the aid of the model-free option-based approach, we make a contribution to the literature on the variance risk premium. Bollerslev, Tauchen, and Zhou (2009) find that their constructed equity variance risk premium is significantly positive and predicts future equity market returns.[2] Consistent with the findings in the equity market, we obtain a positive variance risk premium in the corporate bond market, which robustly predicts future bond and equity returns.

[2]Related papers in the literature include Coval and Shumway (2001), Bakshi and Kapadia (2003), Carr and Wu (2009), Todorov (2010), Aït-Sahalia, Karaman, and Mancini (2015), and Eraker and Wu (2017).

In the credit risk literature, credit derivatives have become increasingly important tools for understanding the underlying firm dynamics behind the high credit spread in the data. For example, based on an estimation with CDS spreads, Du, Elkamhi, and Ericsson (2018) emphasize the role of time-varying asset volatility in resolving the credit spread puzzle.[3] Relatedly, Kelly, Manzo, and Palhares (2018) create a credit-implied volatility surface from CDS spread data via the Merton (1974) model and find a three-factor structure in the surface. Whereas these papers largely focus on asset volatility, our main interest is in extracting bond market volatility.

Our paper also relates to the extensive literature on corporate bond returns. In this literature, empirical analyses are typically based on the time series and the cross section of corporate bond returns. Fama and French (1993) stress the importance of the default and term premia in explaining corporate bond returns.[4] As additional examples, Bai, Bali, and Wen (2016) study the distributional characteristics of historical corporate bond returns, and Bai, Bali, and Wen (2018) investigate the cross-sectional determinants of corporate bond returns. Our paper differs from these papers in that we extract ex-ante risk-neutral moments using CDX swaptions.

CDX swaptions are similar to CDX tranches in that they are both derivative contracts on the CDX. Prior to the subprime mortgage crisis, CDX tranches were much more actively traded than CDX swaptions, and, therefore, received greater attention in the empirical literature.[5] However, the market for CDX tranches abruptly went cold after the crisis: investors became reluctant to trade collateralized debt obligation (CDO) products like CDX

[3]The credit spread puzzle reflects the inability of structural credit risk models in explaining observed credit spreads. See, for example, Eom, Helwege, and Huang (2004), Huang and Zhou (2008), Huang and Huang (2012), and Feldhütter and Schaefer (2018).

[4]See, also, Gebhardt, Hvidkjaer, and Swaminathan (2005), Lin, Wang, and Wu (2011), Acharya, Amihud, and Bharath (2013), Jostova, Nikolova, Philipov, and Stahel (2013), Bongaerts, de Jong, and Driessen (2017), and Choi and Kim (2018).

[5]For example, Coval, Jurek, and Stafford (2009) and Collin-Dufresne, Goldstein, and Yang (2012) debate about the mispricing of CDX tranches during the period preceded by the crisis. Other empirical papers that concern CDX tranches are Longstaff and Rajan (2008), Choi, Doshi, Jacobs, and Turnbull (2018), and Seo and Wachter (2018).

tranches because they were singled out for their role in the crisis. The irony is that this opened up a new opportunity for CDX swaptions. To hedge credit risk, investors started to rely more on CDX swaptions, and as a result, their liquidity has drastically been improving since 2012. To the best of our knowledge, this is the first paper to make use of the information contained in CDX swaptions whose importance in the credit market is on the rise.

The paper proceeds as follows. Section 2 presents an aggregate price index for corporate bonds based on the CDX. In Section 3, we show that puts and calls on this corporate bond index can be replicated using CDX swaptions. Section 4 describes our data sample. Section 5 details our empirical analyses. Section 6 concludes.

2 A Price Index for the Corporate Bond Market

In this section, we synthetically create an aggregate price index of investment grade corporate bonds. What sets corporate bonds apart from government bonds is their credit risk exposures. To focus on the credit component of the corporate bond market, we consider floating rate notes (FRNs), which are immune to fluctuations in risk-free interest rates. In Section 2.1, we first replicate an individual FRN using a default-free FRN and a credit default swap (CDS) contract. In Section 2.2, we show that our synthetic corporate bond index, defined as the average price of synthetic FRNs issued by various investment grade firms, can be constructed using the CDX North American Investment Grade Index (in short, the CDX).

2.1 Synthetic FRNs

Typically, an FRN pays quarterly coupons that are calculated as the sum of (i) the risk-free benchmark interest rate, which resets every 3 months, and (ii) a quoted margin, which is an additional spread that remains fixed as a compensation for credit risk. In our analysis, we consider an FRN issued by a certain firm i with a quoted margin of 1%. As Duffie and Singleton (2003) and Bao and Pan (2013) discuss, this defaultable FRN can be replicated by

(i) investing in a default-free FRN and (ii) selling protection on the firm's CDS in exchange for receiving 1% as the insurance premium.[6] Here, the default-free FRN and the CDS contract should have the same notional value and the same maturity as the given defaultable FRN.

It is evident that this replication strategy works. First, each coupon paid by the defaultable FRN is replicated by the sum of the coupon payment from the default-free FRN (the benchmark interest rate) and the premium from the CDS contract (1%). Second, if the firm survives until maturity, the defaultable FRN expires and pays out its face value. The same payoff is delivered by the replicating portfolio because the default-free FRN contained in it matures at the same time with the same face value. Lastly, in the event of the firm's default, the defaultable FRN experiences a loss. The replicating portfolio suffers from an identical amount of loss due to the protection sell position on the firm's CDS contract.[7]

Therefore, under no arbitrage, the price of the defaultable FRN should equal the cost of implementing the replication strategy.[8] Note that the default-free FRN carries little interest rate risk: since it pays coupons that exactly mirror the risk-free benchmark rate every 3 months, the price of the default-free FRN remains close to the par value.[9] This implies that the price of the defaultable FRN can be calculated as the face value of the bond plus the cost of entering into the firm's CDS contract with a 1% coupon spread. Specifically, by defining

[6] A (single-name) CDS contract transfers the credit risk of a certain reference entity from one party to another. In the event of the reference entity's default (more generally, a credit event), the protection seller should either undertake a defaulted bond from the protection buyer at par (physical settlement), or directly make up for the loss of the protection buyer by paying the difference between the par value and the market value of the defaulted bond (cash settlement). In return, the protection buyer periodically pays an insurance premium to the protection buyer until a credit event occurs or until the CDS contract matures, whichever comes first.

[7] To be more concrete, suppose that the firm's loss rate upon default is L_i. That is, the value of the FRN upon default is $\$\,[1 - L_i]$ for a dollar face value. This is identical to the value of the replicating portfolio: the holder of the portfolio sells the default-free FRN at par ($1) and uses the proceeds to make up for the loss of the CDS counterparty (-$\$L_i$), which will leave the holder $\$\,[1 - L_i]$.

[8] Although, in theory, this replication argument should always hold for every firm, the CDS-bond basis significantly deviated from zero during the Great Recession period, which poses a puzzle (see, e.g., Bai and Collin-Dufresne, 2013). During our sample period, this basis is relatively small and stable. As long as the basis is not too volatile, it should not bias our volatility measure.

[9] Generally, the price of a default-free FRN is exactly par at coupon reset dates. Even between two adjacent coupon reset dates, the price is essentially par because the effective duration is less than 3 months regardless of bond maturity.

$P_{i,t}^{(T)}$ as the time-t defaultable FRN price per a dollar face value, it follows that

$$P_{i,t}^{(T)} \;\; = \;\; 1 + \Big(\text{time-}t \text{ cost of selling protection on a \$1 CDS}\Big) \, .$$

In the market, it is indeed possible to directly observe the cost of entering into such a CDS contract. In April 2009, the International Swap and Derivatives Association (ISDA) introduced the Standard North American Contract (SNAC) in an attempt to standardize CDS transactions and facilitate central clearing. The most distinctive feature of the SNAC is that the coupon spread for an investment grade CDS is always set to be 1%.[10] Since the coupon spread is fixed, market fluctuations are instead captured by the upfront fee exchanged between the two involved parties at the beginning of the contract.

To illustrate this, let $S_{i,t} = S_i(t, t+T)$ denote the T-maturity fair market CDS spread for firm i at time t. We define $\Pi_{i,t} = \Pi_i(t, t+T)$ as the risky PV01, the time-t present value of a risky annuity that pays a series of unit coupons until maturity or firm i's default, whichever occurs first. Under a traditional/non-standardized contract, the coupon spread is determined as the CDS spread, and, therefore, the present value of future premium payments equals $S_{i,t} \times \Pi_{i,t}$. In contrast, under a standardized contract, the present value of future premium payments is $0.01 \times \Pi_{i,t}$ because the coupon spread is fixed at 1%. This discrepancy is resolved by an upfront fee $U_{i,t} = U_i(t, t+T)$ paid by the protection buyer to the protection seller at time t:

$$U_{i,t} + 0.01 \times \Pi_{i,t} = S_{i,t} \times \Pi_{i,t}. \tag{1}$$

In sum, under the SNAC, the protection seller enters into a standardized CDS contract by receiving the quoted upfront fee from the protection buyer.[11] This implies that the time-t

[10]The coupon spread is fixed at 5% for speculative grade entities.

[11]Note that $U_{i,t} = [S_{i,t} - 0.01] \times \Pi_{i,t}$ can also be negative. When the market CDS spread is smaller than 1%, the coupons that the protection seller receives are too high compared to the fair level. Thus, the protection seller should make an upfront payment to the protection buyer, implying a negative $U_{i,t}$.

cost of selling protection on a dollar CDS contract is simply the negative of $U_{i,t}$. Finally, the defaultable FRN price can be expressed as

$$P_{i,t}^{(T)} \;=\; 1 - U_i(t, t+T). \tag{2}$$

Equation (2) is intuitive. Note that the value of the FRN should be par when its 1% quoted margin is a fair compensation for the firm's credit risk. When the fair CDS spread for the firm is 1%, the upfront fee becomes zero according to equation (1), which makes the FRN price indeed equal to 1 in equation (2). If the fair CDS spread becomes higher (lower) than 1%, the 1% quoted margin from the FRN is relatively too low (high) compared to the firm's credit risk, which brings the FRN price below (above) 1. This is confirmed in equation (2) as the upfront fee is quoted as a positive (negative) value.

How does the price of the defaultable FRN evolve as time progresses? After a τ period, the FRN price is denoted as $P_{i,t+\tau}^{(T-\tau)}$ as the time-to-maturity of the bond reduces to $T-\tau$. If the firm survives up to time $t+\tau$, it follows from the same no-arbitrage argument used in equation (2) that

$$P_{i,t+\tau}^{(T-\tau)} \;=\; 1 - U_i(t+\tau, t+T) \quad \text{if firm } i \text{ survives up until time } t+\tau, \tag{3}$$

where $U_i(t+\tau, t+T)$ represents the quoted upfront fee at time $t+\tau$ for the firm's CDS contract maturing at time $t+T$. However, if the firm goes into default by time $t+\tau$, the value of the FRN is expressed as

$$P_{i,t+\tau}^{(T-\tau)} \;=\; 1 - L_i \quad \text{if the firm defaults by time } t+\tau, \tag{4}$$

where L_i is the firm's loss rate given default.

Equations (3) and (4) reveal that the defaultable FRN price varies due to two reasons. First, the firm might go into default, which causes the value of the FRN to fall significantly from its face value. Second, even if the firm survives, fluctuations in the firm's default risk, which are reflected in variations in the firm's CDS upfront fee, can change the FRN price.

2.2 The CDX-Implied Synthetic Corporate Bond Index

Equation (2) suggests a CDS-based approach to create a price index for corporate bonds. For each time t, the synthetic corporate bond index P_t is defined as an equally weighted price index composed of N FRNs issued by N different firms:

$$P_t^{(T)} = \frac{1}{N}\sum_{i=1}^{N} P_{i,t}^{(T)} = 1 - \frac{1}{N}\sum_{i=1}^{N} U_i(t, t+T). \tag{5}$$

From equation (5), we can see that our synthetic corporate bond index can be replicated by purchasing a default-free FRN with \$1 face value and by taking protection sell positions on N single-name CDS contracts, each of which has a notional value of $\$\left[\frac{1}{N}\right]$.

To capture the aggregate behavior of the investment grade corporate bond market, it is important to choose a pool of firms that is representative of the entire market. To this end, we exploit the CDX, a credit index that consists of 125 investment grade debt obligations, evenly distributed across five different industrial sectors (Consumer; Energy; Financial; Industrial; Technology, Media, and Telecommunications). The CDX rolls on a semi-annual basis every March and September to ensure that it tracks the most liquid investment grade entities and to keep the maturity of the index roughly constant. When the new series, so-called "on-the-run" series, is introduced, the previous series then becomes "off-the-run." Most importantly, among various credit derivatives, the CDX is the most popular with the highest trading volume.[12] For these reasons, we believe that the CDX can serve as a good proxy for the average behavior of the credit market.

While the CDX is an index that represents the average CDS spread of 125 firms, it is traded as an independent product whose quoted upfront fee, denoted as U_{CDX}, is readily

[12]For more information about the liquidity and trading volume of this index, see Appendix A.

observable in the market.[13] Since entering into a CDX contract as the protection seller is essentially equivalent to taking equally-weighted protection sell positions on all of the 125 single-name CDS contracts that comprise the index, the synthetic corporate bond index can simply be calculated as

$$P_t^{(T)} \;=\; \frac{1}{N}\sum_{i=1}^{N} P_{i,t}^{(T)} = 1 - U_{\text{CDX}}(t, t{+}T). \tag{6}$$

After a τ period, the synthetic corporate bond index becomes $P_{t+\tau}^{(T-\tau)} = \left[\frac{1}{N}\sum_{i=1}^{N} P_{i,t+\tau}^{(T-\tau)}\right]$, the average of FRN prices in the index at time $t{+}\tau$. Since each FRN price in the index is described by either equation (3), provided the firm survives, or equation (4), provided the firm defaults, we can show that

$$\begin{aligned} P_{t+\tau}^{(T-\tau)} &= \frac{1}{N}\sum_{i\notin\mathbb{D}_{t+\tau}}\Big[1 - U_i(t{+}\tau, t{+}T)\Big] + \frac{1}{N}\sum_{i\in\mathbb{D}_{t+\tau}}\Big[1 - L_i\Big] \\ &= 1 - \frac{1}{N}\sum_{i\notin\mathbb{D}_{t+\tau}} U_i(t{+}\tau, t{+}T) - \frac{1}{N}\sum_{i\in\mathbb{D}_{t+\tau}} L_i, \end{aligned} \tag{7}$$

where $\mathbb{D}_{t+\tau}$ is the set of indices for the firms defaulted by time $t{+}\tau$.

We can reinterpret equation (7) in the context of the CDX. After a total of $N_{t+\tau}^{\text{d}}$ defaults, the CDX represents the average of the remaining $N_{t+\tau}^{\text{s}} = N - N_{t+\tau}^{\text{d}}$ firms in the pool. Accordingly, the upfront fee for the CDX is quoted to capture the average upfront fee for the surviving firms, $\left[\frac{1}{N_{t+\tau}^{\text{s}}}\sum_{i\notin\mathbb{D}_{t+\tau}} U_i(t{+}\tau, t{+}T)\right]$. This implies that the synthetic corporate bond index at time $t{+}\tau$ can be computed as

$$P_{t+\tau}^{(T-\tau)} \;=\; 1 - \left(\frac{N_{t+\tau}^{\text{s}}}{N}\right) U_{\text{CDX}}(t{+}\tau, t{+}T) - L_{\text{CDX},t+\tau}, \tag{8}$$

where $L_{\text{CDX},t+\tau} = \left[\frac{1}{N}\sum_{i\in\mathbb{D}_{t+\tau}} L_i\right]$ is the cumulative loss of the CDX due to defaults of $N_{t+\tau}^{\text{d}}$ firms.

[13] CDX contracts are highly standardized products that comply with the SNAC.

In the data, it is extremely rare to observe a case in which an investment grade entity goes into default within a short period of time. In fact, since the CDX was introduced in 2003, there have been zero defaults in the CDX on-the-run series.[14] In other words, once an investment grade entity entered into a new on-the-run CDX series, it survived for at least 6 months. This means that the realized time series of our synthetic corporate bond index in the data has been entirely driven by fluctuations in default risk, not the occurrence of defaults itself.

However, this does not mean that we can simply disregard future possible realizations of defaults in the index. For instance, the expected loss of the index due to potential defaults can have a non-negligible effect on ex-ante or forward-looking volatility of the index. The impact of default occurrences can play a more significant role when it comes to risk-neutral volatility. Instances in which investment grade firms default within a short period of time are likely to coincide with a very bad economic state with high marginal utility, and, thus, it is possible that the risk-neutral measure puts much more weight on such scenarios.

For this reason, the historical time series of the synthetic corporate bond index alone cannot paint the whole picture of the investment grade corporate bond market. Instruments that can help are option contracts written on the future index level, which we construct in Section 3 using CDX swaptions.

3 Options on the Synthetic Corporate Bond Index

How do we obtain prices of call and put options written on the future level of the synthetic corporate bond index, $P_{t+\tau}^{(T-\tau)}$? In this section, we show that they can be obtained by making use of the time-t prices of CDX swaptions that expire at time $t+\tau$.

CDX swaptions are credit swaptions that allow investors to enter into a CDX contract in

[14] Only two credit events did occur: Fannie Mae and Freddie Mac. However, these events were not really defaults. These two companies were acquired by the government and became default-free entities (i.e. conservatorship).

the future at a given upfront fee. A payer CDX swaption provides the holder the right to enter into a $(T-\tau)$-maturity CDX contract after a τ period from today as the protection buyer (who "pays" insurance premiums) at a strike upfront fee K_{U}. On the other hand, a receiver swaption provides the holder the right to enter into the same contract as the protection seller (who "receives" insurance premiums). These instruments are typically European-style options that can only be exercised at maturity. Under our notation, τ represents the time to maturity of CDX swaptions.

For simplicity, first consider a case in which no firms went into default by time $t+\tau$, the maturity date of CDX swaptions. A payer CDX swaption is only exercised when the quoted upfront fee at maturity, $U_{\text{CDX}}(t+\tau, t+T)$, is larger than the strike K_{U}. This is because in this case, the holder can buy protection for paying a lower-than-the-fair upfront fee. Specifically, the holder can lock in a positive payoff of $[U_{\text{CDX}}(t+\tau, t+T) - K_{\text{U}}]$ at maturity: the holder pays K_{U} when exercising the option to acquire a protection buy position, and receives $U_{\text{CDX}}(t+\tau, t+T)$ when immediately closing out this position by taking an opposite position (i.e. protection sell position) at the market rate. In other words, the payoff of a payer swaption is written as

$$V^{\text{Pay}}_{\text{CDX},t+\tau} = \max\left[U_{\text{CDX}}(t+\tau, t+T) - K_{\text{U}}, 0\right], \quad \text{provided no defaults by time } t+\tau. \quad (9)$$

In contrast, a receiver CDX swaption is only exercised when the quoted upfront fee at maturity is smaller than the strike because in this case, the holder can sell protection for receiving a higher-than-the-fair upfront fee. The payoff of a receiver swaption is expressed as

$$V^{\text{Rcv}}_{\text{CDX},t+\tau} = \max\left[K_{\text{U}} - U_{\text{CDX}}(t+\tau, t+T), 0\right], \quad \text{provided no defaults by time } t+\tau. \quad (10)$$

It is important to note that the payoffs of CDX swaptions in equations (9) and (10) are derived under the assumption that there were zero defaults by the expiration date of

CDX swaptions. What would happen at maturity if some firms in the CDX went into default over the life of CDX swaptions? In such occasions, CDX swaptions provide so-called "front-end" protection. When a payer swaption is exercised, the holder not only obtains the protection buy position on a CDX contract, but also collects the protection payment regarding past defaults from the option writer. When a receiver swaption is exercised, the holder should provide the option writer the protection payment when receiving the protection sell position.[15] Therefore, even when $U_{\text{CDX}}(t+\tau, t+T)$ is smaller than K_{U}, it is possible that a payer option is exercised and a receiver swaption is not, due to front-end protection.

To be concrete, consider a scenario in which five out of the 125 firms (i.e. 4% of the index) went into default before the exercise date. To simplify the example, assume that each firm's bond price after its default was 50% of the par value. This means that the CDX as a whole experienced a loss of $4\% \times (1-0.5) = 2\%$. Without any defaults, the holder of a payer (receiver) CDX swaption would pay (receive) K_{U} dollars and enter into a dollar CDX contract as the protection buyer (seller). However, since 4% of the index was already defaulted in this example, the holder enters into a CDX contract only with a notional amount of 96 cents. Additionally, due to the loss caused by the defaulted firms, the holder receives (pays) an immediate compensation, or front-end protection, of 2 cents.

We can mathematically formulate these payoffs as follows:

$$\begin{aligned} V^{\text{Pay}}_{\text{CDX},t+\tau} &= \max\left[\left(\frac{N^{\text{s}}_{t+\tau}}{N}\right) U_{\text{CDX}}(t+\tau, t+T) + L_{\text{CDX},t+\tau} - K_{\text{U}}, 0\right], \\ V^{\text{Rcv}}_{\text{CDX},t+\tau} &= \max\left[K_{\text{U}} - \left(\frac{N^{\text{s}}_{t+\tau}}{N}\right) U_{\text{CDX}}(t+\tau, t+T) - L_{\text{CDX},t+\tau}, 0\right], \end{aligned}$$

where $\left[\frac{N^{\text{s}}_{t+\tau}}{N}\right]$ is the fraction of firms that survived by the options' maturity, and $L_{\text{CDX},t+\tau}$ is the cumulative loss of the CDX, consistent with the notation in Section 2.2. These equations reduce to equations (9) and (10) when there are no defaults, as $N^{\text{s}}_{t+\tau} = N$ and $L_{\text{CDX},t+\tau} = 0$.

[15] This front end protection is contingent on options being exercised: if options are not exercised, the protection payment is not made in either type of options.

These payoff structures clearly indicate that using CDX swaptions, it is possible to replicate payoffs of hypothetical option contracts written on our synthetic corporate bond index. Since equation (8) implies that

$$\left(\frac{N^{s}_{t+\tau}}{N}\right) U_{\mathrm{CDX}}(t+\tau, t+T) + L_{\mathrm{CDX},t+\tau} \;\; = \;\; 1 - P^{(T-\tau)}_{t+\tau},$$

the payoffs of CDX swaptions can be rewritten in terms of the synthetic corporate bond index at time $t+\tau$:

$$V^{\mathrm{Pay}}_{\mathrm{CDX},t+\tau} \;\; = \;\; \max\left[\left(1 - K_{\mathrm{U}}\right) - P^{(T-\tau)}_{t+\tau}, 0\right], \tag{11}$$

$$V^{\mathrm{Rcv}}_{\mathrm{CDX},t+\tau} \;\; = \;\; \max\left[P^{(T-\tau)}_{t+\tau} - \left(1 - K_{\mathrm{U}}\right), 0\right]. \tag{12}$$

Essentially, equation (11) establishes that each payer CDX swaption with strike K_{U} has the same payoff as the European put option on the synthetic corporate bond index with the corresponding strike $K_{\mathrm{P}} = 1 - K_{\mathrm{U}}$. Since the two options generate identical payoffs at maturity, their prices at time t should also be identical under no arbitrage. Similarly, equation (12) demonstrates that the price of each receiver CDX swaption with strike K_{U} should be the same as that of the European call option on the synthetic corporate bond index with the corresponding strike $K_{\mathrm{P}} = 1 - K_{\mathrm{U}}$.

In sum, the prices of CDX swaptions enable us to directly acquire the prices of puts and calls written on an aggregate price index for the corporate bond market, as long as we adjust the strike dimension.

4 Data on CDX Swaptions

The daily on-the-run CDX and CDX swaptions data are obtained from a major investment bank. Our sample period is from March 2012 to September 2018, as CDX swaptions started actively trading from 2012. CDX swaptions in our sample are with a 1-month maturity

($\tau = 1/12$). Once exercised, option contracts deliver an on-the-run CDX contract that expires approximately 5 years from today ($T = 5$).[16]

In the market, CDX swaptions are quoted in terms of Black-implied volatilities: for each swaption, the volatility term in the Black formula (equations (B.3) and (B.4) in Appendix B) is backsolved to match its market price, and the resulting volatility is called the Black-implied volatility. Note that this is simply the standard market practice for quoting credit swaptions, which has nothing to do with the validity of the Black model.

Appendix B describes the derivation of the Black formula in detail. It is worth noting that the Black model concerns non-standardized single-name credit swaptions. Recall that under the SNAC, when credit swaptions are exercised, the holder enters into a standardized CDS contract with a fixed deal spread of 1% and receives/pays the strike upfront fee K_{U} at the beginning of the CDS contract. However, in the case of non-standardized credit swaptions, the holder enters into a non-standardized CDS contract at a given strike spread K_{S} without an upfront payment. In other words, the Black formula ignores the adoption of the SNAC and assumes that if swaptions are exercised, the holder enters into a CDS contract with a deal spread of K_{S}.[17] Therefore, under the Black model, the moneyness of credit swaptions is expressed in terms of the strike spread K_{S}.

Black-implied volatilities in our sample are for at-the-money (ATM) and out-of-the-money (OTM) CDX swaptions across a wide range of strike spreads (K_{S}). Specifically, for each day, strike spreads are from 65% to 135% of the spot spread with 5% intervals. We convert Black-implied volatilities into CDX swaption prices using the Black formula in

[16]In fact, the maturity of each on-the-run CDX series experiences small variations from 5.25 years (when it is first introduced) to 4.75 years (when it becomes off-the-run). For computational simplicity, we assume that the maturity of the CDX is always 5 years.

[17]Furthermore, the Black formula assumes knockout swaptions: when the firm defaults before maturity, it assumes that swaptions disappear without any payments. In the U.S., standard credit swaptions are non-knockout swaptions that do not cancel at default. In the case of a non-knockout payer swaption, the holder is still able to exercise the option at maturity and enter into a CDS contract as the protection buyer. Since the reference entity is already defaulted, this protection buy position immediately allows the holder to collect a protection payment from the counterparty or deliver a defaulted bond at par. In contrast, a non-knockout receiver swaption would never be exercised because it is not profitable to sell protection on an already defaulted entity.

equations (B.3) and (B.4). We also convert each strike spread (K_{S}) into the corresponding strike upfront fee (K_{U}). Note that CDX swaptions are under the SNAC, so the correct strike dimension is K_{U}. However, since the Black model ignores the SNAC, Black-implied volatilities are with K_{S}. Fortunately, the mapping from K_{S} to K_{U} is simple: we can calculate the corresponding K_{U} as the CDX upfront fee (under the SNAC) when the CDX spread is K_{S}.[18]

Through this entire process, for each day, we are able to collect the cross section of CDX swaption prices across various strike upfront fees. To signify that CDX swaption prices depend on the strike upfront fee K_{U}, we denote the time-t CDX swaption prices as $V^{\mathrm{Pay}}_{\mathrm{CDX},t}(\tau; K_{\mathrm{U}})$ and $V^{\mathrm{Rcv}}_{\mathrm{CDX},t}(\tau; K_{\mathrm{U}})$. Using the prices of CDX swaptions, we finally obtain the prices of puts and calls written on the synthetic corporate bond index, which are denoted as $V^{\mathrm{Put}}_t(\tau; K_{\mathrm{P}})$ and $V^{\mathrm{Call}}_t(\tau; K_{\mathrm{P}})$. As we discuss in Section 3, they can be found from CDX swaption prices based on the following mapping:

$$\begin{aligned} V^{\mathrm{Put}}_t(\tau; K_{\mathrm{P}}) &= V^{\mathrm{Pay}}_{\mathrm{CDX},t}(\tau; K_{\mathrm{U}}), \\ V^{\mathrm{Call}}_t(\tau; K_{\mathrm{P}}) &= V^{\mathrm{Rcv}}_{\mathrm{CDX},t}(\tau; K_{\mathrm{U}}) \quad \text{where the strike price } K_{\mathrm{P}} = 1 - K_{\mathrm{U}}. \end{aligned}$$

5 Empirical Results

Equipped with the prices of options written on the synthetic corporate bond index, we are able to investigate the corporate bond market in a model-free way. We first show that the prices of these option contracts make it possible to create an aggregate volatility measure for corporate bonds (Section 5.1). Based on this volatility index, we compute the variance risk premium to study the role of variance risk in the corporate bond market (Section 5.2). Following Martin (2017), we also construct the simple corporate bond VIX and examine its implications (Section 5.3). Taking a further step, we consider higher-order moments of the synthetic corporate bond index by estimating its risk-neutral skewness and kurtosis as well

[18] To be concrete, $K_{\mathrm{U}} = [K_{\mathrm{S}} - 0.01] \times \Pi(K_{\mathrm{S}})$, where we define $\Pi(K_{\mathrm{S}})$, by slight abuse of notation, as the risky PV01 when the CDX term structure is flat at K_{S}.

as physical, risk-neutral, and state-price densities (Section 5.4).

5.1 The Corporate Bond VIX

We create a model-free volatility index for the corporate bond market, which we call the corporate bond VIX (in short, CBVIX). The CBVIX measures the risk-neutral expectation of future 1-month volatility on the 5-year synthetic corporate bond index defined in Section 2.

For the remainder of this paper, we assume that $\tau = 1/12$ and $T = 5$. Since there is no ambiguity, we drop the superscripts from $P_t^{(T)}$ and $P_{t+\tau}^{(T-\tau)}$ and denote them as P_t and $P_{t+\tau}$ to simplify the notation. For the sake of brevity, the term "bond index" refers to the 5-year synthetic corporate bond index.

As the first step, we calculate the risk-neutral expectation of the realized log bond index variance over the next month:

$$E_t^{\mathbb{Q}}\left[RV_{t\to t+\tau}\right] \;=\; E_t^{\mathbb{Q}}\left[\int_t^{t+\tau} d\left[\log P\right]_u\right],$$

where $\mathbb{Q}$ represents the risk-neutral measure, and $[\log P]_u$ refers to the quadratic variation of the log bond index up to time u. We calculate this risk-neutral expectation using calls and puts, under the assumption that the bond index follows an Ito process, as in the case of the equity VIX.[19] In Appendix C, using the general spanning formula in Bakshi and Madan (2000) and Carr and Madan (2001), we show that the risk-neutral expectation of the bond index variance is expressed as

$$E_t^{\mathbb{Q}}\left[RV_{t\to t+\tau}\right] \;=\; 2e^{r_f\tau}\left(\int_0^{F_t}\frac{V_t^{\text{Put}}(\tau;K)}{K^2}dK + \int_{F_t}^{\infty}\frac{V_t^{\text{Call}}(\tau;K)}{K^2}dK\right), \qquad (13)$$

where r_f is the risk-free rate, and $F_t = E_t^{\mathbb{Q}}\left[P_{t+\tau}\right]$ is the τ-maturity forward price of the bond

[19] An extensive literature discusses the calculation of the VIX. Examples include, but are not limited to, Dupire (1994), Neuberger (1994), Carr and Madan (1998), Britten-Jones and Neuberger (2000), Jiang and Tian (2005), and Carr and Wu (2006).

index.[20] Since this quantity is calculated using options, it is often referred to as implied variance. We also use this term throughout the paper. Following other volatility indices, the CBVIX is expressed in annualized percentage volatility:

$$\text{CBVIX}_t = 100 \times \sqrt{\frac{1}{\tau} E_t^{\mathbb{Q}} \left[RV_{t \to t+\tau} \right]}.$$

Panel A of Figure 1 plots the resulting time series of the CBVIX during our sample period. We can see that the CBVIX has significant time variations: while the sample average is at 1.76%, it fluctuates from 0.86% to 5.42% depending on the market condition.

The beginning of the sample corresponds to the post Great Recession period. During this period, the CBVIX maintained a high level, reflecting fears of the eurozone debt crisis. The CBVIX started to decline as uncertainty reduced after the European Central Bank took an aggressive measure to support eurozone countries. The reduced level of the CBVIX started to rise again from mid-2014. Besides the resurfacing of the Greek government debt issue, China's economic slowdown resulted in the turmoil of global financial markets, and, as a result, the CBVIX increased to almost 4% in February 2016. Nevertheless, the steady recovery of the U.S. economy stabilized the financial market and pushed the CBVIX to a lower level during 2016 and 2017. Despite minor spikes due to events such as Brexit in June 2016 and the presidential election in November 2016, the CBVIX reached 1% by the end of 2017. While the CBVIX temporarily went up as high as 2.5% in February 2018 when high inflation concerns caused the stock market to plummet, it has since maintained a lower level between 1% and 2%.

Panel B of Figure 1 shows the time series of the 5-year constant maturity synthetic corporate bond index. Clearly, we can see that the level of the index negatively comoves

[20] The forward price is calculated as

$$F_t = P_t e^{r_f \tau} - C_{t+\tau},$$

where $C_{t+\tau}$ is the accrued coupon from the index between times t and $t+\tau$. Although the coupon is paid out at time $t+\tau$, this quantity is known at time t; it is calculated based on the coupon rate that is reset at time t.

with the CBVIX. In the data, the two time series exhibit a negative correlation of -0.89. This implies that a positive shock to the CBVIX tends to be associated with a negative shock to the aggregate bond price, creating asymmetric volatility. This finding can be explained as the volatility feedback effect, which is well-documented in the stock market.[21] If volatility risk is priced, a higher level of volatility induces investors to demand a higher premium for holding corporate bonds, which leads to lower corporate bond prices.

Panel A of Figure 2 compares the time series of the CBVIX with that of the equity (S&P 500) VIX. We can see that the CBVIX has a much smaller magnitude compared to the equity VIX. During our sample period, the average equity VIX is 14.86%, which is roughly 8 times larger than the average CBVIX. This is intuitive for two reasons: first, the bond is a senior claim on a firm's income or asset while the equity is a residual claim. Thus, when shocks to the firm arrive, the bond should respond to them with less sensitivity. Furthermore, while the equity has an infinite maturity (resulting in a large magnitude of the equity duration), the bond index we consider for the CBVIX is with a 5-year maturity, which also leads to lower price volatility.

Despite a substantial difference in their levels, the CBVIX and the equity VIX show fairly similar patterns: they fluctuate in a consistent manner, peaking or dipping around the same times. In line with this, these two volatility indices are significantly correlated at 0.71. Such a strong association between the two volatility indices is not so surprising if we regard the S&P 500 and our bond index as the equity and bond securities issued by the same hypothetical representative firm. Under a structural credit risk framework, a default event occurs when the firm's assets fall below a certain threshold. The firm's floating rate bond is a senior claim whose value is entirely driven by the firm's credit risk, and the equity is a residual claim whose payoff resembles a call option on the firm's assets. Therefore, both bond and equity dynamics originate from the same source: the firm's asset dynamics.

Based on this insight, one might wonder if the volatility of the corporate bond market

[21] See French, Schwert, and Stambaugh (1987), Campbell and Hentschel (1992), Bekaert and Wu (2000), and Bollerslev, Litvinova, and Tauchen (2006) for more details.

can reasonably be estimated based on the equity VIX with the aid of a structural model. To address this question, we calculate the corporate bond volatility implied by the equity VIX through the lens of the Merton (1974) model.[22] In Panel B of Figure 2, we plot this model-implied volatility together with the CBVIX.

Compared to the CBVIX, the Merton-implied corporate bond volatility is substantially smaller, with an average of 0.01%. This is mainly because we consider firms that are relatively stable with low leverage.[23] As a result, the Merton model generates nearly zero FRN volatility except for a few instances when the equity volatility is abnormally high. While this failure might be attributable to the oversimplified assumptions of the model, the finding nevertheless suggests that the CBVIX provides nontrivial information about the corporate bond market.

In fact, the CBVIX and the equity VIX occasionally show different sensitivities to shocks to the economy. In response to the economic slowdown and stock market bubble burst in China, the U.S. stock market crashed in August 2015 and the equity VIX sharply increased, exceeding 40%. However, this event had a smaller effect on the corporate bond market as well as the CBVIX, as can be seen in Panel A of Figure 2. A similar pattern can be observed in February 2018.

In Figure 3, we also plot the time series of the Treasury VIX (or TYVIX), together with the CBVIX. The TYVIX reflects the expected volatility of 10-year Treasury note future prices under the risk-neutral measure. To facilitate comparison with the CBVIX, which is based on a 5-year bond, we first divide the TYVIX by 2. This allows us to roughly approximate the TYVIX that corresponds to a 5-year maturity because the duration of 5-year Treasury

[22]We first find the face value of the firm's debt as the total sum of short-term and long-term debts of S&P 500 firms. Then, there are two unknown model quantities: the value and volatility of the firm's assets. For each date, we calibrate them so that the value and volatility of the firm's equity in the model exactly match the total market capitalization of the S&P 500 and the equity VIX, respectively. The model-implied FRN volatility is calculated as $\sigma_P = \sigma_A(\partial P/P)(\partial A/A)$, where σ_A is the firm's asset volatility, and the elasticity term is found in closed form.

[23]All firms in the synthetic corporate bond index and nearly 90% of the firms in the S&P 500 are investment grade.

notes is roughly half of the duration of 10-year Treasury notes.[24]

In the beginning of the sample, the scaled TYVIX exhibits a lower level and a lower variability compared to the CBVIX. While the eurozone debt crisis heavily influenced the corporate bond market, it had a limited impact on the Treasury market because U.S. government bonds were regarded as the safest asset. However, the TYVIX doubled in June 2013, when Ben Bernanke alluded that the Federal Reserve may reduce the size of quantitative easing policies. From this point in time, the TYVIX consistently showed a higher magnitude than the CBVIX, reflecting higher uncertainty regarding "tapering" and post-crisis monetary policies.

The average scaled TYVIX is 2.54% in our sample. That is, compared with the equity VIX, the TYVIX has a level much closer to the CBVIX. This is intuitive because both the CBVIX and the TYVIX are based on bonds whose volatilities are much smaller than stocks. However, in terms of patterns, the CBVIX has a weaker correlation with the TYVIX (0.43) compared to that with the equity VIX. Considering that the CBVIX measures the future volatility of FRNs, which are insensitive to interest rate risk, such a small correlation is not surprising because Treasury notes are sensitive to interest rate risk alone. Nonetheless, the two volatility indices are still correlated because the level of interest rates is directly and indirectly associated with the level of credit risk in the economy.

5.2 Variance Risk in the Corporate Bond Market

5.2.1 The Realized Variance Measure

While implied variance is estimated using options, realized variance is estimated using the realized time series of the bond index. Specifically, we construct the monthly realized variance measure $\hat{RV}_{t \to t+\tau}$ between times t and $t+\tau$ by tracking the daily levels of the bond index

[24] The (modified) duration is a price sensitivity measure with respect to parallel shifts in the yield curve. Thus, if the duration is half, the price volatility should also be approximately half. Of course, this approximation ignores second- and higher-order effects of parallel shifts as well as the effects of nonparallel shifts, such as changes in the slope and curvature of the yield curve.

whose maturity starts out as 5 years at time t. To simplify our computation, we assume that each monthly horizon corresponds to 22 trading days and that each interval between two adjacent trading days is $\delta = \tau/22$. Then, following French, Schwert, and Stambaugh (1987), our realized variance measure is estimated as the sum of squared daily log price relatives of the bond index, adjusted for the first-order autocorrelation:[25]

$$\hat{RV}_{t\to t+\tau} = \sum_{n=1}^{22} p_{t+\delta n}^2 + 2\sum_{n=1}^{21} p_{t+\delta n} \cdot p_{t+\delta(n+1)},$$

where $p_{t+\delta n} = \log\left(\frac{P_{t+\delta n}}{P_{t+\delta(n-1)}}\right)$.

Figure 4 shows the daily time series of our realized variance measure together with the model-free implied variance. Note that the value of $RV_{t\to t+\tau}$ is displayed at time $t+\tau$ (not time t) because this realized variance is observed at time $t+\tau$. That is, the value plotted at time t is $RV_{t-\tau\to t}$, the realized variance from time $t-\tau$ up to time t. We multiply the two time series by 10^4 in order to express them in monthly percentage squared terms, consistent with the literature.

Figure 4 reveals that the time series of the realized variance has a fairly similar pattern compared to the implied variance. More importantly, we can see that the level of the implied variance is significantly higher than the realized variance for the majority of our sample period. The daily average of the implied variance in our sample is 0.29 whereas that of the realized variance is 0.19. To gauge how large this difference is, we express the two variables in annual percentage volatility terms: the average implied volatility (i.e. CBVIX) is 1.76% while the average realized volatility is 1.35%. In other words, the implied volatility of the bond index is 30% larger than its realized counterpart, on average. The substantial gap between the implied variance and the realized variance suggests that variance risk in the corporate bond market is priced.

[25] In the case of the stock market, it is well known that realized variance can be accurately estimated using high-frequency (typically, 5-minute) time series (e.g. Andersen, Bollerslev, Diebold, and Labys, 2003; Andersen, Fusari, and Todorov, 2015). Unfortunately, high-frequency data on the CDX are not available.

5.2.2 The Variance Risk Premium

To quantitatively assess the importance of variance risk in the corporate bond market, we construct the monthly time series of the corporate bond variance risk premium, which is defined as the difference between the risk-neutral and physical expectations of future bond index variance:

$$\text{VRP}_t = E_t^{\mathbb{Q}}\left[RV_{t\to t+\tau}\right] - E_t^{\mathbb{P}}\left[RV_{t\to t+\tau}\right]. \tag{14}$$

The monthly time series of the risk-neutral expectation in equation (14) is simply obtained by extracting the implied variance series on the last day of each month. In contrast, constructing the monthly time series of the physical expectation is subject to various approaches. For example, Bollerslev, Tauchen, and Zhou (2009) assume a unit root process for the realized variance so that the previous month's realized variance serves as a proxy for the physical expectation of the upcoming month's realized variance. Drechsler and Yaron (2011) run a linear forecast model where the realized variance is projected onto the lagged realized variance and the lagged implied variance. Lastly, Zhou (2018) additionally suggests methods based on a moving average and on an autoregressive model. In our paper, we estimate the physical expectation in equation (14) as the exponentially weighted average of the last 12 monthly realized variances. As Zhou (2018) discusses, this smoothing method is simple as it does not require parameter estimation.

Panel A of Table 1 contains the summary statistics for the variables used in our empirical analyses. The first three variables are the variance risk premium, the implied variance, and the realized variance in the corporate bond market. The next three variables are the corresponding variables in the equity market.[26] The last two variables are the monthly percentage excess log returns on the synthetic bond index and on the CRSP value-weighted index.[27]

[26] We obtain these three variables from Hao Zhou's webpage.

[27] Returns on the synthetic bond index include not only capital gains/losses but also coupon payments.

Our sample consists of 79 months from March 2012 to September 2018. From Panel A, we can observe that the variance risk premium in the corporate bond market has a sample mean of 0.01 and a sample standard deviation of 0.13. This variable is positively skewed with its median lesser than its mean, and is fat-tailed with a large kurtosis. In addition, the corporate bond variance risk premium has a monthly AR(1) coefficient of 0.22, which suggests that this variable quickly mean-reverts with relatively low persistence. From this table, we can also see the characteristics of the variance risk premium in the equity market. The equity variance risk premium has a sample mean of 9.59 and a sample standard deviation of 6.36. This variable has a skewness near zero, albeit slightly negative, and a kurtosis of 4.73, which indicates that the equity variance risk premium also has a fat-tailed sample distribution.

Looking at the summary statistics for implied variances and realized variances, we can see that in both markets, on average, the implied variance has a larger magnitude than the realized variance, while the standard deviations are about the same. However, in terms of kurtosis and skewness, the patterns are different in the two markets. In the equity market, the realized variance has a larger skewness and kurtosis compared to those of the implied variance. In the case of the corporate bond market, this pattern is reversed such that the implied variance has a larger skewness and kurtosis.

The bottom two rows in Panel A summarize synthetic bond returns and equity returns in our sample. Bond returns are, on average, 0.12% per month (1.43% annually), with a standard deviation of 0.36% (1.25% annually). The skewness value is close to zero but the kurtosis value is large at 6.15. The average equity return in our sample period is approximately 1.12% per month (13.39% annually), which is higher than the post-war sample, but the standard deviation is 2.83% per month (9.80% annually), which is lower than the post-war sample. This is because our sample coincides with the post Great Recession period when the stock market has steadily recovered from the financial crisis.

Panel B of Table 1 lists the monthly correlations among our eight variables of interest. The positive correlations among the six variance-related variables imply that they tend to comove in the same direction (with the exception of the equity variance risk premium and the equity realized variance). For example, the corporate bond variance risk premium and the equity variance risk premium are moderately correlated at 0.27.

We can also see from this panel that the returns on the bond and the returns on the equity are positively correlated at 0.77. As anticipated, the first six variance-related variables exhibit negative contemporaneous correlations with the two return time series: during bad times when markets suffer, variance risk as well as the compensation for variance risk typically go up. For instance, the corporate bond variance risk premium has a correlation of -0.64 with the bond return and a correlation of -0.59 with the equity return.

5.2.3 Return Predictability

Table 1 shows that the corporate bond variance risk premium is positive on average and is substantially time-varying. Furthermore, it has negative contemporaneous correlations with bond and equity market returns. These characteristics are similar to those of the equity variance risk premium. Given that the literature finds that the equity variance risk premium predicts future equity returns, a natural question that follows is: can the corporate bond variance risk premium predict future bond returns? Moreover, is it capable of predicting future equity returns as well? What is the joint predictability of the two variance risk premium measures?

Although our sample is relatively short with 79 monthly time series, it is still possible to run predictability regressions to address the above questions. First, we investigate the predictability of 1-month ahead bond market returns based on the corporate bond variance risk premium as well as other variance-related variables, as shown in Table 2.[28] From column (1), we can see that higher levels of the corporate bond variance risk premium significantly predicts higher future returns over the next month. The slope coefficient is 0.79 with a t-statistic of 2.96. In other words, a one standard deviation increase in the corporate bond

[28]Note that all standard errors in Table 2 and Table 3 are corrected according to Newey and West (1987) with 4 lags.

variance risk premium (i.e. 0.13) leads to a roughly $0.13 \times 0.79 = 0.1\%$ point higher bond return the next month. This regression generates a sizable adjusted R^2 over 6%.

Moreover, higher levels of the corporate bond implied variance and realized variance also predict larger future bond returns, as can be seen in columns (2) and (3). The coefficients are roughly similar in size and are highly significant. In the case of the corporate bond implied variance, it generates an even higher adjusted R^2 of 9.45%. The fact that both the implied and realized variances predict future returns is unique to the bond market. In the equity market, as Bollerslev, Tauchen, and Zhou (2009) document, the variance risk premium predicts future equity returns, but the implied and realized variances do not, which we later confirm in our sample. The fact that all three variance-related variables in the corporate bond market predict future bond returns suggests that variance risk is an important source of risk that drives bond returns.

Column (4) of Table 2 indicates that the equity variance risk premium predicts future bond returns, despite the significance level only being marginal (t-statistic of 1.68). The slope coefficient is 0.01, implying that a one standard deviation increase in the equity variance risk premium (i.e. 6.36) leads to a $0.01 \times 6.36 = 0.06\%$ higher bond return the next month, approximately half compared to the corporate bond variance risk premium. This predictive regression has an adjusted R^2 of around 2%, which is also smaller than that of the corporate bond variance risk premium. The equity implied and realized variances do not predict future bond returns: their coefficients are insignificant, as shown in columns (5) and (6).

We also run multiple regressions to assess the joint predictability of our variables of interest. In column (7), we enter the corporate bond variance risk premium and the equity variance risk premium into the same regression. The result shows that the two slope coefficients are both positive. However, the corporate bond variance risk premium remains significant with a t-statistic of 2.07 whereas the equity variance risk premium becomes insignificant with a t-statistic of 0.84. Comparing this regression with the univariate regression solely based on the corporate bond variance risk premium (column (1)), the addition of the equity variance risk premium causes the adjusted R^2 to drop from 6.49 to 6.43. This shows that although the equity variance risk premium itself predicts bond returns, when the corporate bond variance risk premium is present, it does not add any extra explanatory power. We can observe a similar pattern in column (8) when the corporate bond implied variance and the equity variance risk premium are entered into the same regression.

Now, we turn to the predictability of equity returns. In our sample, we find that higher levels of the equity variance risk premium predict higher equity returns the next month whereas higher levels of the implied and realized variances do not, consistent with prior studies. This is summarized in columns (4), (5), and (6) of Table 3. What we newly discover and add to the literature is that the corporate bond variance risk premium also predicts future equity returns. For instance, column (1) shows that the corporate bond variance risk premium has a positive slope coefficient of 5.40. This implies that a one standard deviation increase in the corporate bond variance risk premium leads to a $5.40 \times 0.13 = 0.70\%$ point higher equity return the next month. The adjusted R^2 is fairly high at 4.60. While the corporate bond implied variance also predicts future equity returns (column (2)), the corporate bond realized variance does not (column (3)), with a t-statistic of only 0.39.

We examine the equity return predictability based on the corporate bond variance risk premium and the corporate bond implied variance, controlling for the equity variance risk premium. In column (8), when the corporate bond implied variance is entered into the same regression with the equity variance risk premium, it becomes insignificant. This is the exact opposite of what we observed with the bond return predictability: the equity variance risk premium is driven out by the corporate bond implied variance when predicting future bond returns. In contrast, as can be seen in column (7), when we put the two variance risk premium measures from both markets into the same regression, they both remain statistically significant (with the corporate bond variance risk premium at 5% and the equity variance risk premium at 10%). The two variables jointly generate high explanatory power with an adjusted R^2 of 7.34.

In sum, Tables 2 and 3 consistently find that the corporate bond variance risk premium positively predicts future returns in both markets. This result signifies that variance risk in the corporate bond market captures an important source of systematic risk, shared not only by the bond market but also the equity market.

5.3 The Simple Corporate Bond VIX (SCBVIX)

Inspired by Martin (2017), we create another volatility index called the simple CBVIX (in short, SCBVIX). This measures the risk-neutral volatility of the simple price relative, scaled by the gross risk-free rate.[29] The SCBVIX is defined as

$$\text{SCBVIX}_t = 100 \times \sqrt{\frac{1}{\tau}\text{Var}_t\left(e^{-r_f\tau}\frac{P_{t+\tau}}{P_t}\right)},$$

which is computed in a model-free manner using call and put options. Unlike the CBVIX, the calculation of this quantity does not require the assumption that the bond index follows an Ito process. Appendix C shows that the general spanning formula in equation (C.1) directly leads to the expression for the conditional variance of the price relative:

$$\text{Var}_t\left(\frac{P_{t+\tau}}{P_t}\right) = \frac{2e^{r_f\tau}}{P_t^2}\left(\int_0^{F_t} V_t^{\text{Put}}(\tau;K)dK + \int_{F_t}^{\infty} V_t^{\text{Call}}(\tau;K)dK\right).$$

Figure 5 shows the times series of the resulting SCBVIX, expressed in terms of annual percentage volatility. In Panel A, we can observe that the SCBVIX appears to have a similar level as well as a similar time series pattern compared to the CBVIX. Thus, we overlay the time series of the CBVIX and find that the two time series are almost indistinguishable. This is not unexpected: the SCBVIX should not be drastically different from the conditional

[29]Equivalently, the SCBVIX can be interpreted as the risk-neutral volatility of the simple holding period return on the index, scaled by the gross risk-free rate. Note that the holding period return on the index is determined by the following two components: (i) the price relative ($P_{t+\tau}/P_t$), and (ii) the coupon yield ($C_{t+\tau}/P_t$). While the former is realized in the future at time $t+\tau$, the latter is known at time t because $C_{t+\tau}$ depends on the coupon rate that is determined at time t. Therefore, it is straightforward that central moments of the holding period return are exactly identical to those of the price relative.

volatility of the log price relative, which is roughly what the CBVIX measures.

Panel B calculates the difference between the CBVIX and the SCBVIX. Although their difference is as high as 0.07% during the eurozone debt crisis, generally it is very small with a sample mean of 0.01%. Nonetheless, the interesting aspect of the gap between the CBVIX and the SCBVIX is not in its magnitude, but in its sign. According to Martin (2017), this gap should be negative if the return and the pricing kernel are conditionally log-normal at the 1-month horizon. Consistent with the findings of Martin (2017) in the stock market, we can see that the difference is always positive, implying that the CBVIX is higher than the SCBVIX for every single day of our sample. Therefore, Panel B clearly indicates a rejection of conditionally log-normal models.

5.4 Higher Moments

Having studied variance risk in the corporate bond market, the natural next step is to examine the significance of higher-order moments, such as skewness and kurtosis. Again, we exploit the general spanning formula to estimate risk-neutral conditional higher-order moments using option prices on the bond index. In Appendix C, we show that the n-th order non-central moment of the future price relative is determined as

$$
\begin{aligned}
m_{n,t} &= E_t^{\mathbb{Q}}\left[\left(\frac{P_{t+\tau}}{P_t}\right)^n\right] \\
&= \left(\frac{F_t}{P_t}\right)^n + \frac{n(n-1)e^{r_f\tau}}{P_t^n}\left(\int_0^{F_t} V_t^{\text{Put}}(\tau;K)K^{n-2}dK + \int_{F_t}^{\infty} V_t^{\text{Call}}(\tau;K)K^{n-2}dK\right).
\end{aligned}
$$

Note that this formula is distinct from the one in Bakshi, Kapadia, and Madan (2003): while Bakshi, Kapadia, and Madan (2003) consider the moments of the log price relative, we consider the simple price relative. For the detailed derivation, refer to Appendix C. Once we estimate the first four non-central moments of the price relative based on the equation above, the risk-neutral conditional skewness and kurtosis are calculated as

$$\begin{aligned} \text{Skew}_t\left(\frac{P_{t+\tau}}{P_t}\right) &= \left[m_{3,t} - 3m_{1,t}m_{2,t} + 2m_{1,t}^3\right] / \left[m_{2,t} - m_{1,t}^2\right]^{\frac{3}{2}}, \\ \text{Kurt}_t\left(\frac{P_{t+\tau}}{P_t}\right) &= \left[m_{4,t} - 4m_{1,t}m_{3,t} + 6m_{1,t}^2 m_{2,t} - 3m_{4,t}\right] / \left[m_{2,t} - m_{1,t}^2\right]^2. \end{aligned}$$

In Figure 6, the solid blue lines represent the time series of the risk-neutral conditional return skewness (Panel A) and excess kurtosis (Panel B). For comparison, the dotted red lines plot the corresponding time series, calculated under the log-normal assumption. If the conditional distribution indeed follows a log-normal distribution, the solid blue lines and the dotted red lines should be close.

As can be seen in Figure 6, the log-normal assumption generates nearly zero (but slightly positive) skewness and excess kurtosis. This is clearly not the case in the data. First of all, the actual skewness values are substantially negative over the entire sample. The conditional skewness is, on average, -1.72, fluctuating between -3.67 and -0.39. Furthermore, the actual excess kurtosis values far exceed zero with a mean of 4.82. Especially in the early part of the sample, the risk-neutral excess kurtosis goes even beyond 15, implying an extremely fat-tailed distribution. In sum, we can conclude that risk-neutral conditional distributions of the price relative are highly skewed to the left and are heavily fat-tailed, which set them far apart from a log-normal distribution.

Taking a further step, we now turn to distributions of the price relative, which are affected not just by the first four moments, but by all moments. We follow Aït-Sahalia and Duarte (2003) to nonparametrically estimate conditional densities of the price relative. Specifically, we first solve the optimization problem under conic constraints, as proposed in Dykstra (1983), to ensure that option prices are arbitrage-free across different strike values.[30] As Breeden and Litzenberger (1978) and other subsequent studies show, the risk-neutral density can be calculated as the second-order partial derivative of the call price with respect to the

[30]For the details of the algorithm, see Dykstra (1983), and Aït-Sahalia and Duarte (2003).

strike. Since we are interested in the price relative $P_{t+\tau}/P_t$ rather than the future index level $P_{t+\tau}$, we normalize the price of each call option and its strike by the current index level P_t. Then, the density of the price relative is calculated as

$$f^{\mathbb{Q}}(m) = e^{r_f \tau} \frac{\partial^2 V_n^{\text{Call}}(\tau; m)}{\partial m^2}, \tag{15}$$

where $V_n^{\text{Call}}(\tau; m)$ is the normalized price of a τ-maturity call option whose moneyness is m. The second-order partial derivative in equation (15) is estimated by running a locally linear regression.[31]

While the approach of Aït-Sahalia and Duarte (2003) can be used to generate conditional densities by only choosing options from certain dates, our main objective is to obtain the unconditional, or average, density over our sample period. We, therefore, apply their estimation methodology to average normalized option prices.[32] The dotted red line in Figure 7 shows the resulting risk-neutral density function with the normal kernel and a bandwidth of 0.50%.

We also estimate the density of the price relative under the physical measure. Using the daily time series of the price relative, which is calculated according to equations (6) and (8), we run kernel density estimation. The solid blue line in Figure 7 shows the resulting physical density function, or $f^{\mathbb{P}}(\cdot)$, based on the normal kernel with a bandwidth of 0.50%.

Lastly, we estimate the state-price density (or pricing kernel) projected on the 1-month future price relative as the ratio between the risk-neutral and physical densities. We overlay the resulting state-price density with a dashed green line in Figure 7.

Comparing the solid blue line and the dotted red line, it is apparent that the risk-neutral

[31] The locally linear estimator of the second-order partial derivative is provided in Appendix B of Aït-Sahalia and Duarte (2003).

[32] Specifically, for each moneyness, we take the time series average of normalized option prices. Since moneyness values of available options do not coincide every day, we use cubic spline interpolation. Note that aside from using average option prices, an alternative way to obtain the unconditional distribution is to run the estimation based on all available options during our sample period. This is computationally very challenging due to the conic-constrained optimization problem.

distribution has a larger variance. Furthermore, it is more skewed to the left and exhibits fatter tails on both sides, compared to the physical distribution. More importantly, as the future price moves away from its mode, the physical density converges to zero much faster than the risk-neutral density does, which makes the ratio between the risk-neutral and physical densities sharply rise in both directions. As a result, we can observe that the state price density is U-shaped.

Prior studies such as Rosenberg and Engle (2002) consistently document a non-monotonic or U-shaped pricing kernel, when projected on the future stock market return.[33] This finding is not limited to the stock market. Li and Zhao (2009), Song and Xiu (2016), and Christoffersen and Pan (2017) discover similar patterns of the pricing kernel using options on interest rates, the VIX, and crude oil prices, respectively. We confirm this pattern in the corporate bond market using options on the synthetic corporate bond index.

6 Conclusion

The credit derivatives market experienced a setback during the subprime mortgage crisis. For example, the trading volume of tranche products decreased significantly, as they were stigmatized for instigating the crisis. Ironically, this provided an opportunity for CDX swaptions: market participants began to actively use CDX swaptions to hedge credit risk, in place of tranche products.

In this paper, we highlight the utility of CDX swaptions whose trading volume has steadily been rising. CDX swaptions enable us to obtain prices of option contracts written on an aggregate corporate bond index. Based on these option prices, we create the corporate bond VIX and the simple corporate bond VIX, two model-free volatility measures for the corporate bond market. We further estimate the corporate bond variance risk premium,

[33] There is an extensive literature on examining the properties of the pricing kernel estimated based on equity index options. Examples include Aït-Sahalia and Lo (1998), Jackwerth (2000), Chabi-Yo, Garcia, and Renault (2007), Bakshi, Madan, and Panayotov (2010), Chabi-Yo (2012), and Christoffersen, Heston, and Jacobs (2013).

which demonstrates how much compensation investors demand to take variance risk in the corporate bond market. Our model-free option-based analyses extend beyond the second moment: synthetic bond index options make it possible to investigate higher-order conditional moments as well as whole probability distributions.

Our results have important implications for structural credit risk models. So far, the validity of these models mainly relied on their abilities to match the average credit spread in the data. The conditional moments we estimate in this paper can serve as extra grounds for testing structural models. In line with this, we present a simple exercise that shows that the Merton model fails to generate reasonable conditional corporate bond volatilities. It would be interesting to investigate which ingredients we need from firms' asset dynamics in order to explain conditional corporate bond moments.

In this paper, our findings are based on 1-month CDX swaptions. However, swaptions with longer maturities (such as 3 months and 6 months) also trade in the market, and they can be used to study the term structure of risks in the corporate bond market. We can further extend our analyses to the market for speculative grade bonds by exploiting the swaptions on the CDX North American High Yield index. We plan to explore these dimensions in future research.

Appendix

A Liquidity of Credit Derivatives

Credit derivatives are financial instruments that allow investors to manage their credit risk exposures.[34] The market has observed a stratospheric increase in the trading volume of credit derivatives, ever since their introduction in the mid-1990s. Although these products were blamed for instigating and worsening the financial crisis that led to the Great Recession, they are still actively traded. According to a report by BIS (2016), the total outstanding notional amount of credit derivatives exceeded 12 trillion dollars as of December 2015.

Figure A.1 depicts the liquidity of on-the-run CDX series based on data from the Depository Trust and Clearing Corporation (DTCC). Panel A shows the average of weekly open interests over the 6-month period when each CDX series was on the run. In 2015, the weekly open interest was over $350 billion in terms of gross notional principal (left y-axis, the blue bars on the left), and over 3,000 contracts in terms of number of contracts (right y-axis, the red bars on the right). Panel B presents the average of weekly trading volumes over the 6-month period when each CDX series was on the run (left y-axis, the blue bars). In 2015, the average weekly trading volume was over $750 billion. This corresponds to 15% of entire credit derivatives trades that were cleared through the DTCC in terms of gross notional principal (right y-axis, the dotted red line), and 4% in terms of number of contracts (right y-axis, the dashed red line).

Figure A.2 displays the liquidity of credit index options. Each blue bar (left y-axis) represents the average of weekly trading volumes over the 6-month period when the underlying index was on the run. We can see that the average trading volume has steadily increased since 2012, exceeding $300 billion per week, on average. In 2015, transactions on credit index

[34] Regardless of their complexity, various credit derivatives are built upon a single-name CDS structure. For example, a credit index is created by collecting a large number of single-name CDS contracts with different reference entities. When such a large pool of single-name CDS contracts is divided into several tranches based on seniority, each tranche is called a synthetic CDO product. Moreover, forward and option contracts can be constructed not only on a single-name CDS, but also on a credit index.

options account for around 8% of entire credit derivatives trades cleared through the DTCC in terms of gross notional principal (right y-axis, the dotted red line), and around 3% in terms of number of contracts (right y-axis, the dashed red line).

B The Black Formula for Credit Swaptions

A regular CDS contract is often called a spot CDS contract because protection immediately starts to apply as soon as a trade is made. In contrast, protection takes effect a certain period of time after the trade date in the case of a forward CDS contract. In a forward CDS contract, investors enter into a $(T-\tau)$-maturity CDS contract after a τ period from today either as a protection buyer (i.e. long protection forward) or as a protection seller (i.e. short protection forward) at a deal spread pre-established today. That is, while the deal spread is determined at time t, protection against credit events begins at future time $t+\tau$ and lasts until maturity $t+T$.

The deal spread should be a quantity that the protection seller and buyer can both agree upon. In other words, the deal spread is chosen so the value of the forward contract is zero at the beginning of the contract. This spread is observable in the market and is called the forward CDS spread. We denote the time-t forward CDS spread for firm i as $F_{i,t} = F_i(t, t+\tau, t+T)$. The forward CDS spread $F_{i,t}$ is the main object of interest in the Black model.

As discussed in Section 4, the Black formula is derived based on the payoff of a non-standardized single-name credit swaption. Let $V^{\text{Pay}}_{i,t+\tau}$ and $V^{\text{Rcv}}_{i,t+\tau}$ denote the payoffs of non-standardized payer and receiver swaptions at their maturity. It follows that

$$\begin{aligned} V^{\text{Pay}}_{i,t+\tau} &= \Pi_i(t+\tau, t+T) \max\left[S_i(t+\tau, t+T) - K_{\text{S}}, 0\right], \\ V^{\text{Rcv}}_{i,t+\tau} &= \Pi_i(t+\tau, t+T) \max\left[K_{\text{S}} - S_i(t+\tau, t+T), 0\right]. \end{aligned}$$

A payer swaption is only exercised when the spot spread is higher than the strike spread.

In this case, the holder obtains credit protection by paying a cheaper premium than the fair spot level. On the other hand, a receiver swaption is only exercised when the spot spread is lower than the strike spread. This is because the holder can sell credit protection for receiving a higher-than-the-fair spread. In either case, the gap between the spot spread and the strike spread is converted into the corresponding dollar value when it is multiplied by $\Pi_i(t+\tau, t+T)$, the risky PV01 at time $t+\tau$.

Note that a spot CDS contract can be viewed as a special case of a forward contract that immediately becomes effective. Thus, we can re-express the spot CDS spread at time $t+\tau$ as

$$S_i(t+\tau, t+T) = F_i(t+\tau, t+\tau, t+T),$$

which implies that

$$V_{i,t+\tau}^{\text{Pay}} = \Pi_i(t+\tau, t+T) \max\left[F_i(t+\tau, t+\tau, t+T) - K_{\text{S}}, 0\right], \quad \text{(B.1)}$$

$$V_{i,t+\tau}^{\text{Rcv}} = \Pi_i(t+\tau, t+T) \max\left[K_{\text{S}} - F_i(t+\tau, t+\tau, t+T), 0\right]. \quad \text{(B.2)}$$

The Black model calculates the time-t values of payer and receiver swaptions by assuming that between times t and $t+\tau$, the forward spread $F_{i,t}$ follows a geometric Brownian motion under the τ-forward survival measure.[35] Coupled with the payoff structures in equations (B.1) and (B.2), this assumption results in the Black formula for payer and receiver credit swaptions:

$$V_{i,t}^{\text{Pay}} = \Pi_i(t, t+\tau, t+T)\left[F_i(t, t+\tau, t+T)\Phi(d_1) - K_{\text{S}}\Phi(d_2)\right], \quad \text{(B.3)}$$

$$V_{i,t}^{\text{Rcv}} = \Pi_i(t, t+\tau, t+T)\left[K_{\text{S}}\Phi(-d_2) - F_i(t, t+\tau, t+T)\Phi(-d_1)\right], \quad \text{(B.4)}$$

[35] For more details about the forward survival measure and its relation with other probability measures, see, for instance, O'Kane (2011).

where

$$d_1 = \frac{\log(F_i(t, t+\tau, t+T)/K_{\mathrm{S}}) + \sigma^2\tau/2}{\sigma\sqrt{\tau}} \quad \text{and} \quad d_2 = d_1 - \sigma\sqrt{\tau}.$$

The function $\Phi(\cdot)$ represents the cumulative distribution function of the standard normal distribution, and $\Pi_i(t, t+\tau, t+T)$ the time-t risky PV01 between times $t+\tau$ and $t+T$.[36]

C Spanning Formulas Based on Option Prices

Bakshi and Madan (2000), Carr and Madan (2001), and Bakshi, Kapadia, and Madan (2003) show that the risk-neutral expectation of a twice-differentiable payoff function $H(\cdot)$ can be spanned by prices of European calls and puts. Expanding $H(P_{t+\tau})$ around the forward price F_t results in

$$E^{\mathbb{Q}}[H(P_{t+\tau})] = H(F_t) + E^{\mathbb{Q}}[H'(F_t)(P_{t+\tau} - F_t)] + e^{r_f\tau}\left(\int_0^{F_t} H''(K)V_t^{\mathrm{Put}}(\tau;K)dK + \int_{F_t}^{\infty} H''(K)V_t^{\mathrm{Call}}(\tau;K)dK\right). \quad \text{(C.1)}$$

C.1 The CBVIX

By setting $H(P_{t+\tau}) = \log(P_{t+\tau}/F_t)$, equation (C.1) implies that

$$\underbrace{E_t^{\mathbb{Q}}\left[\frac{P_{t+\tau} - F_t}{F_t}\right]}_{=0} - E_t^{\mathbb{Q}}\left[\log\left(\frac{P_{t+\tau}}{F_t}\right)\right] = e^{r_f\tau}\left(\int_0^{P_t} \frac{V_t^{\mathrm{Put}}(\tau;K)}{K^2}dK + \int_{P_t}^{\infty} \frac{V_t^{\mathrm{Call}}(\tau;K)}{K^2}dK\right), \quad \text{(C.2)}$$

[36] Typically, the risky PV01 is estimated from market CDS spreads. Specifically, we estimate the risky PV01 to exactly fit the term structures of spot and forward CDS spreads every day.

because $H'(F_t) = (1/F_t))$ and $H''(K) = -(1/K^2)$. By assuming that the price process follows an Ito process, the left-hand side of equation (C.2) equals

$$\log F_t - E_t^{\mathbb{Q}}[\log P_{t+\tau}] = \frac{1}{2} E_t^{\mathbb{Q}} \left[\int_t^{t+\tau} d\left[\log P\right]_u \right].$$

Therefore, it follows that

$$\tau \cdot \text{CBVIX}_t^2 \;=\; E_t^{\mathbb{Q}} \left[\int_t^{t+\tau} d\left[\log P\right]_u \right] = 2e^{r_f \tau} \left(\int_0^{P_t} \frac{V_t^{\text{Put}}(\tau;K)}{K^2} dK + \int_{P_t}^{\infty} \frac{V_t^{\text{Call}}(\tau;K)}{K^2} dK \right),$$

which provides us the expression in equation (13).

C.2 The Simple CBVIX

Now, consider the case in which $H(P_{t+\tau}) = (P_{t+\tau}/F_t)^2$. It follows from this definition that $H'(P_t) = 2P_t/F_t^2$ and $H''(K) = 1/F_t^2$. By plugging the expressions for $H(P_t + \tau)$, $H'(P_t)$, and $H''(K)$ into equation (C.1), we can show that

$$E_t^{\mathbb{Q}} \left[\left(\frac{P_{t+\tau}}{F_t} \right)^2 \right] = 1 + 2 \underbrace{\left(E_t^{\mathbb{Q}} \left[\frac{P_{t+\tau}}{F_t} \right] - 1 \right)}_{=0} + \frac{2e^{r_f \tau}}{F_t^2} \left(\int_0^{F_t} V_t^{\text{Put}}(\tau;K) dK + \int_{F_t}^{\infty} V_t^{\text{Call}}(\tau;K) dK \right).$$

We obtain the expression for the conditional variance of the price relative by multiplying both sides of this equation by $(F_t/P_t)^2$:

$$\underbrace{E_t^{\mathbb{Q}} \left[\left(\frac{P_{t+\tau}}{F_t} \right)^2 \right] - \left(\frac{F_t}{P_t} \right)^2}_{=\text{Var}_t(P_{t+\tau}/P_t)} \;=\; \frac{2e^{r_f \tau}}{F_t^2} \left(\int_0^{F_t} V_t^{\text{Put}}(\tau;K) dK + \int_{F_t}^{\infty} V_t^{\text{Call}}(\tau;K) dK \right).$$

Therefore, the simple CBVIX can be expressed in a model-free way using options prices on the bond index:

$$\tau \cdot \text{SCBVIX}_t^2 \;=\; \text{Var}_t \left(e^{-r_f \tau} \frac{P_{t+\tau}}{P_t} \right) = \frac{2}{e^{r_f \tau} P_t^2} \left(\int_0^{F_t} V_t^{\text{Put}}(\tau;K) dK + \int_{F_t}^{\infty} V_t^{\text{Call}}(\tau;K) dK \right).$$

C.3 Risk-Neutral Non-Centralized Moments

More generally, we consider the case in which $H(P_{t+\tau}) = (P_{t+\tau}/F_t)^n$. This implies that $H'(P_t) = nP_t^{n-1}/F_t^n$ and $H''(K) = n(n-1)K^{n-2}/F_t^n$. By plugging the expressions for $H(P_{t+\tau})$, $H'(P_t)$, and $H''(K)$ into equation (C.1), we obtain

$$E_t^{\mathbb{Q}}\left[\left(\frac{P_{t+\tau}}{F_t}\right)^n\right] = 1 + \frac{n(n-1)e^{r_f\tau}}{F_t^n}\left(\int_0^{F_t} V_t^{\text{Put}}(\tau;K)dK + \int_{F_t}^{\infty} V_t^{\text{Call}}(\tau;K)dK\right).$$

Multiplying both sides of this equation by $(F_t/P_t)^n$ generates the expression for the n-th order non-central moment.

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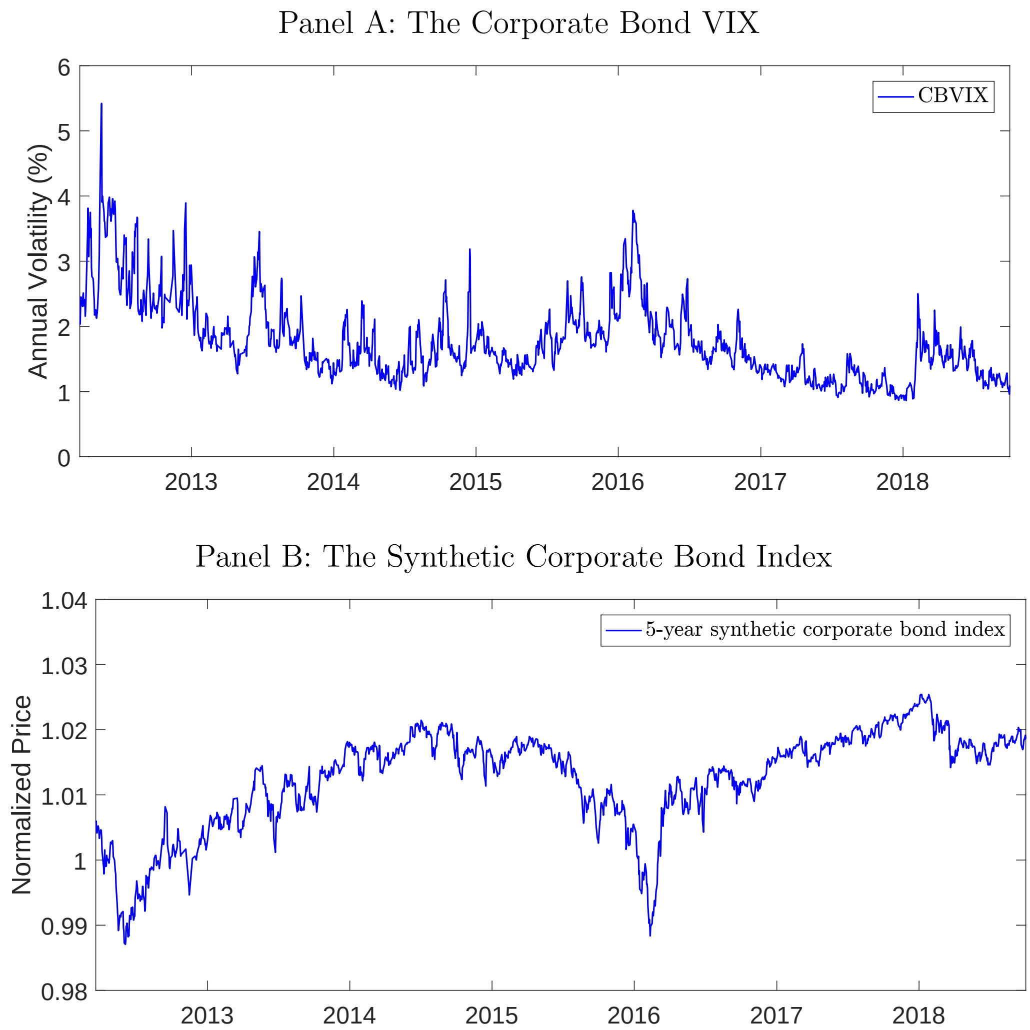
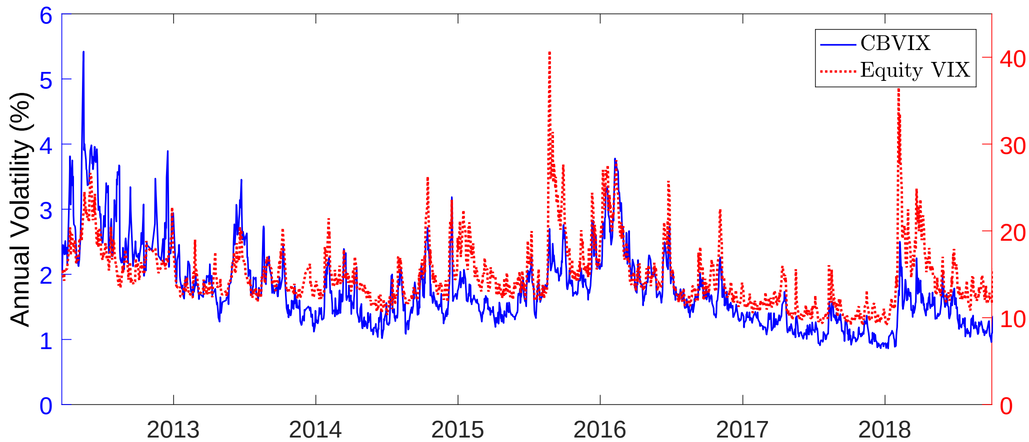
Figure 1: Time Series of the Corporate Bond VIX



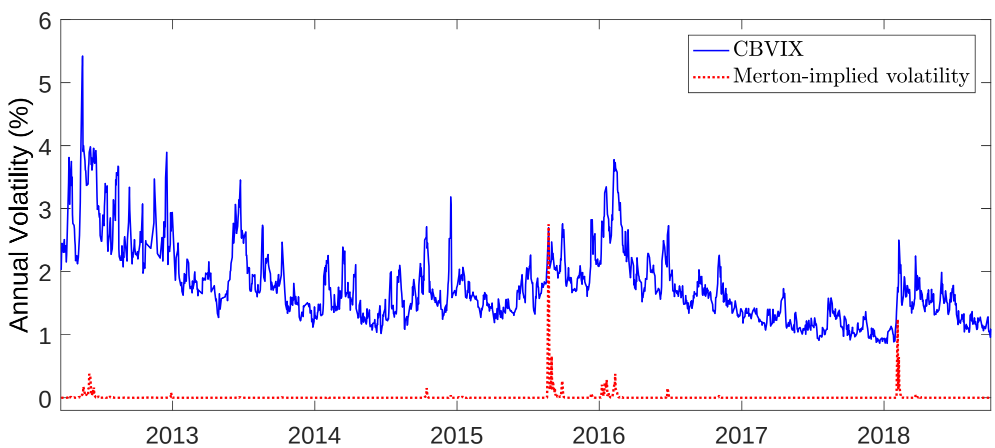
Notes: This figure presents the daily time series of the CBVIX (Panel A) and the 5-year constant maturity synthetic corporate bond index during our sample period (from March 2012 to September 2018). At each point in time, the CBVIX represents the risk-neutral expectation of future 1-month volatility on the 5-year synthetic corporate bond index. In Panel A, the CBVIX is expressed in terms of annual percentage volatility. In Panel B, the index represents the average price of 125 synthetic FRNs, each of which is with a dollar face value.

Figure 2: A Comparison of the CBVIX and the Equity VIX

Panel A: The Equity (S&P 500) VIX

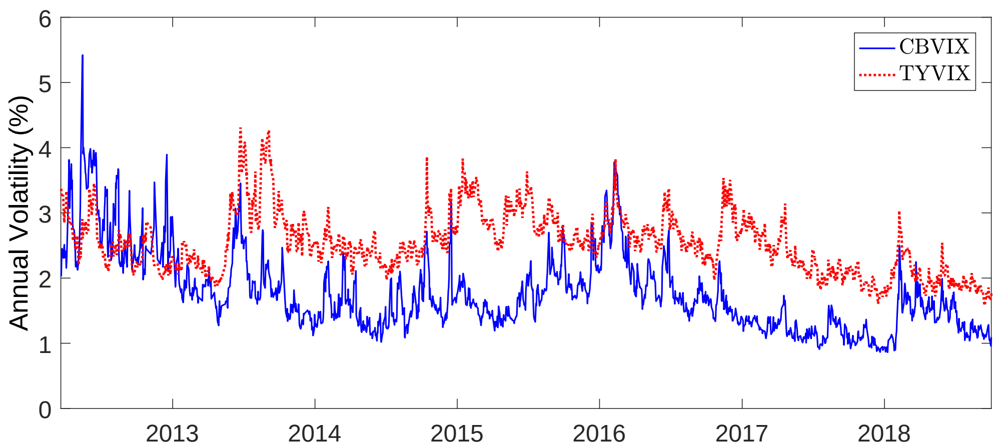


Panel B: The Merton-Implied Bond Volatility



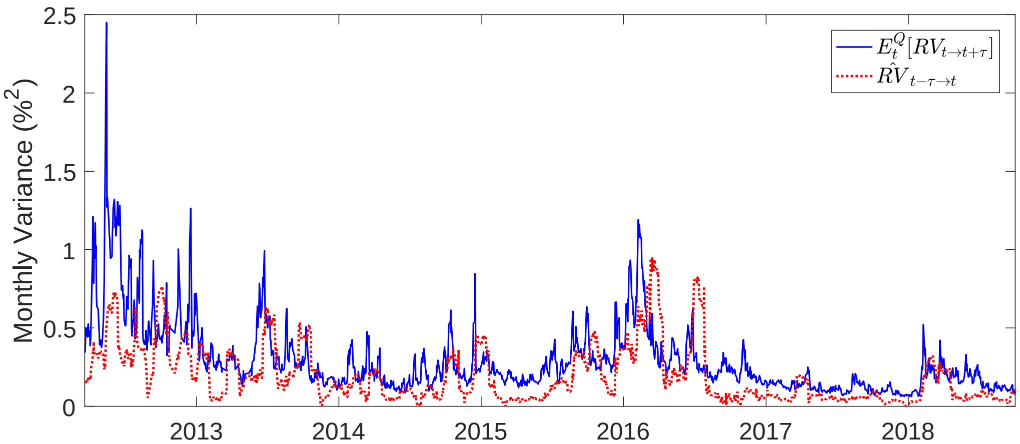
Notes: This figure contains side-by-side comparisons of the time series of the CBVIX with that of the equity VIX (Panel A) and that of the Merton-implied corporate bond volatility (Panel B), from March 2012 to September 2018. The solid blue lines represent the CBVIX and the dotted red lines represent the equity VIX (right y-axis in Panel A) and the Merton-implied corporate bond volatility (Panel B). All time series are in terms of annual percentage volatility.

Figure 3: A Comparison of the CBVIX and the TYVIX



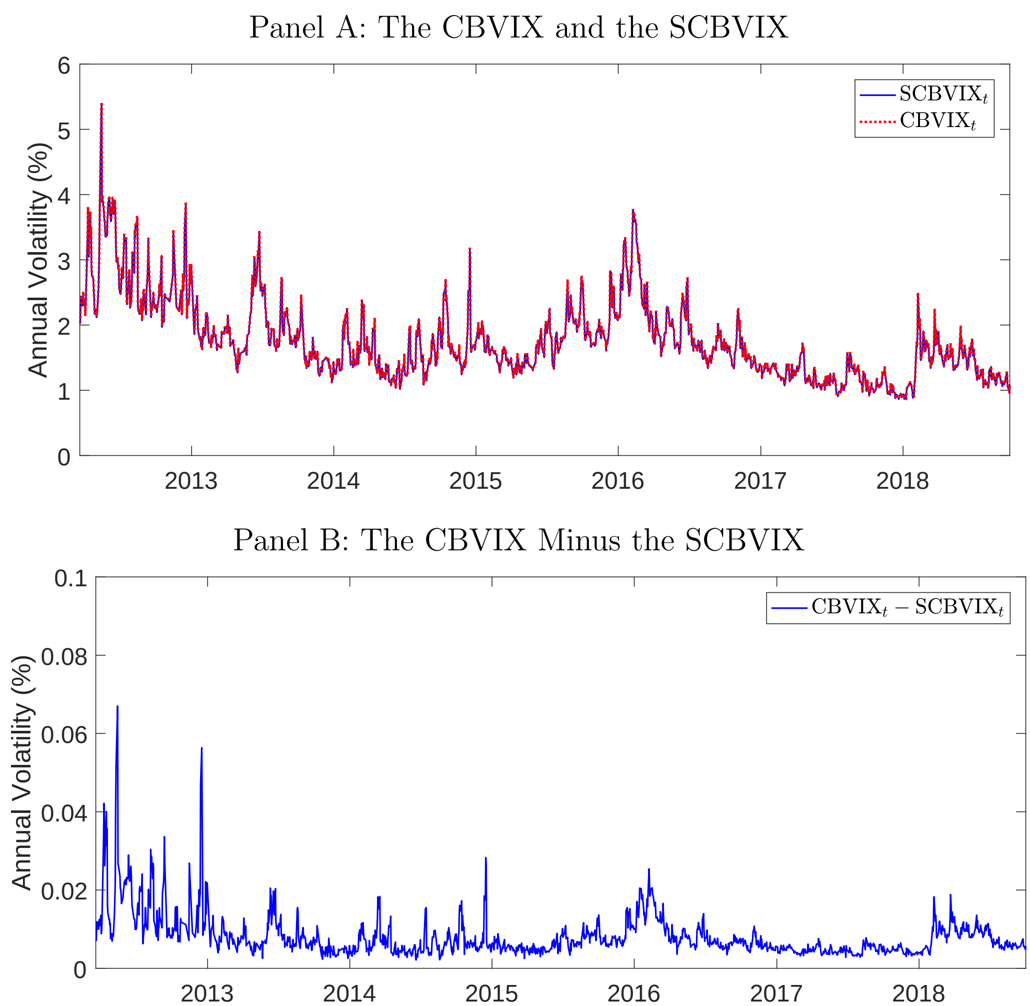
Notes: This figure contains a side-by-side comparison of the time series of the CBVIX with that of the TYVIX, from March 2012 to September 2018. The solid blue line represents the CBVIX and the dotted red line represents the TYVIX. Both time series are in terms of annual percentage volatility. When plotted, the original TYVIX is scaled by 2 so that the TYVIX and the CBVIX are based on bonds that have roughly the same duration.

Figure 4: Time Series of the Implied and Realized Variance Measures



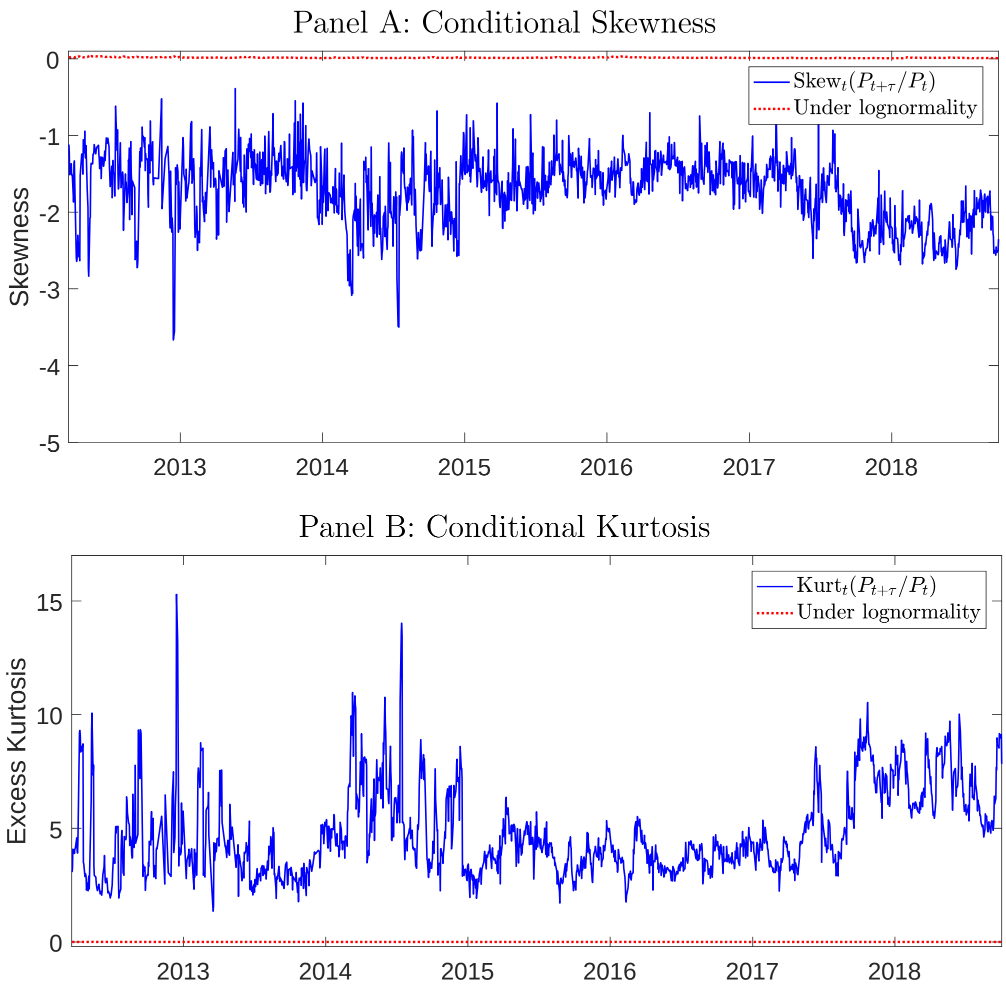
Notes: This figure plots the time series of the model-free implied variance measure (the solid blue line), and that of the realized variance measure (the dotted red line), from March 2012 to September 2018. The implied variance measure is estimated using call and put options on the synthetic corporate bond index. The realized variance measure is estimated from daily log price relatives of the synthetic corporate bond index, following French, Schwert, and Stambaugh (1987). Both time series are expressed in monthly percentage squared terms.

Figure 5: A Comparison of the CBVIX and the SCBVIX



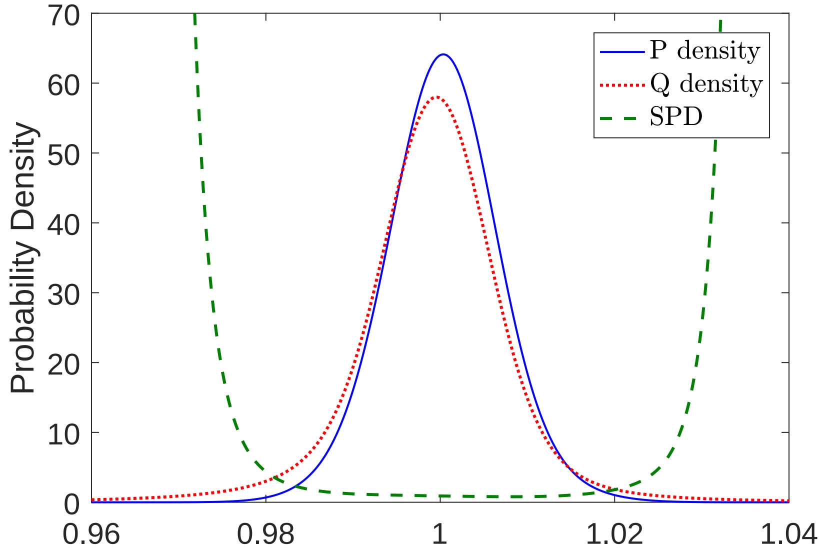
Notes: Panel A of this figure displays the SCBVIX (the solid blue line) along with the CBVIX (the dotted red line), from March 2012 to September 2018. Both time series are in terms of annual percentage volatility. Panel B shows the time series of the CBVIX subtracted by the SCBVIX.

Figure 6: Higher Moments of the Synthetic Corporate Bond Index



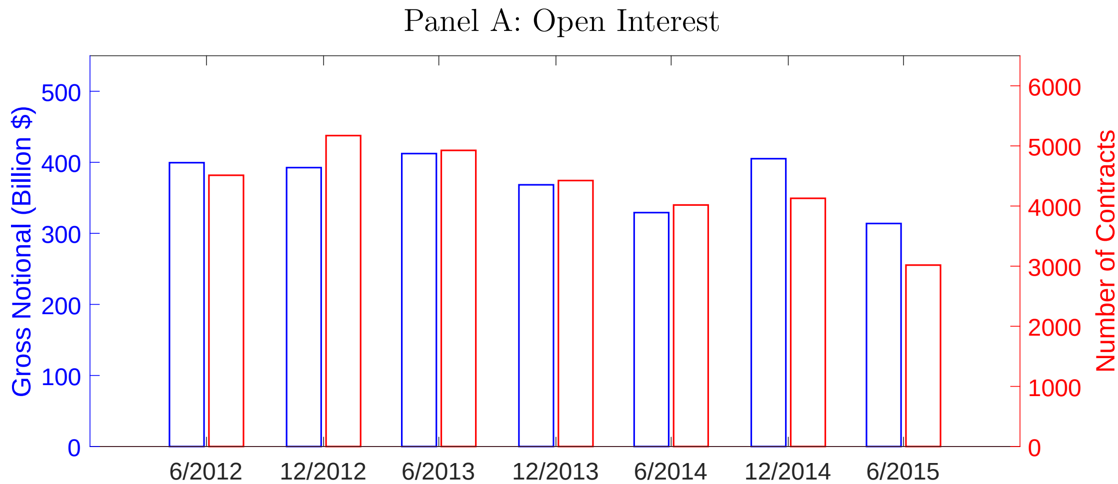
Notes: This figure depicts the higher moments of the price relative of the synthetic corporate bond index, from March 2012 to September 2018. The solid blue lines represent the time series of the 1-month risk-neutral conditional skewness (Panel A) and excess kurtosis (Panel B). The dotted red lines plot the corresponding time series of skewness and kurtosis under log-normality.

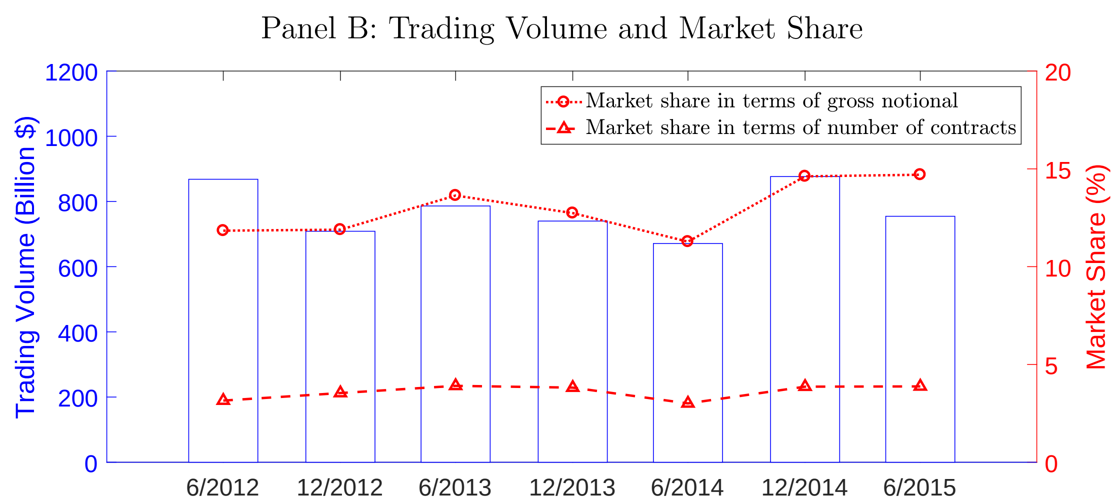
Figure 7: The Risk-Neutral, Physical, and State-Price Densities



Notes: This figure shows the results from our nonparametric density estimations. The solid blue line and the dotted red line indicate the physical and risk-neutral density functions of the 1-month price relative of the synthetic corporate bond index. Both density functions are estimated with the normal kernel and a bandwidth of 0.50%. The dashed green line represents the state-price density, which is the ratio between these two functions.

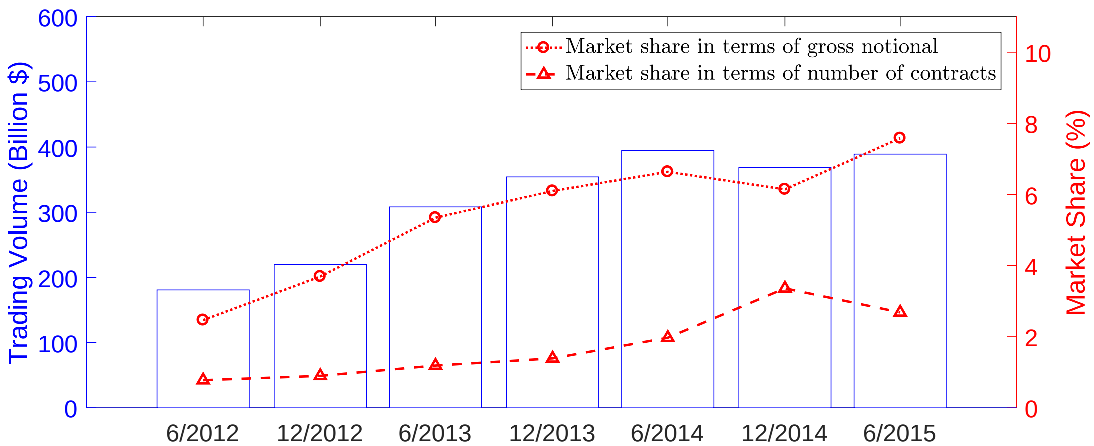
Figure A.1: Liquidity of On-The-Run CDX Series





Notes: This figure illustrates the liquidity of on-the-run CDX (North American Investment Grade) series based on data from the DTCC. Panel A shows the average of weekly open interests over the 6-month period when each CDX series was on the run. Open interest is represented in terms of gross notional principal (left y-axis, the blue bars on the left) and in terms of number of contracts (right y-axis, the red bars on the right). Panel B presents the average of weekly trading volumes over the 6-month period when each CDX series was on the run (left y-axis, the blue bars). We also plot market shares in terms of gross notional principal (right y-axis, the dotted red line) and in terms of number of contracts (right y-axis, the dashed red line).

Figure A.2: Liquidity of Credit Index Options



Notes: This figure displays the liquidity of credit index options. Each blue bar (left y-axis) indicates the average of weekly trading volumes over the 6-month period when the underlying index was on the run. We also plot market shares in terms of gross notional principal (right y-axis, the dotted red line) and in terms of number of contracts (right y-axis, the dashed red line).

Table 1: Descriptive Statistics

Panel A: Summary Statistics							
	N	Mean	Median	Std.	Skew.	Kurt.	AR
CB VRP	79	0.04	0.01	0.13	1.82	7.88	0.22
CB IV	79	0.27	0.22	0.18	2.04	9.55	0.58
CB RV	79	0.19	0.13	0.19	1.63	5.03	0.26
Equity VRP	79	9.59	9.01	6.36	-0.14	4.73	-0.09
Equity IV	79	20.06	16.78	9.99	2.03	8.98	0.39
Equity RV	79	10.47	7.08	10.50	3.38	17.73	0.38
$\log R_B^e$ (%)	79	0.12	0.13	0.36	0.10	6.15	-0.13
$\log R_E^e$ (%)	79	1.12	1.17	2.83	-0.42	3.29	-0.16

Panel B: Correlations							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
(1) CB VRP	—						
(2) CB IV	0.69	—					
(3) CB RV	0.23	0.61	—				
(4) Equity VRP	0.27	0.39	0.02	—			
(5) Equity IV	0.71	0.69	0.42	0.24	—		
(6) Equity RV	0.51	0.42	0.39	-0.38	0.81	—	
(7) $\log R_B^e$	-0.64	-0.35	-0.04	-0.26	-0.53	-0.34	—
(8) $\log R_E^e$	-0.59	-0.40	-0.14	-0.16	-0.67	-0.54	0.77

Notes: This table shows the descriptive statistics for the variables used in our empirical analyses. The first three variables pertain to the corporate bond market: namely, the corporate bond variance risk premium (CB VRP), the corporate bond implied variance (CB IV), and the corporate bond realized variance (CB RV). The next three variables are the corresponding variables in the equity market: the equity variance risk premium (Equity VRP), the equity implied variance (Equity IV), and the equity realized variance (Equity RV). The last two variables are the monthly percentage excess log returns on the synthetic corporate bond index ($\log R_B^e$) and the monthly percentage excess log returns on the CRSP value-weighted index ($\log R_E^e$). Panel A lists the summary statistics, and Panel B, the correlations.

Table 2: Bond Return Predictability

	Excess Return on the Synthetic Corporate Bond Index							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	0.08	-0.07	0.02	0.02	0.03	0.12	0.03	-0.09
	(2.00)	(-1.11)	(0.45)	(0.26)	(0.43)	(3.03)	(0.29)	(-1.07)
CB VRP	0.79						0.70	
	(2.96)						(2.07)	
CB IV		0.67						0.62
		(3.42)						(2.58)
CB RV			0.51					
			(2.98)					
Equity VRP				0.01			0.01	0.00
				(1.68)			(0.84)	(0.45)
Equity IV					0.00			
					(1.02)			
Equity RV						0.00		
						(0.03)		
Adj. R^2 (%)	6.49	9.45	6.19	1.94	0.11	-1.31	6.43	8.57

Notes: This table reports the results of predictability regressions of 1-month ahead bond market returns. Predictor variables include variance-related variables in the corporate bond market (the corporate bond variance risk premium, the corporate bond implied variance, and the corporate bond realized variance) and the corresponding variables in the equity market (the equity variance risk premium, the equity implied variance, and the equity realized variance). The dependent variable is the monthly percentage excess log returns on the synthetic corporate bond index. All standard errors are Newey-West corrected with 4 lags.

Table 3: Equity Return Predictability

	Excess Return on the CRSP Value-Weighted Index							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	0.85	0.16	0.98	0.00	0.46	1.23	0.03	-0.36
	(2.73)	(0.34)	(2.71)	(0.00)	(0.84)	(4.25)	(0.05)	(-0.57)
CB VRP	5.50						4.15	
	(3.68)						(2.39)	
CB IV		3.38						2.09
		(2.41)						(1.21)
CB RV			0.56					
			(0.39)					
Equity VRP				0.11			0.09	0.09
				(2.35)			(1.69)	(1.72)
Equity IV					0.03			
					(1.18)			
Equity RV						-0.01		
						(-0.62)		
Adj. R^2 (%)	4.60	3.15	-1.17	5.32	-0.07	-1.06	7.34	5.52

Notes: This table reports the results of predictability regressions of 1-month ahead equity returns. Predictor variables include variance-related variables in the corporate bond market (the corporate bond variance risk premium, the corporate bond implied variance, and the corporate bond realized variance) and the corresponding variables in the equity market (the equity variance risk premium, the equity implied variance, and the equity realized variance). The dependent variable is the monthly percentage excess log returns on the CRSP value-weighted index. All standard errors are Newey-West corrected with 4 lags.