Common Idiosyncratic Volatility and Carry Trade Returns*

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Abstract

I provide new evidence that incomplete consumption risk sharing across countries is an important determinant of carry trade returns. I show that there is a strong co-movement in idiosyncratic volatilities over time, and that shocks to the common idiosyncratic volatility (CIV) factor, defined as the equally weighted average of the idiosyncratic volatilities in the cross-section, are priced. I find that high-interest rate currencies deliver low returns when the CIV increases, which are bad times for investors. Low-interest rate currencies provide a hedge by yielding positive returns. CIV shocks remain an empirically powerful risk factor in explaining the cross-section of carry trade returns after controlling for global foreign exchange (FX) volatility risk. Furthermore, CIV risk is correlated with cross-country income risk faced by households. My findings are consistent with a heterogeneous-agent model with persistent, uninsurable idiosyncratic shocks in consumption growth. The calibrated model quantitatively accounts for the cross-sectional differences in average returns across CIV-beta sorted portfolios for plausible market prices of CIV risk.

JEL classification: E3, E20, F31, G12, G15.

Keywords: Carry Trade, Idiosyncratic Volatility, Heterogeneous-Agent Model, Currency Excess Returns

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1 Introduction

The carry trade anomaly refers to the fact that lending in currencies that have high-interest rates – "investment currencies" – while borrowing in currencies that have low-interest rates – "funding currencies" – is a profitable trading strategy. This anomaly is a major stylized fact in the international finance literature and reflects the failure of the uncovered interest rate parity (UIP) condition. The UIP condition states that the interest rate differential between riskless assets denominated in two currencies is equal to the rate at which the higher interest rate currency is expected to depreciate against the lower interest rate currency. Thus, if the UIP condition holds, an investor engaged in the carry trade would expect a zero net payoff. Empirically, however, high-yielding currencies tend to appreciate instead of depreciate, adding to the profitability of the carry trade.[1] Understanding the properties of carry trade strategies is important not just for asset pricing, but also for macroeconomics, since it is widely believed that this strategy is partly responsible for excessive exchange-rate volatility, which is often viewed as problematic by financial regulators and policymakers (Brunnermeier et al., 2008).

In this paper, I provide new evidence that incomplete consumption risk sharing across countries is an important determinant of the cross-section of expected currency returns. In particular, I show that a country's exposure to changes in the cross-country dispersion of consumption growth is key to understanding currency risk premia. The economic intuition is simple: If markets are complete, agents can fully insure each other against country-specific shocks in consumption. In this case, the cross-country dispersion of consumption growth is zero. However, while household-specific shocks within a given country can be partially insured through the tax and transfer system (e.g., unemployment insurance), such mechanisms are absent for across-country idiosyncratic shocks. As a consequence, the cross-country dispersion of consumption growth is no longer zero because agents experience persistent income shocks that they are unable to completely insure against across countries. Because an increase in the variability of consumption growth across countries represents an increase in consumption risk for the average household within each country, it adversely affects its marginal utility. If high-interest rate currencies perform poorly in bad times – which are times of high cross-country consumption growth variability – investors demand a risk premium for holding these currencies.

My findings support a risk-based explanation for the carry trade profitability based

[1] Investors speculating on high-yield currencies are thus likely to earn a double profit, namely from interest rate differentials as well as from exchange rate movements in their favor.

on the existence of a time-varying risk-premium originating from cross-country variation in consumption growth. First, I document a strong factor structure in the volatilities of factor model residuals for currency excess returns. I define the common factor – termed the common idiosyncratic volatility (CIV) factor – as the cross-sectional average of idiosyncratic volatilities of all currencies available at each point in time. One natural question is whether this CIV factor explains the profitability of the carry trade strategy. I find that it does: Carry trade returns are negatively related to CIV shocks. This implies that high-interest rate currencies perform poorly during periods of unexpectedly high CIV shocks when low-interest rate currencies provide a hedge by yielding positive returns. Further analysis shows that exposure to CIV shocks is priced in the cross-section of currency returns. Even after controlling for global foreign exchange (FX) volatility risk, CIV shocks remain an empirically powerful risk factor in explaining the cross-section of carry trade returns. To connect carry returns to more fundamental risk sources, I provide new evidence linking CIV risk to country-level income risk faced by households. My results indicate that the CIV factor does, indeed, capture important aspects of incomplete cross-country risk sharing. Finally, to rationalize my empirical findings and quantitatively account for them, I propose a multi-country heterogeneous-agent model with incomplete markets that features uninsurable, persistent idiosyncratic shocks in consumption growth. The calibrated model quantitatively accounts for the cross-sectional differences in average returns across CIV-beta sorted portfolios for plausible market prices of CIV risk.

In my first analysis, I document a high degree of co-movement among the idiosyncratic volatilities of currency excess returns. A single factor – the CIV factor – explains more than 30% of the cross-sectional variation in idiosyncratic return volatilities. To the extent that the factor model regression that I use to measure idiosyncratic volatility is misspecified, it is possible that an omitted common factor whose importance has changed over time might account for the observed pattern in idiosyncratic volatilities. However, even after saturating the factor regression with up to five principal components, residual return volatilities continue to display the same co-movement seen in raw return volatilities. To further support the hypothesis that volatility co-movement is not driven by omitted factors, I examine the cross-sectional relationship between idiosyncratic volatility and expected returns. If the factor model specification is correct, forming currency portfolios by sorting on idiosyncratic volatility will provide no difference in average returns. But, if the factor model is misspecified, sorting in this way potentially provides a set of currencies that may have different exposures to the omitted factor and hence different average returns. Empirically, however, I do not find a significant spread in mean returns when forming portfolios based on idiosyncratic volatilities.

To study the relation between carry trade returns and CIV shocks, I divide the sample into four quartiles based on the value of the CIV shock. I find that high-interest rate currencies are negatively related to CIV shocks and thus deliver low returns in bad times for investors, when low-interest rate currencies provide a hedge by yielding positive returns. This negative relationship between CIV risk and carry trade returns is also observed in the data if I use CIV shocks based on consumption growth, instead of CIV shocks computed from currency excess returns. These findings suggest that carry trade returns are indeed a compensation for time-varying risk originating from macroeconomic fundamentals.

I empirically test whether CIV risk explains the cross-section of currency excess returns using a standard, linear asset pricing framework. If idiosyncratic volatility risk is a priced factor, then it is reasonable to assume that currencies sorted according to their exposure to idiosyncratic volatility innovations yield a cross-section of portfolios with a significant spread in mean returns. I sort currencies into five portfolios depending on their past beta with innovations to CIV factor. I use 36-month rolling window estimates of beta and rebalance portfolios every month. I refer to a currency's exposure to the CIV shock as its CIV-beta. I find that investing in currencies with positive CIV-beta (i.e., hedges against cross-country idiosyncratic volatility risk) leads to a significantly lower return than investing in negative CIV-beta currencies. The spread between portfolio 1 (low CIV-beta, that is, high idiosyncratic volatility risk) and portfolio 5 (high CIV-beta, that is, low idiosyncratic volatility risk) exceeds 4% per annum for the developed countries sample. Using traditional Fama and MacBeth (1973) regressions, I find that CIV risk is priced in the cross-section of currency excess returns. The Fama-MacBeth analysis also shows that the CIV factor does not merely capture global currency volatility. It retains its explanatory power for the cross-section of CIV-beta portfolios and drives out the global FX volatility factor of Menkhoff et al. (2012).

I attempt to connect CIV risk to more fundamental macroeconomic risks by showing that CIV risk appears strongly related to country-level income risk that is orthogonal to global income risk faced by households. I find that annual changes in CIV share a correlation of 75% with changes in the cross-sectional volatility of unemployment growth and a correlation of 29% with changes in the cross-sectional volatility of GDP growth for the sample that includes all countries (after removing a common component from unemployment and from GDP growth). For the developed countries sample, the correlations are 50% and 49%, respectively. These two new pieces of evidence, combined with the fact that carry trade returns are negatively related to CIV shocks derived from consumption growth, suggest that the CIV factor is a plausible proxy for country-level income risk faced by households, the key ingredient in heterogeneous agent models such as the ones presented by Constantinides and Duffie (1996), Storesletten et al. (2004, 2007), and Herskovic et al. (2016). The advantage of CIV is that it is constructed from currency returns that are observable at high frequencies, instead of using macroeconomic fundamentals such as consumption growth, released at low frequencies and subject to large revisions.

Finally, to rationalize my empirical findings regarding volatility co-movement and currency excess returns, I develop a heterogeneous-agent asset pricing model with incomplete markets. I follow the approach of Constantinides and Duffie (1996) and especially Herskovic et al. (2016) by modeling households with an incentive to hedge against increases in idiosyncratic volatility. However, my model has two novel features. First, I assume that there are two state variables: the cross-sectional volatility of consumption growth *across* countries and the cross-sectional volatility of consumption growth *within* each country. The dispersion in consumption across countries is consistent with my empirical evidence regarding the correlation between CIV shocks and the cross-country volatility of consumption growth. The within-country dispersion turns out to be critically important when I derive the asset pricing implications of this model. Second, I assume that the world consumption growth is driven by persistent shocks whose conditional volatilities are time-varying. The model gives rise to the common component in idiosyncratic volatility as a priced state variable: Currencies with more negative exposure to this innovation (a more negative CIV-beta) earn a higher risk premium, suggesting that returns to the carry trade are indeed compensation for time-varying fundamental risk. Another key feature of the model is that it generates expected currency returns related to two risk factors: a dollar risk factor and the idiosyncratic, but not diversifiable, risk proxied by the cross-country variation of consumption growth.

The high profitability of carry trades is a long-standing conundrum in the field of finance, fueling the search for risk factors driving these returns. Despite its importance, very few studies step outside the representative agent/complete markets paradigm and investigate the extent to which certain deviations from frictionless Arrow-Debreu markets can help to account for the carry trade anomaly. The main contribution of this study is to enrich the pricing implications of the representative-agent model by assuming that an agent's consumption process contains an idiosyncratic component that cannot be perfectly diversified across countries due to incomplete markets. The joint hypothesis of incomplete consumption insurance and consumer heterogeneity across countries gives rise to a common factor in idiosyncratic volatilities of currency excess returns. This common factor is the key state variable driving both residual currency volatility and dispersion in consumption growth across countries. Because consumption growth is negatively related to the cross-country variability of consumption growth, the common factor acts as a potential new source of currency risk premia.

Related literature. This paper is related to several strands of research. First, it builds on the vast literature that tries to explain the carry trade profitability based on the existence of a time-varying risk premium.[2] Intuitively, if investment in high-interest rate currencies delivers low returns during bad times for investors, when the marginal utility is very high, carry trade profits are just compensation for investor's higher risk exposure. Some risks of the carry trade are well known. There is a vast literature that looks to crash risk and its relation to carry trade returns. For example, Brunnermeier and Pedersen (2009) show that carry trades are related to low conditional skewness, indicating that they are subject to crash risk, a result confirmed in further analysis by Farhi and Gabaix (2016). Jurek (2014) finds that currency carry trades are strongly exposed to the returns of the Jurek and Stafford (2015) downside risk index (DRI) portfolio, and he concludes that the abnormal returns of currency carry trades are indistinguishable from zero after controlling for their exposure to DRI and other equity risk factors. Lustig et al. (2011) show that the risk of carry trades across currency pairs is not completely diversifiable, so there is a systematic risk component to the strategy. They form two empirically motivated risk factors and show that they explain the carry premium: The average currency excess return of a large set of currencies against the US dollar ("Dollar" risk factor) and the return to the carry trade portfolio itself (the "Carry" factor). Carry also performs worse when there are liquidity squeezes (Brunnermeier et al., 2008; Mancini et al., 2013), increases in foreign exchange volatility (Menkhoff et al., 2012), and increases in global imbalances (Della Corte et al., 2016). Empirically, however, few papers have attempted to connect carry trade returns to more fundamental risk sources. The novel insight of this paper is thus to uncover a new source of currency risk premia that emerges because countries are

[2] There is also a second strand of the currency literature that looks for non-risk-based explanations. Bacchetta and Van Wincoop (2010) attribute the failure of UIP to infrequent revisions of investor portfolio decisions. Burnside et al. (2007) find that the return of the carry trade portfolio is uncorrelated to standard risk factors, attributing instead the forward premium to market frictions. Another possible explanation of the carry premium is that carry trade reflects out of sample risk: Periods of extreme risk aversion that have not been observed in the data (peso events) can explain the returns to the carry trade (Barro, 2006; Farhi and Gabaix, 2016; Gourio et al., 2013; Burnside, 2011). Using options to hedge away the "peso risk" reduces abnormal returns, lending some support to this view, but the remaining returns depend crucially on the particular option strategy used for hedging (Jurek, 2014). Even so, Burnside et al. (2011) argues that the recent financial crisis was not the "peso event" needed to rationalize the carry trade previous returns, since carry trades are still profitable after covering most of the downside risk through the use of derivatives.

unable to share their idiosyncratic consumption risk.

Another branch of the literature explores the ability of market incompleteness to address various puzzles in international finance.[3] In incomplete markets, the exchange rate return is in general different from the ratio of international stochastic discount factors, a feature that can be captured by a stochastic exchange rate wedge as in Backus et al. (2001). Lustig and Verdelhan (2019) adopt a preference-free approach and use Euler equations to derive restrictions on this wedge. They show that, in a no-arbitrage setting, some of the constraints imposed on the wedge to jointly address international finance puzzles may be difficult to reconcile with the data. Favilukis et al. (2015) show that a standard two-period general equilibrium model of a two-country (home and foreign), two-sector (tradable and nontradable) economy with market incompleteness induced by restrictions in financial trade also cannot resolve multiple exchange rate puzzles simultaneously. In this paper, unlike Favilukis et al. (2015) and Lustig and Verdelhan (2019), I provide a multi-country, fully specified dynamic asset pricing model that starts from a complete description of preferences and endowments. In the model, market incompleteness arises because agents are not able to fully insure each other against country-specific shocks. My model can resolve two exchange rate puzzles simultaneously, namely, the currency risk-premium puzzle of Fama (1984) and the cyclicality puzzle of Backus and Smith (1993): the CIV factor accounts for most of the cross-sectional variation in average excess returns between high- and low-interest rate currencies, and the model also induces a positive correlation between currency appreciation and consumption growth in high-interest rate countries, while most complete markets models (counterfactually) imply a negative relationship between them.[4]

My work also connects to a literature that is at the intersection of consumption risk sharing and asset pricing. For example, Backus et al. (1992) and Lewis (1996) show that international consumption correlations are too low to be explained by models with complete markets. Brandt et al. (2006) find that stochastic discount factors derived from stock prices are highly correlated across countries, indicating significant international risk sharing, while marginal utility growth derived from aggregate consumption is weakly corre-

[3]There are three puzzles regarding exchange rates and risk sharing emphasized in studies of international finance: (i) the currency risk premium of Fama (1984); (ii) the exchange rate cyclicality puzzle of Backus and Smith (1993); and (iii) the volatility puzzle of Brandt et al. (2006). Along those three dimensions, the data are at odds with the standard models in the academic literature that typically assume that financial markets are complete.

[4]Notable recent theoretical contributions include the work by Alvarez et al. (2002, 2009), Bacchetta and Van Wincoop (2006), Corsetti et al. (2008), Pavlova and Rigobon (2010, 2012), Bruno and Shin (2015), Maggiori (2017), Gabaix and Maggiori (2015), Bakshi et al. (2018), and Sandulescu and Vedolin (2020).

lated across countries, indicating much less risk sharing. Brav et al. (2002), Cogley (2002), Sarkissian (2003), Storesletten et al. (2004, 2007), Lustig and Van Nieuwerburgh (2005, 2010), and Chien et al. (2011) follow Mankiw (1986) and Constantinides and Duffie (1996) and study models in which assets with higher expected returns perform poorly at times of higher cross-sectional labor income risk.

Finally, more closely related to this paper, there are a few studies showing that there is a lot of commonality in the volatilities of asset returns. For example, Bekaert et al. (2010) study idiosyncratic volatility for individual firms from 23 developed countries (including the US) from 1980 to 2008. They show that idiosyncratic return volatilities commove substantially in developed countries. Herskovic et al. (2016) show that US firm's idiosyncratic volatility obeys a strong factor structure and that shocks to a common idiosyncratic volatility factor are priced. Christoffersen et al. (2018) show that implied volatilities of equity options display a strong factor structure.

The remainder of the paper is organized as follows. In Section 2, I describe the data and present some variable definitions. In Section 3, I provide an empirical investigation of idiosyncratic risk in currency markets. In Section 4, I present a heterogeneous-agent model with CIV as a priced state variable. Section 5 concludes. The Appendix contains all proofs. A separate Internet Appendix provides further details on the data, robustness tests, and additional results.

2 Data and currency portfolios

This section describes the main data employed in the empirical analysis, the construction of currency portfolios and their associated excess returns, and how idiosyncratic volatilities are computed in a panel of currency returns. Following Lustig et al. (2011), I focus on investments in forward and spot currency markets. I consider currency portfolios that include developed and emerging market countries for which forward contracts are traded. I start by setting up some notation. Then, I describe the data and the methodology used to compute idiosyncratic volatilities. I conclude by providing a summary of the carry trade portfolio returns.

2.1 Currency excess returns for a US investor

Following the extant literature since Fama (1984), I work in logarithms of spot and forward rates for ease of exposition and notation. Let $s_{j,t}$ denote the log of the spot exchange rate for currency j in units of foreign currency per US dollar, and $f_{j,t}$ the log forward exchange rate one month ahead, also in units of foreign currency per US dollar. An increase in s means an appreciation of the home currency. The log excess return rx on buying a foreign currency in the forward market and then selling it in the sport market after one month is given by

$$rx_{j,t+1} = f_{j,t} - s_{j,t+1}.$$

According to the CIP condition, under no arbitrage, the forward premium is approximately equal to the interest rate differential:

$$f_{j,t} - s_{j,t} \approx r_t - r_{j,t},$$

where $r_{j,t}$ and r_t denote the foreign and domestic nominal risk-free rates over the maturity of the contract. The log currency excess return can then be expressed as

$$rx_{j,t+1} = (f_{j,t} - s_{j,t}) - \Delta s_{j,t+1} \approx r_{j,t} - r_t - \Delta s_{j,t+1}. \quad (1)$$

Therefore, if the CIP condition holds, the log currency excess return is approximately equal to the interest rate differential less the rate of depreciation of the home currency.[5] I report results without transaction costs because the inclusion of bid and ask quotes inflates the volatility of the excess returns, giving more weight to less traded and illiquid currencies.[6]

[5] Akram et al. (2008) study high-frequency deviations from covered interest-rate parity (CIP). They conclude that CIP holds at daily and lower frequencies. On the other hand, Du et al. (2018) document persistent deviations from the CIP condition after the global financial crisis in 2008/2009, leading to significant arbitrage opportunities in currency and fixed income markets. My results are quantitatively similar if I drop the data after the 2008/2009 financial crisis.

[6] Neely and Weller (2013) argue, based on interviews with traders, that posted bid-ask spreads may systematically overestimate the spreads available in practice and suggest using one-third of the posted one-month forward rate bid-ask spread as a more reliable estimate of one-way transactions costs.

2.1.1 Daily currency excess returns

While the literature has almost exclusively focused on the characteristics of carry trade returns at the monthly frequency, to be able to compute idiosyncratic volatilities, I need carry trade returns at the daily frequency, while retaining the monthly decision interval.[7]

To calculate daily returns for a monthly carry trade strategy, I follow Daniel et al. (2017) and assume traders borrow and lend at the prorated one-month foreign-currency interest rates, and I infer foreign interest rates from the US interest rate and covered interest rate parity when foreign interest rates are not available. Let $P_{j,t,\tau}$ represent the cumulative carry trade profit realized on day τ during month t, with D_t being the number of trading days within the month. The excess daily return can be calculated as follows:

$$RX_{j,t,\tau} = \frac{P_{j,t,\tau}}{P_{j,t,\tau-1}} - e^{r_t \frac{1}{D_t}}, \tag{2}$$

where

$$P_{j,t,\tau} = e^{r_{j,t} \frac{\tau}{D_t}} \frac{S_{j,t,\tau}}{S_{j,t-1,D_{t-1}}}$$

is the cumulative carry trade profit.

2.2 Data

I use daily spot and one-month forward exchange rates vis-à-vis the US dollar (USD) spanning the time period November 1983-June 2020. The data were collected by Barclays and Reuters and are available on Datastream. My data set nests the data used in recent studies on currency returns, including Lustig et al. (2011), Burnside et al. (2011), and Hassan and Mano (2019). The main data set contains at most 37 currencies (referred to as my "All Countries" sample below): Australia, Austria, Canada, Czech Republic, Denmark, euro area, Finland, France, Germany, Greece, Hong Kong, Hungary, India, Indonesia, Ireland, Italy, Japan, Kuwait, Malaysia, Mexico, Netherlands, New Zealand, Norway, Philippines, Poland, Portugal, Saudi Arabia, Singapore, South Africa, South Korea, Spain, Sweden, Switzerland, Taiwan, Thailand, United Kingdom, and the United Arab Emirates.

[7] While traders in foreign exchange markets can easily adjust their carry trade strategies at the daily frequency with minimal transaction costs, I choose to examine the daily returns to carry trades that are rebalanced monthly to maintain consistency with the academic literature and because I do not have quotes on forward rates for arbitrary maturities that are necessary to close out positions within the month.

Some of these currencies pegged their exchange rates partly or completely to the US dollar over the course of the sample. I keep them in my sample because forward contracts were easily accessible to investors. After the euro introduction in January 1999, I exclude the euro area countries and keep only the euro series.

Since many currencies in this broad sample are pegged or subject to capital restrictions at various points in time, I also consider a subset of 15 countries that I refer to as "Developed Countries". This sample comprises: Australia, Belgium, Canada, Denmark, euro area, France, Germany, Italy, Japan, Netherlands, New Zealand, Norway, Sweden, Switzerland, and the United Kingdom.

Throughout the main text, I take the perspective of a US investor and calculate all returns in US dollars. The Internet Appendix lists the coverage of individual currencies and describes the data-selection and data-cleaning process in detail.

Descriptive statistics. Table 1 reports descriptive statistics for the six carry trade portfolios, the dollar factor (*Dollar*), and the carry trade factor (*Carry*). The top panel shows results for the sample of all 37 currencies, and the lower panel shows results for the sample of 15 developed countries.

Consistent with results in the literature, I find that the carry trade strategy applied to portfolios of currencies yields high average payoffs, as well as Sharpe ratios that are substantially higher than those associated with the US stock market. As can be seen from the table, average returns monotonically increase when moving from portfolio 1 to portfolio 6. The annualized average carry trade return is 6.21% for the full sample and 4.88% for the developed countries sample. Ignoring transaction costs, the Sharpe ratio of the equally weighted carry trade portfolio is 0.70 for the full sample and 0.50 for the developed countries sample. The unconditional average excess return from holding an equally weighted portfolio of foreign currencies (i.e., the "Dollar" portfolio) is about 1.4% per annum, which suggests that US investors demand a modest but positive risk premium for holding foreign currency.

2.3 Estimating idiosyncratic return volatility

The idiosyncratic volatility of a currency is estimated relative to the systematic returns of the currency, and hence model dependent. In this paper, I study the behavior of idiosyncratic volatilities defined relative to three different factor model specifications for currency excess returns.

Table 1: Descriptive statistics

The table reports annualized summary statistics of currency portfolios sorted monthly on time $t-1$ forward discounts. I also report risk-adjusted returns with respect to the MM model (Alpha MM), the LRV model (Alpha LRV), and the PCA model (Alpha PCA). Portfolio 1 contains currencies with the lowest forward discounts, whereas portfolio 6 contains currencies with the highest forward discounts. All returns are excess returns in US dollars. *Dollar* denotes the average return of the six currency portfolios and *Carry* denotes a long-short portfolio that is long in portfolio 6 and short in portfolio 1. I report excess returns without transaction cost adjustments. Newey and West (1987) HAC t-statistics are reported in parentheses. Panel A gives results for the sample that includes all countries. Panel B reports summary statistics for the subsample of developed countries. Returns are monthly and the sample period is November 1983 to June 2020. *, **, and *** denotes significance at 10%, 5%, and 1%, respectively.

Portfolio	1	2	3	4	5	6	*Carry*	*Dollar*
Panel A: All Countries								
Mean	-1.49	-0.05	0.88	2.63	2.58*	4.72***	6.21***	1.43
	(1.35)	(0.88)	(1.18)	(1.60)	(1.43)	(1.70)	(1.61)	(1.20)
Std. Dev.	7.31	6.69	7.24	8.05	8.22	9.96	8.87	6.87
Sharpe Ratio	-0.20	-0.01	0.12	0.33	0.31	0.47	0.70	0.21
Alpha MM	-2.72***	-1.22**	-0.46	1.11**	1.06	2.96***	5.69***	
	(0.76)	(0.49)	(0.38)	(0.53)	(0.75)	(1.14)	(1.75)	
Alpha LRV	-0.28	-0.44	-0.14	1.00*	0.81	-0.28		
	(0.36)	(0.52)	(0.38)	(0.54)	(0.79)	(0.36)		
Alpha PCA	-1.49	-0.05	0.88	2.62	2.57*	4.73***	6.22***	
	(1.35)	(0.88)	(1.18)	(1.60)	(1.43)	(1.70)	(1.61)	
Panel B: Developed Countries								
Mean	-0.89	0.14	1.01	0.90	1.64	3.99*	4.88***	1.29
	(1.65)	(2.09)	(1.57)	(1.76)	(1.85)	(2.05)	(1.39)	(1.64)
Std. Dev.	9.13	9.98	9.08	10.56	10.33	11.01	9.77	8.49
Sharpe Ratio	-0.10	0.01	0.11	0.09	0.16	0.36	0.50	0.15
Alpha MM	-2.39**	-1.49	-0.65	-0.92	-0.12	2.05**	4.44***	
	(0.96)	(1.21)	(0.60)	(0.77)	(0.87)	(1.00)	(1.49)	
Alpha LRV	0.39	0.19	0.17	-0.89	-0.32	0.65		
	(0.84)	(1.02)	(0.69)	(0.76)	(0.88)	(0.87)		
Alpha PCA	-0.90	0.13	1.00	0.90	1.64	3.99**	4.89***	
	(1.64)	(2.09)	(1.58)	(1.76)	(1.85)	(2.04)	(1.35)	
N	441	441	441	441	441	441	441	441

2.3.1 Factor model specification

Idiosyncratic returns are constructed within a one-year window by estimating a factor model using daily observations within that year. The factor model takes the form

$$rx_{j,t} = \gamma_{0j} + \gamma_{1j}' \boldsymbol{F}_t + \epsilon_{j,t}, \tag{3}$$

where t denotes a daily observation and $\boldsymbol{F}_t$ is a vector of common factors. I follow Ang et al. (2006) and Herskovic et al. (2016) and define the idiosyncratic volatility as $\sqrt{\text{Var}\left(\epsilon_{j,t}\right)}$ in equation (3). Empirically, the idiosyncratic volatility is computed as the standard deviation of residuals $\hat{\epsilon}_{j,t}$ within the calendar year. The result of this procedure is a panel of currency-year idiosyncratic volatility estimates.

The first return factor model that I consider is the "market" factor model, denoted *MM model*. This model specifies $\boldsymbol{F}_t = [Dollar_t]$, where $Dollar_t$ is the market risk factor or average excess return from Lustig et al. (2011). The dollar factor is defined as the equally weighted average return for a US investor who goes long in all N currencies available in the sample. The *Dollar* factor has also a natural interpretation as the currency market return in USD available to a US investor and is driven by the fluctuations of the USD against a broad basket of currencies.

The second factor model specifies $\boldsymbol{F}_t = [Dollar_t, Carry_t]$, where $Carry_t$ is defined as the average excess return for a currency portfolio that is long in high-interest rate currencies and short in low-interest rate currencies. This factor was also proposed by Lustig et al. (2011), who argue that the carry trade factor explains the common variation in carry trade returns. They suggest that this risk factor captures "global risk" for which carry traders earn a risk premium. I refer to this model as the *LRV model*.

Finally, the third return factor model, referred to as the *PCA model*, is purely statistical and specifies $\boldsymbol{F}_t$ as the first three principal components (PCs) of the cross-section of returns within a one-year window.[8]

2.3.2 Common factor in volatilities

I use a straightforward measure to proxy for the common factor in idiosyncratic return volatilities. More specifically, I define the CIV factor as the cross-sectional average of idiosyncratic volatilities of different currencies at a point in time. My CIV proxy in year t

[8] Robustness checks using five principal components as statistical factors produce quantitatively similar results.

is thus given by

$$CIV_t = \frac{1}{N} \sum_{j=1}^{N} \sqrt{\text{Var}_t \left(\epsilon_{j,t} \right)}, \tag{4}$$

where N denotes de number of available currencies on year t. I also calculate the proxy CIV_t^{DEV} based on the developed countries' returns.

This proxy is the FX analog of the CIV factor introduced by Herskovic et al. (2016) for pricing the cross-section of US stock returns. It is approximately equal to the first principal component of the idiosyncratic volatility panel but avoids complications arising with the computation of principal components in unbalanced panels. The CIV factor is also similar to measures of realized volatility (see, for example, Andersen et al., 2001; Della Corte et al., 2016). I do not weight currencies, for example, according to shares in international reserves or trade, but provide robustness on this issue in the Internet Appendix.

For comparison purposes, I also report results using a proxy for the total return volatility of the market:

$$Total_t = \frac{1}{N} \sum_{j=1}^{N} \sqrt{\text{Var}_t \left(rx_{j,t} \right)}. \tag{5}$$

This proxy is similar to the aggregate FX volatility factor proposed by Menkhoff et al. (2012), with two main differences. First, I am using currency excess returns instead of exchange rates. Second, the volatility is defined as the standard deviation of raw returns, as opposed to the absolute values of exchange rates.[9]

Volatility shocks. For the empirical analysis, I focus on idiosyncratic volatility shocks. I estimate an AR(1) for the volatility and idiosyncratic volatility levels and take the residuals as my main proxy for volatility innovations since the AR(1) residuals are, in fact, uncorrelated with their own lags. A similar approach was also employed in Menkhoff et al. (2012) and Mancini et al. (2013).[10]

[9] Robustness checks using a volatility proxy based on the absolute value of changes in exchange rates produce quantitatively similar results.

[10] Ang et al. (2006) and Herskovic et al. (2016) define volatility shocks as the first difference of the volatility series. They argue that the first difference has a negligible serial correlation. As part of their robustness checks, they also measure volatility innovations by specifying a stationary time series model for the conditional mean of the volatility factor and find the results to be similar to those using simple first differences.

3 Empirical Results

This section summarizes my empirical results. First, I show that there is a strong factor structure in idiosyncratic volatilities of currency excess returns. Next, I present cross-sectional asset pricing tests for currency portfolios and the common factor in idiosyncratic volatilities – the CIV factor – and empirically document that CIV risk is priced in a broad cross-section of currency portfolios. I find that CIV risk is priced even when controlling for the global FX volatility factor of Menkhoff et al. (2012). Finally, I provide evidence linking the common factor in idiosyncratic volatilities to country-level income risk faced by households.

3.1 The factor structure in volatility

Residuals from linear factor-based asset pricing models exhibit statistical independence, but a strong factor structure in the squared terms, i.e., a strong co-movement in idiosyncratic volatilities. This section provides empirical evidence of idiosyncratic volatility co-movement in currency returns and rules out alternative explanations for this factor structure based on omitted factors.

3.1.1 Common patterns in currency-level volatilities

There is a strong commonality in currency volatilities and idiosyncratic volatilities over time. Figure 1 plots annual currency-level total return volatilities and idiosyncratic volatilities averaged within start-of-year per capita GDP groups. Panels (a) and (c) report the average total return volatility by GDP group for all countries and for the subset of developed countries, respectively. Panels (b) and (d) report the same within-group averages for idiosyncratic volatilities – the standard deviation of residuals – instead of total volatilities.

Currencies in all quartiles demonstrate very similar time series volatility patterns. If there is a factor structure in returns, i.e., if we can explain the cross-section of currency returns using common factors, and the volatilities of those common factors are time-varying, then currency-level volatilities also inherit this factor structure. In this case, it would be reasonable to expect that there is a lot of commonality in total return volatilities over time. However, the fact that we observe the same dynamics for idiosyncratic volatilities after removing common factors in returns is surprising. This suggests that

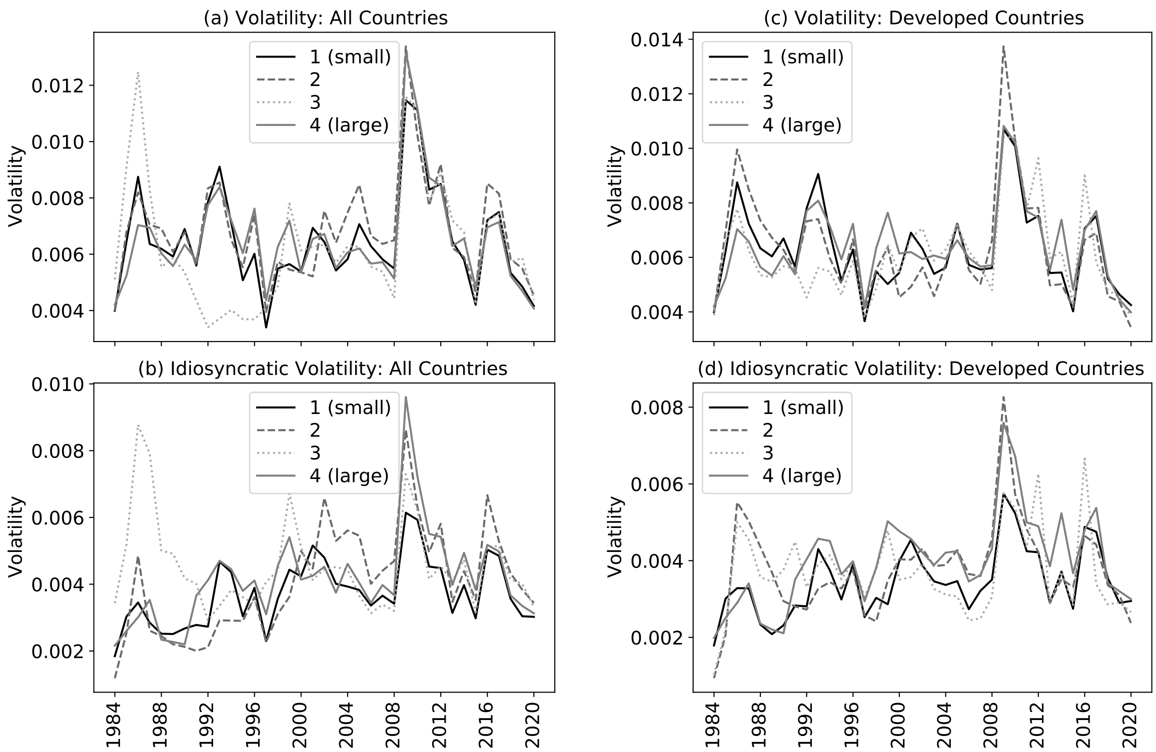

Figure 1: Total and idiosyncratic return volatility by per capita GDP group.

The figure plots annualized currency-level volatilities averaged within GDP group. Within each calendar year, volatilities are estimated as the standard deviation of daily returns for each currency. Panels (a) and (c) show total return volatilities averaged within GDP quartiles for all countries and for the developed country sample, respectively. Panels (b) and (d) report the same within-group averages for currency idiosyncratic volatilities. Idiosyncratic volatility is defined as the standard deviation of residuals from a three-factor principal components model for daily currency excess returns. The sample period is November 1983 to June 2020.

idiosyncratic return volatility is driven largely by shocks that affect the FX market as a whole, rather than individual FX rates.

3.1.2 Is it due to omitted common factors in the factor model specification?

A possible explanation for this common pattern in idiosyncratic volatilities is the co-movement among factor model residuals, for instance, due to omitted common factors in the factor model specification (3). I examine this possibility considering three pieces of evidence.

First, I compute average pairwise correlation for returns and for residuals. Panel (a) of Figure 2 reports the average pairwise correlation within each year for returns and residuals for all countries, and Panel (b) reports the same results for the subset of developed countries. As can be seen in the figure, raw returns share substantial common variation, occasionally exceeding 60%. The average pairwise correlation over the 1983-2020 sample is 34% for all countries and 51% for the sample that only includes developed countries. However, the factor models capture nearly all of this common variation at the daily frequency, as factor model residuals are virtually uncorrelated on average. The MM model and the LRV model appear to absorb all of the co-movement in currency returns, making omitted factors an unlikely explanation for the high degree of commonality in idiosyncratic return volatilities.

Second, I consider three different factor model specifications. Panels (c) and (d) of Figure 2 report the cross-sectional average of volatilities and idiosyncratic volatilities each year for total and idiosyncratic returns. Although the average idiosyncratic volatilities from various factor models are different, their dynamics over time are very similar. In a typical year, only 50% of the average total volatility is explained by the three principal components model, with idiosyncratic volatility inheriting the remaining 50%. The same is true for the MM model and the LRV model, with 35% and 40% of average total volatility explained by common factors, respectively. Even if I saturate the model with up to five principal components, the residual return volatility continues to display the same dynamics we observe in total return volatilities.

Finally, I form currency portfolios by sorting on idiosyncratic volatilities. If there is a missing factor in the factor model specification, the sensitivity of currency returns to the missing factor times the movement in the missing factor will show up in the residuals of the model. Currencies with greater sensitivities to the missing factor should therefore have larger idiosyncratic volatilities relative to the factor model, everything else being equal. If the factor model is correct, forming portfolios by sorting on idiosyncratic volatility will obviously provide no difference in average returns. Table 2 reports average returns of portfolios sorted on total volatility in Panel A and of portfolios sorted on idiosyncratic volatility in Panel B. At the end of every month, I simply sort currencies into quintile portfolios based on their level of idiosyncratic volatility computed using daily returns over the past month, and I hold these equally weighted portfolios for 1 month. The portfolios are rebalanced each month.

Panel A shows that average returns for the sample that includes all countries increase from 0.16% per year going from quintile 1 (low total return volatility currencies) to 1.8% per year for quintile 5, which contains currencies with the highest total return volatility. For the subset of developed countries, the results go in the opposite direction: av-

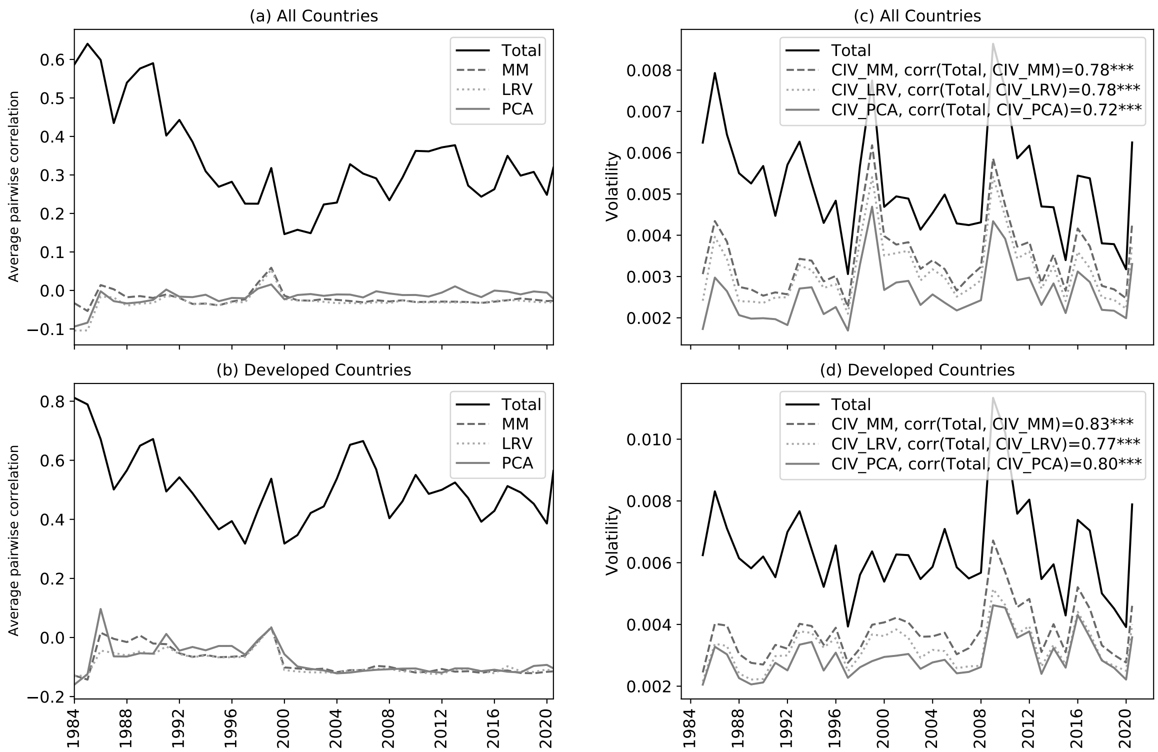

Figure 2: Volatility and correlation of factor model residuals.

Panels (a) and (b) show the average pairwise correlation for total and idiosyncratic returns within each calendar year for all countries and for the subsample of developed countries, respectively. Panels (c) and (d) show the cross-sectional average annualized currency-level volatility each year for total and idiosyncratic returns. Idiosyncratic volatility is defined as the standard deviation of residuals from a three-factor principal components model for daily currency excess returns. The sample period is November 1983 to June 2020.

erage returns decrease from 1.56% per month going from quintile 1 to 0.98% per month for quintile 5, which is consistent with early evidence for stock markets from Ang et al. (2006). For both sets of countries, the average excess return for the long-short portfolio (1-5) is not statistically significant. I obtain similar patterns in Panel B, where the portfolios are sorted on idiosyncratic volatility. Therefore, forming portfolios by sorting on idiosyncratic volatility provides no difference in average returns, implying that omitted factors are partially or totally incompatible with this factor structure in idiosyncratic return volatilities.

Table 2: Portfolios sorted by volatility

I form equally weighted quintile portfolios every month by sorting currencies based on total volatility and idiosyncratic volatility relative to the PCA model. Portfolios are formed every month, based on volatility computed using daily data over the previous month. Portfolio 1 (5) is the portfolio of currencies with the lowest (highest) volatilities. The statistics in the rows labeled Mean and Std. Dev. are measured in annual percentage terms and applied to excess returns. The column (1-5) refers to the difference in returns between portfolio 1 and portfolio 5. The Alpha rows report Jensen's alpha with respect to the MM model, the LRV model, and the PCA model. Robust Newey and West (1987) standard errors with optimal lag selection according to Andrews (1991) are reported in parentheses. The sample period period is October 1983 to June 2020. *, **, and *** denotes significance at 10%, 5%, and 1%, respectively.

Panel A: Portfolios sorted by Total volatility

	All Countries						Developed Countries					
Portfolio	1	2	3	4	5	(1-5)	1	2	3	4	5	(1-5)
Mean	0.17	2.33**	1.52	1.27	1.80	1.63	1.56	0.92	1.14	1.22	0.91	-0.65
	(0.35)	(1.03)	(1.38)	(1.66)	(1.83)	(1.64)	(1.17)	(1.52)	(1.76)	(1.70)	(1.76)	(1.37)
Std. Dev.	1.86	5.12	7.76	9.20	10.73	10.04	6.50	8.99	10.18	10.13	10.88	9.17
Sharpe Ratio	0.09	0.46	0.20	0.14	0.17	0.16	0.24	0.10	0.11	0.12	0.08	-0.07
alpha MM	-0.09	1.43**	-0.07	-0.68	-0.41	-0.32	0.82	-0.24	-0.12	-0.16	-0.51	-1.33
	(0.26)	(0.56)	(0.51)	(0.55)	(0.70)	(0.79)	(0.70)	(0.67)	(0.90)	(0.67)	(0.81)	(1.24)
alpha LRV	0.03	1.72***	0.36	-0.16	-1.79***	-1.82**	1.13	-0.06	0.13	-0.15	-1.20	-2.33*
	(0.26)	(0.57)	(0.52)	(0.58)	(0.65)	(0.74)	(0.70)	(0.68)	(0.95)	(0.70)	(0.82)	(1.22)
alpha PCA	0.17	2.33**	1.52	1.27	1.80	1.63	1.56	0.92	1.14	1.22	0.91	-0.65
	(0.35)	(1.03)	(1.38)	(1.65)	(1.83)	(1.64)	(1.17)	(1.51)	(1.76)	(1.69)	(1.77)	(1.38)

Panel B: Portfolios sorted by idiosyncratic volatility relative to the PCA model

	All Countries						Developed Countries					
Portfolio	1	2	3	4	5	(1-5)	1	2	3	4	5	(1-5)
Mean	0.79	1.65	1.68	0.77	2.04	1.24	1.57	1.62	0.53	2.01	-0.25	-1.82
	(0.81)	(1.37)	(1.38)	(1.35)	(1.41)	(1.17)	(1.70)	(1.70)	(1.74)	(1.40)	(1.40)	(1.38)
Std. Dev.	4.27	7.32	7.67	7.70	8.14	7.24	9.59	9.49	10.68	8.65	8.58	8.23
Sharpe Ratio	0.19	0.23	0.22	0.10	0.25	0.17	0.16	0.17	0.05	0.23	-0.03	-0.22
alpha MM	0.12	0.17	0.10	-0.80	0.46	0.34	0.28	0.33	-0.76	0.91	-1.28	-1.56
	(0.54)	(0.52)	(0.51)	(0.52)	(0.69)	(1.01)	(0.68)	(0.61)	(0.91)	(0.68)	(0.88)	(1.31)
alpha LRV	-0.01	0.84	0.44	-0.78	-0.52	-0.52	0.66	0.29	-1.19	0.88	-1.37	-2.02
	(0.58)	(0.52)	(0.53)	(0.54)	(0.68)	(1.04)	(0.68)	(0.63)	(0.98)	(0.72)	(0.91)	(1.34)
alpha PCA	0.79	1.65	1.69	0.77	2.04	1.24	1.57	1.62	0.53	2.01	-0.25	-1.82
	(0.81)	(1.37)	(1.39)	(1.35)	(1.41)	(1.15)	(1.68)	(1.70)	(1.76)	(1.39)	(1.41)	(1.36)
N	9544	9544	9544	9544	9544	9544	9544	9544	9544	9544	9544	9544

3.1.3 Quantifying the factor structure in volatilities

I next estimate factor regression models for currency-level volatilities. I consider total volatilities as well as idiosyncratic volatilities estimated from the MM model, the LRV model, or the PCA model. In all cases, for each currency j, I run a time series regression with volatility as the left-hand-side variable. The factor in each set of regressions is defined as the equally weighted average of the left-hand-side volatility measure.

Table 3: Volatility factor model estimates

The table reports estimates of annual volatility one-factor regression models. The volatility is defined as the equally weighted, cross-sectional average of currency volatility within each year. That is, all estimated volatility factor models take the form: $\sigma_{j,t} = intercept_i + loading_i \cdot \bar{\sigma}_t + e_{i,t}$. Columns represent different volatility measures. The columns report estimates for a factor model of total return volatility and idiosyncratic volatility based on residual returns from the MM model, the LRV model, and the three principal components models. For the OLS model, I report cross-sectional averages of loadings and intercepts as well as time series regression R^2 averaged over all currencies. I also report a pooled factor model R^2 and a fixed effect factor model R^2, which compares the estimated factor model with a model with only currency-specific intercepts and no factor. The sample period is November 1983 to June 2020.

	Total	MM	LRV	PCA
Panel A: All Countries				
Loading (average)	1.002	1.018	1.000	0.979
Intercept (average)	0.000	0.000	0.000	0.000
R2 (average univariate)	0.422	0.337	0.323	0.323
R2 (pooled)	0.178	0.159	0.170	0.187
R2 (fe)	0.245	0.249	0.274	0.308
Panel B: Developed Countries				
Loading (average)	1.037	0.903	0.945	0.939
Intercept (average)	0.000	0.000	0.000	0.000
R2 (average univariate)	0.670	0.407	0.435	0.415
R2 (pooled)	0.541	0.302	0.336	0.348
R2 (fe)	0.596	0.366	0.394	0.404

Table 3 reports volatility factor models' results for quarterly return volatilities. Columns correspond to the method used to construct return residuals. For the sample that includes all countries, the average univariate time series R^2 is around 42% for the total volatility model and 32% for the idiosyncratic volatility models for all countries. For the subset of developed countries, the average univariate time series R^2 is 67% for the total volatility model and around 41% for the idiosyncratic volatility models. For developed countries, the pooled panel ordinary least square R^2 is between 30% and 34%, whereas it ranges from 15% to 19% in all countries, suggesting that there is a lot of heterogeneity across countries in the part of the sample that includes emerging markets.

3.2 CIV shocks and expected currency returns

This section presents cross-sectional asset pricing tests for currency portfolios and the common idiosyncratic volatility factor and documents that CIV risk is priced in a broad cross-section of currency portfolios.

3.2.1 Relation between CIV shocks and carry trade returns

I first provide a very simple graphical analysis to visualize the relationship between CIV shocks and carry trade returns. To do so, I divide the sample into four subsamples based on the realization of volatility shocks. The first subsample contains the 25% months with the lowest realizations of volatility shocks and the fourth subsample contains the 25% months with the highest realizations of volatility shocks. I then calculate the average carry trade excess return for these subsamples. Results are shown in Figure 3. The left panel shows results for all countries, whereas the right panel gives the corresponding results for the subset of 15 developed countries. Each bar shows the annualized mean return of the carry trade portfolio for that specific subsample. The top panels show the results using *Total* volatility shocks, and the bottom panels use CIV shocks.

As can be seen in the figure, high-interest rate currencies perform well when CIV shocks are low and vice versa. Average excess returns for the carry trade factor decrease monotonically when moving from the low to the high idiosyncratic volatility states for both the sample that includes all countries and the developed countries sample. Although the results using *Total* volatility shocks and CIV shocks go in the same direction, the spread in returns between the first and the fourth subsample for the subset of developed countries is much larger when I do the sorting based on CIV shocks rather than *Total* volatility shocks.

Figure 4 repeats the same exercise, but instead of using volatility shocks computed from returns, I use volatility shocks based on consumption growth. The approach is the same as in equation (3), with the exception that the left-hand-side variable is now consumption growth, F_t is the first principal component of private consumption growth, and the data frequency is quarterly. Factor regressions are estimated in a 30-quarter rolling window.

Consistent with my previous results, high-interest rate currencies perform really well when common idiosyncratic volatility shocks in consumption growth are low. As can be seen in the top panels of Figure 4, there is no clear relationship between *Total* volatility

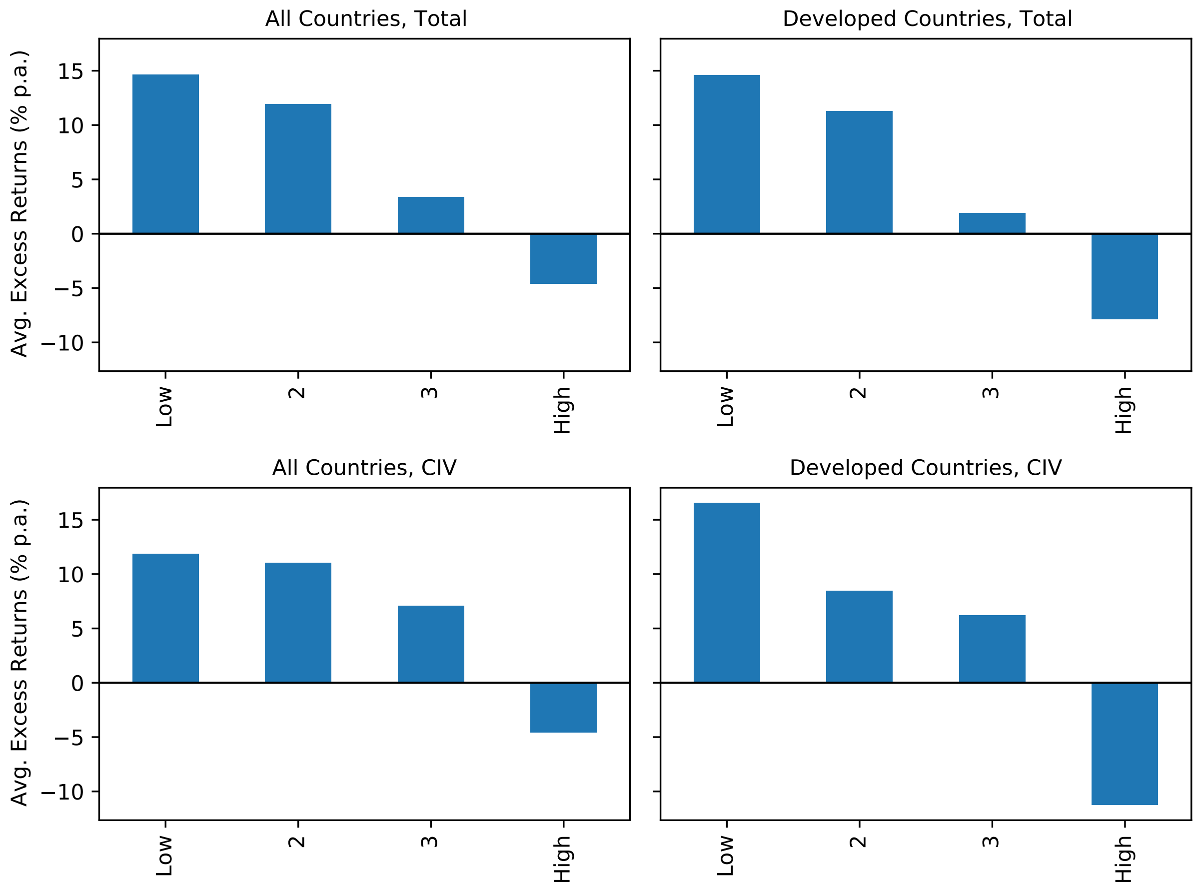

Figure 3: Carry trade excess returns and return volatility innovations.

The figure shows annualized mean excess returns for carry trade portfolios conditional on currency return volatility innovations being within the lowest to highest quartile of its sample distribution (four categories from "Low" to "High" shown on the x-axis of each panel). The bars show average excess returns for being long in portfolio 6 (largest forward discounts) and short in portfolio 1 (lowest forward discounts). The top panels report average carry trade returns depending on the quartile of the distribution of Total volatility shocks, whereas the bottom panels show average carry trade excess returns depending on the quartile of the distribution of CIV shocks. The left panels show results for all countries, while the right panels shows results for developed countries. The sample period is November 1983 to June 2020.

shocks and carry trade returns. I only observe the negative relationship between carry trade returns and volatility shocks when I divide the sample based on the realization of CIV shocks, but not when I use *Total* volatility shocks. This finding suggests that carry trade returns are related to country-level income risk that is orthogonal to global income risk.

While this analysis is intentionally simple, it intuitively demonstrates a clear relationship between CIV shocks and returns to carry trade portfolios, and this relation is even

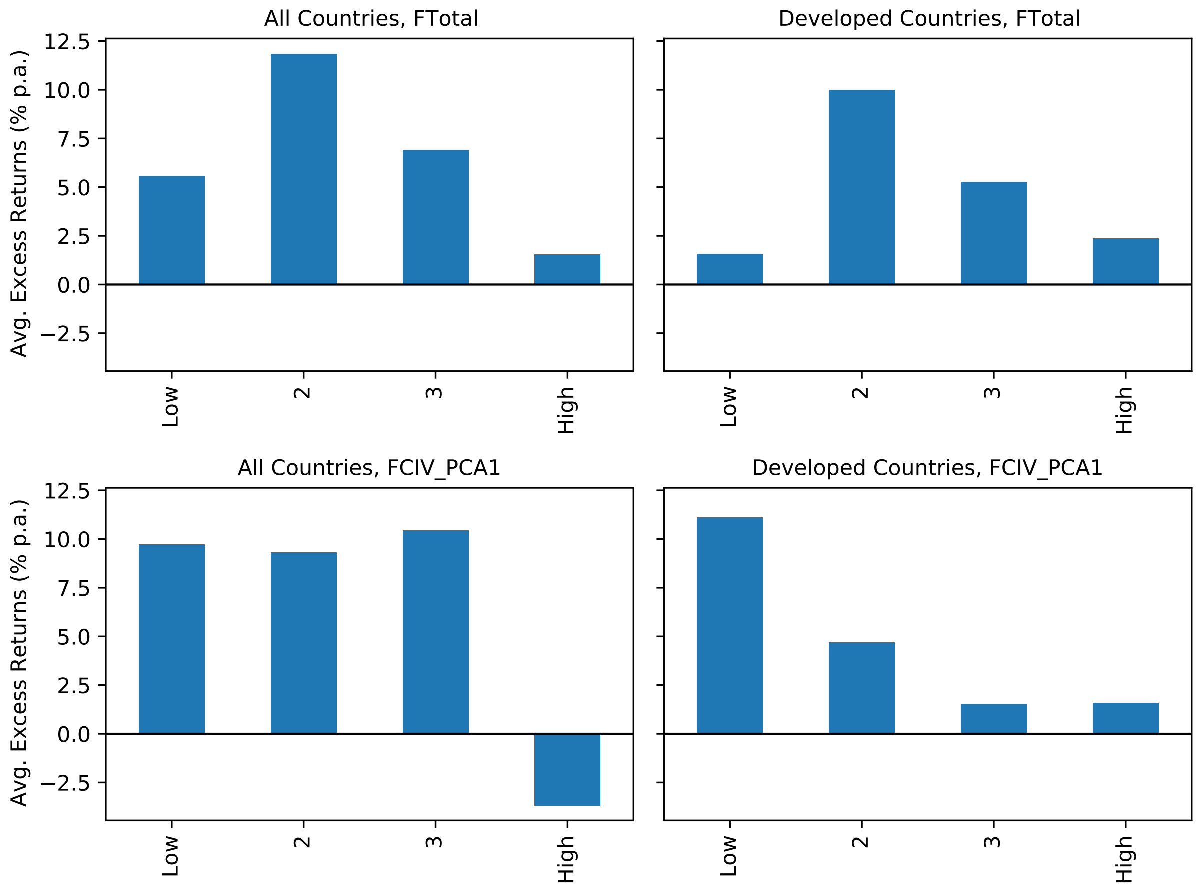

Figure 4: Carry trade excess returns and consumption volatility innovations.

The figure shows annualized mean excess returns for carry trade portfolios conditional on consumption growth volatility innovations being within the lowest to highest quartile of its sample distribution (four categories from "Low" to "High" shown on the x-axis of each panel). The bars show average excess returns for being long in portfolio 6 (largest forward discounts) and short in portfolio 1 (lowest forward discounts). The top panels report average carry trade returns depending on the quartile of the distribution of *Total* volatility shocks (FTotal), whereas the bottom panels show average carry trade excess returns depending on the quartile of the distribution of CIV shocks (FCIV_PCA1). The left panels show results for all countries, while the right panels shows results for developed countries. The sample period is November 1983 to June 2020.

more stronger in developed countries. The carry trade performs poorly when CIV shocks are high. Consequently, low-interest rate currencies provide a hedge in times of high common idiosyncratic volatility. The following sections test this finding more rigorously.

3.2.2 CIV-beta-sorted portfolios

I now examine the explanatory power of volatility risk for currencies from another perspective. If idiosyncratic volatility risk is a priced factor, then it is reasonable to assume that currencies sorted according to their exposure to idiosyncratic volatility innovations yield a cross-section of portfolios with a significant spread in mean returns. Currencies that hedge against volatility risk should trade at a premium, whereas currencies that yield low returns when volatility is high should yield a higher return in equilibrium.

My asset pricing analysis is conducted using monthly returns, so the results of this section use a monthly version of the CIV factor. I construct CIV shocks as the residuals of an AR(1) model. For each month from November 1983 to June 2020, I regress monthly individual currency excess returns on CIV shocks using a trailing 36-month window. I refer to a currency's exposure to the CIV shock as its CIV-beta. Next, I sort currencies into quintiles based on their CIV-beta each month, form an equally weighted portfolio of currencies in each quintile, and hold that portfolio for one month (as in Lustig et al., 2011). Panel B of Table 4 reports the annualized average excess return of each portfolio, as well as the return on a strategy that goes long the lowest CIV-beta quintile and short the highest CIV-beta quintile. Panel A of Table 4 reports the results for portfolios sorted on *Total* volatility shocks.

Panel B of Table 4 shows that investing in currencies with positive exposure to CIV shocks (i.e., hedges against CIV risk) leads to a significantly lower return than investing in negative CIV-beta currencies. For developed countries, the spread between portfolio 1 (low volatility beta, that is, high idiosyncratic volatility risk) and portfolio 5 (high volatility beta, that is, low idiosyncratic volatility risk) exceeds 4% per annum. For the same set of countries, portfolio sortings based on the exposure to *Total* volatility shocks do not produce a statistically significant spread in returns. There is a monotonically decreasing pattern of average returns of the CIV-beta quintile portfolios. Table 4 also reports pre-formation CIV-beta loadings. CIV-beta loadings increase from negative to positive from portfolio 1 to 5, consistent with a risk story.

Table 5 reports the results for portfolios sorted on CIV shocks orthogonalized against *Total* volatility shocks. For the developed countries sample, results are only slightly weaker if CIV changes are orthogonalized against changes in aggregate FX volatility (right panel). On the other hand, for the full sample (left panel), the spread in returns between portfolio 1 and portfolio 5 is larger when I form portfolios based on the exposure to orthogonalized CIV shocks.

Table 4: Portfolios sorted on betas with common volatility factors

The table reports statistics for portfolios sorted on volatility betas, that is, currencies are sorted according to their beta in a rolling time-series regression of individual currencies' excess returns on volatility innovations using a 36-month rolling window. Portfolio 1 contains currencies with the lowest betas, whereas portfolio 5 contains currencies with the highest betas. I report the average pre-formation betas (pre-f. betas) and forward discounts for each portfolio. Pre-formation averages are calculated at the end of the month just prior to portfolio formation. Robust Newey and West (1987) standard errors with optimal lag selection according to Andrews (1991) are reported in parentheses. Panel A reports summary statistics for portfolios based on the exposure to Total volatility shocks, while Panel B presents summary statistics for portfolios formed based on the exposure to CIV shocks. The sample period is November 1983 to June 2020. *, **, and *** denotes significance at 10%, 5%, and 1%, respectively.

Panel A: Portfolios sorted by Total volatility shocks												
	All Countries						Developed Countries					
Portfolio	1	2	3	4	5	(1-5)	1	2	3	4	5	(1-5)
Mean	2.28	0.84	0.74	-0.02	-0.57	2.85**	1.35	1.04	0.25	-0.13	-0.61	1.96
	(1.56)	(1.23)	(1.02)	(1.03)	(1.60)	(1.20)	(1.92)	(1.52)	(1.83)	(1.53)	(1.95)	(1.29)
Std. Dev.	9.79	7.99	7.18	5.73	6.31	9.19	10.33	9.36	10.36	9.10	8.55	9.48
Sharpe Ratio	0.23	0.11	0.10	-0.00	-0.09	0.31	0.13	0.11	0.02	-0.01	-0.07	0.21
pre-f. beta	-9.33	-4.92	-2.36	-0.34	3.38		-6.16	-2.81	-1.10	0.09	3.88	
pre-f. f-s	0.25	0.15	0.12	0.03	-0.03		0.17	0.09	0.09	-0.00	-0.08	
alpha MM	1.21	-0.05	-0.07	-0.60	-1.12	2.34	0.96	0.69	-0.15	-0.49	-0.90	1.86
	(0.85)	(0.57)	(0.60)	(0.68)	(1.01)	(1.56)	(0.80)	(0.96)	(0.87)	(0.82)	(1.02)	(1.40)
alpha LRV	-0.46	-0.64	-0.26	0.57	0.85	-1.31	-0.68	-0.08	-0.21	0.08	0.91	-1.59
	(0.54)	(0.52)	(0.68)	(0.49)	(0.79)	(0.92)	(0.73)	(0.82)	(0.94)	(0.88)	(0.98)	(1.20)
alpha PCA	2.29	0.84	0.74	-0.02	-0.57	2.86**	1.35	1.04	0.25	-0.13	-0.61	1.96
	(1.56)	(1.23)	(1.01)	(1.02)	(1.59)	(1.18)	(1.92)	(1.52)	(1.82)	(1.53)	(1.94)	(1.28)
Panel B: Portfolios sorted by CIV shocks												
	All Countries						Developed Countries					
Portfolio	1	2	3	4	5	(1-5)	1	2	3	4	5	(1-5)
Mean	2.37	1.43	0.24	0.13	-0.77	3.13**	3.11	0.34	-0.43	-0.40	-1.07	4.18**
	(1.66)	(1.07)	(1.12)	(1.16)	(1.46)	(1.59)	(1.92)	(1.62)	(1.75)	(1.62)	(1.90)	(1.70)
Std. Dev.	9.32	7.65	7.06	6.45	6.57	9.08	9.83	9.44	10.55	9.30	8.73	9.71
Sharpe Ratio	0.25	0.19	0.03	0.02	-0.12	0.34	0.32	0.04	-0.04	-0.04	-0.12	0.43
pre-f. beta	-18.05	-9.36	-4.45	-0.23	6.39		-12.88	-4.74	-0.03	1.40	9.24	
pre-f. f-s	0.28	0.14	0.11	0.01	-0.04		0.20	0.10	0.09	-0.01	-0.11	
alpha MM	1.37	0.60	-0.55	-0.57	-1.36	2.73*	2.75***	-0.02	-0.83	-0.77	-1.37	4.12**
	(0.86)	(0.62)	(0.43)	(0.53)	(1.00)	(1.65)	(1.01)	(0.79)	(0.54)	(1.00)	(1.04)	(1.73)
alpha LRV	-0.24	0.24	-0.34	0.24	0.32	-0.55	1.00	-0.61	-1.03**	-0.24	0.53	0.47
	(0.62)	(0.61)	(0.63)	(0.43)	(0.59)	(0.91)	(0.80)	(0.62)	(0.48)	(1.07)	(0.81)	(1.05)
alpha PCA	2.37	1.43	0.24	0.13	-0.77	3.14**	3.10	0.34	-0.44	-0.40	-1.07	4.17**
	(1.65)	(1.07)	(1.12)	(1.15)	(1.44)	(1.57)	(1.92)	(1.64)	(1.74)	(1.60)	(1.88)	(1.68)
N	391	391	391	391	391	391	391	391	391	391	391	391

Table 5: Portfolios sorted on betas with CIV orthogonalized

The table reports statistics for portfolios sorted on volatility betas, that is, currencies are sorted according to their beta in a rolling time-series regression of individual currencies' excess returns on CIV shocks orthogonalized against *Total* volatility shocks using a 36-month window. Portfolio 1 contains currencies with the lowest betas, whereas portfolio 5 contains currencies with the highest betas. I report the average pre-formation betas (pre-f. betas) and forward discounts for each portfolio. Robust Newey and West (1987) standard errors with optimal lag selection according to Andrews (1991) are reported in parentheses. Pre-formation averages are calculated at the end of the month just prior to portfolio formation. The sample period is November 1983 to June 2020. *, **, and *** denotes significance at 10%, 5%, and 1%, respectively.

Panel C: Portfolios sorted by CIV-orthogonalized shocks

	All Countries						Developed Countries					
Portfolio	1	2	3	4	5	(1-5)	1	2	3	4	5	(1-5)
Mean	2.78	0.59	0.85	-0.38	-0.51	3.30*	3.07	0.09	-1.35	0.01	-0.76	3.83**
	(1.72)	(0.96)	(1.24)	(1.33)	(1.33)	(1.82)	(1.99)	(1.81)	(1.70)	(1.70)	(1.67)	(1.82)
Std. Dev.	8.62	7.42	7.13	6.98	7.19	8.51	9.48	9.53	10.19	9.41	9.11	9.71
Sharpe Ratio	0.32	0.08	0.12	-0.05	-0.07	0.39	0.32	0.01	-0.13	0.00	-0.08	0.39
pre-f. beta	-19.87	-9.40	-3.51	1.12	8.81		-12.48	-4.26	0.81	2.30	10.32	
pre-f. f-s	0.25	0.13	0.10	0.05	-0.01		0.17	0.07	0.08	0.01	-0.06	
alpha MM	1.90**	-0.18	0.06	-1.11*	-1.23	3.13*	2.74**	-0.29	-1.73*	-0.36	-1.08	3.82**
	(0.97)	(0.51)	(0.41)	(0.64)	(0.87)	(1.75)	(1.07)	(0.86)	(0.95)	(0.67)	(1.04)	(1.80)
alpha LRV	0.78	0.14	0.13	-0.34	-0.54	1.32	1.38*	-0.56	-1.51	-0.33	0.33	1.05
	(0.94)	(0.53)	(0.44)	(0.40)	(0.81)	(1.68)	(0.81)	(0.84)	(1.03)	(0.80)	(0.88)	(1.24)
alpha PCA	2.79	0.59	0.85	-0.38	-0.51	3.30*	3.07	0.08	-1.35	0.01	-0.77	3.83**
	(1.71)	(0.96)	(1.24)	(1.33)	(1.31)	(1.79)	(2.00)	(1.81)	(1.70)	(1.70)	(1.62)	(1.78)
N	391	391	391	391	391	391	391	391	391	391	391	391

Overall, this section shows that cross-country idiosyncratic volatility risk, as measured by the covariance of a portfolio's return with CIV shocks, matters for understanding the cross-section of currency excess returns. This empirical relation is in line with theoretical arguments that assets offering high payoffs in times of (unexpectedly) high aggregate volatility, and hence, that serve as a volatility hedge, trade at a premium in equilibrium and vice versa.

3.2.3 Pricing carry trade returns

Next, I formally test whether CIV risk affects carry trade returns. I assume that variation in the cross-section of carry trade returns is driven by different exposures to a small number of risk factors (Ross, 1976).[11] I empirically document that CIV risk is priced in a broad cross-section of currency portfolios.

[11] The same approach is used by Lustig et al. (2011), Menkhoff et al. (2012), Della Corte et al. (2016), and Mancini et al. (2013), among others. For additional details, see Cochrane (2009).

Methodology. Let the excess returns on portfolio j in period t be denoted as $RX_{j,t}$. In the absence of arbitrage opportunities, risk-adjusted excess returns have a price of zero and satisfy the following Euler equation:

$$\mathrm{E}_t \left[M_{t+1} RX_{j,t+1} \right] = 0, \tag{6}$$

with a stochastic discount factor (SDF) linear in the pricing factors f_{t+1}, given by

$$M_{t+1} = 1 - b'(f_{t+1} - \mu), \tag{7}$$

where b is the vector of factor loadings, and μ denotes the factor means. This specification implies a beta pricing model in which the expected excess return on portfolio j is equal to the factor risk price λ times the risk quantities β_j. The beta pricing model is defined as

$$\mathrm{E} \left[RX_j \right] = \lambda' \beta_j, \tag{8}$$

where the market price of risk $\lambda = \Sigma_f b$ can be obtained via the factor loadings b. $\Sigma_f = \mathrm{E}\left[(f_t - \mu)(f_t - \mu)' \right]$ is the variance-covariance matrix of the risk factors, and β_j are the regression coefficients of each portfolio's excess return $RX_{j,t+1}$ on the risk factors f_{t+1}.

The factor loadings b in equation (7) are estimated via the generalized method of moments (GMM) of Hansen (1982). To implement GMM, I use the pricing errors as a set of moments and a prespecified weighting matrix. Since the objective is to test whether the model can explain the cross-section of expected currency excess returns, I only rely on unconditional moments and do not employ instruments other than a constant and a vector of ones. Factor means and the individual elements of the covariance matrix of risk factors Σ_f are estimated simultaneously with the SDF parameters by adding the corresponding moment conditions to the asset pricing moment conditions implied by equation (6). The first-stage GMM estimation used here employs an identity weighting matrix, which tells us how much attention to pay to each moment condition. With an identity matrix, GMM attempts to price all currency portfolios equally well. This one-step approach ensures that we adequately incorporate estimation uncertainty associated with the fact that factor means and the factor covariance matrix have to be estimated (see, for example, Burnside et al., 2011).

Risk factors and pricing kernel. Economic theory provides several reasons why the price of risk of shocks in market volatility should be negative. For example, Campbell (1992, 1996) shows that investors want to hedge against changes in market volatility because positive volatility innovations (i.e., unexpectedly high volatility) represent a deterioration in future investment opportunities. Moreover, unexpectedly high volatility typically coincides with low returns so that assets that covary positively with market volatility innovations provide a good hedge and are therefore expected to earn a lower expected return.

The most recent literature on cross-sectional asset pricing in currency markets has considered a two-factor SDF. The first risk factor is the expected market excess return, approximated by the average excess return on a portfolio strategy long in all foreign currencies with equal weights and short in the domestic currency – the *Dollar* factor of Lustig et al. (2011). For the second risk factor, the literature has employed several return-based factors, such as the slope factor (*Carry*) of Lustig et al. (2011), the global FX volatility factor of Menkhoff et al. (2012), the liquidity risk factor of Mancini et al. (2013), or the global imbalance factor of Della Corte et al. (2016). Following this literature, I consider a two-factor SDF with *Dollar* and CIV shocks as risk factors to assess whether currencies more exposed to CIV risk offer a higher risk premium. In fact, given that volatility is known to exhibit substantial persistence, it is reasonable to consider volatility innovations as a pricing factor.

Table 6 shows cross-sectional pricing results using carry trade portfolios as test assets. Panel A reports the estimate of λ, as well as cross-sectional R^2s and the (HJ) distance measure – the maximum pricing error per one unit of the payoff norm (Hansen and Jagannathan, 1997). Statistical significance at the 10%, 5%, and 1% level, respectively, is based on Newey and West (1987) standard errors with optimal lag length selection according to Andrews (1991). The simulated p-values for the null hypothesis that the HJ distance is equal to zero are reported in brackets.

Starting from Panel A of Table 6, I focus on the sign and statistical significance of λ_{CIV}, the market price of risk attached to the CIV risk factor. As expected, given the portfolio analysis in the previous section, I find a significantly negative estimate for λ_{CIV}. In fact, λ_{CIV} is estimated to be negative for the full sample (left part of the table) and for the subsample of developed countries (right part of the table). The estimated factor price is -0.05% for the full sample and -0.02% for the developed country sample.

A negative estimate of the factor price of CIV risk implies lower risk premia for currency portfolios whose returns positively comove with CIV shocks (i.e., hedges against CIV risk), and higher risk premia for currency portfolios exhibiting a negative covariance with CIV shocks. The price of risk associated with CIV shocks is highly statistically significant for both the full sample and for the subsample of developed countries. Equation (8)

Table 6: Cross-sectional asset pricing results

The table reports cross-sectional pricing results for the linear factor model based on the dollar risk factor (Dollar) and CIV innovations. The test assets are excess returns to carry trade portfolios based on currencies from all countries (left panel) or developed countries (right panel). Panel A shows factor risk prices λ obtained by GMM (in percentage points). Standard errors of coefficient estimates obtained by the Newey and West (1987) procedure with optimal lag selection according to Andrews (1991) are reported in parentheses, as well as p-values for the Hansen-Jagannathan distance (HJ) and the χ^2 test statistics for the null that all pricing errors are jointly equal to zero. Panel B reports results for time-series regressions of excess returns on a constant (*alpha*), the dollar risk factor (*Dollar*), and common idiosyncratic volatility innovations (CIV shocks). The sample period is November 1983 to June 2020. *, **, and *** denotes significance at 10%, 5%, and 1%, respectively.

Panel A: Factor Prices

	All Countries	Developed Countries
Dollar	0.18***	0.12
	(0.1)	(0.14)
CIV shocks	-0.05***	-0.02**
	(0.01)	(0.01)
N	439	439
R^2	0.76	0.74
HJ	2.58	3.18
P-value	[0.63]	[0.53]

Panel B: Factor Betas

	All Countries						Developed Countries					
Portfolio	1	2	3	4	5	6	1	2	3	4	5	6
alpha	0.00	-0.00	0.00	0.00	-0.00	0.00	0.00	-0.00	-0.00	-0.00	-0.00	0.00
	(0.00)	(0.00)	(0.06)	(0.70)	(0.04)	(0.64)	(0.00)	(0.00)	(0.05)	(1.22)	(0.04)	(1.08)
Dollar	0.92***	0.82***	0.96	1.08***	1.03	1.17***	0.93***	1.00***	0.98	1.03***	0.99	1.08***
	(0.00)	(0.00)	(0.75)	(0.04)	(0.89)	(0.05)	(0.00)	(0.00)	(1.04)	(0.03)	(1.14)	(0.05)
CIV shocks	6.43***	1.96***	1.69***	-1.15*	-1.89***	-4.95***	7.86***	3.05***	1.05***	-1.91**	-3.70***	-7.31***
	(0.00)	(0.00)	(0.05)	(0.60)	(0.05)	(1.02)	(0.00)	(0.00)	(0.06)	(0.74)	(0.05)	(0.88)

provides a good fit to the data with adjusted-R^2s around 75%, implying that the vast majority of monthly variation in carry trade returns can be explained by exposure to two risk factors. Finally, further support in favor of the pricing power of CIV shocks comes from the fact that the HJ distance is insignificant (I cannot reject the null that the HJ distance is equal to zero).

Panel B of Table 6 shows which portfolios of currencies provide insurance against idiosyncratic volatility risk and which do not. I report the time series beta estimates for the six carry trade portfolios based on the sample of all countries and the subsample of developed countries. For currencies with low interest rate differentials (portfolio 1), the estimates of β_{CIV} are large and positive, whereas countries with high interest rate differentials co-move negatively with CIV shocks. There is a monotonic decline in betas when moving from portfolio 1 to portfolio 6 and it is precisely this monotone relationship that produces the large spread in mean excess returns shown in Table 4. Moreover, no currency portfolio exhibits a significant alpha.

Table 7: Cross-sectional asset pricing results with additional controls

The table reports cross-sectional pricing results for the linear factor model based on the dollar risk factor (Dollar), CIV shocks, and global FX volatility shocks (Total shocks). The test assets are excess returns to carry trade portfolios based on currencies from all countries (left panel) or developed countries (right panel). Panel A shows factor risk prices λ obtained by GMM (in percentage points). Standard errors of coefficient estimates obtained by the Newey and West (1987) procedure with optimal lag selection according to Andrews (1991) are reported in parentheses, as well as p-values for the Hansen-Jagannathan distance (HJ) and the χ^2 test statistics for the null that all pricing errors are jointly equal to zero. Panel B reports results for time-series regressions of excess returns on a constant (*alpha*), the dollar risk factor (*Dollar*), and common idiosyncratic volatility innovations (CIV shocks). The sample period is November 1983 to June 2020. *, **, and *** denotes significance at 10%, 5%, and 1%, respectively.

Panel A: All Countries								
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Dollar	0.12						0.18*	0.16
	(0.10)						(0.10)	(0.11)
Total shocks		-0.10***			-0.03	-0.03		-0.02
		(0.02)			(0.06)	(0.05)		(0.06)
CIV shocks			-0.04***		-0.04***		-0.05***	-0.05***
			(0.01)		(0.01)		(0.01)	(0.01)
CIV shocks (orthogonalized)				-0.04***		-0.03		
				(0.01)		(0.02)		
HJ	19.31	3.63	2.80	2.17	1.52	1.42	2.58	1.83
P-value	[0.00]	[0.60]	[0.73]	[0.83]	[0.68]	[0.70]	[0.63]	[0.61]
R^2	0.74	0.03	0.05	0.02	0.06	0.06	0.76	0.76

Panel B: Developed Countries								
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Dollar	0.11						0.12	0.10
	(0.14)						(0.14)	(0.14)
Total shocks		-0.06**			0.00	0.00		-0.00
		(0.03)			(0.03)	(0.03)		(0.03)
CIV shocks			-0.03**		-0.03**		-0.02***	-0.03**
			(0.01)		(0.01)		(0.01)	(0.01)
CIV shocks (orthogonalized)				-0.03**		-0.03*		
				(0.01)		(0.01)		
HJ	9.97	4.43	2.56	1.68	1.71	1.67	3.18	1.76
P-value	[0.08]	[0.49]	[0.77]	[0.89]	[0.64]	[0.64]	[0.53]	[0.63]
R^2	0.72	0.03	0.04	0.01	0.04	0.04	0.74	0.74
No. Factors:	1	1	1	1	2	2	2	3
No. Test Portfolios:	6	6	6	6	6	6	6	6
No. Observations:	440	439	439	439	439	439	439	439

The estimates for the CIV price of risk are similar if we consider alternative SDF specifications. Table 7 reports CIV prices of risk for SDFs that also include *Total* volatility shocks, and CIV shocks orthogonalized against monthly changes in *Total* volatility shocks.

The results in Table 7 are stark: CIV risk is priced even when controlling for the aggregate FX volatility factor of Menkhoff et al. (2012) and the dollar factor of Lustig et al. (2011). This result also corroborates the graphical illustration in Figures 3 and 4 in the previous section: Investors demand a high return on high-interest rate currencies in the carry trade portfolio (investment currencies) since they perform poorly in periods of unexpectedly high CIV shocks, whereas investors are willing to accept low returns on low-interest rate currencies (funding currencies) since these currencies provide a hedge against periods of increased cross-country dispersion of consumption growth.

3.3 CIV as a proxy for income risk faced by households

Given the empirical evidence regarding the negative relation between CIV shocks computed from GDP growth and carry trade returns presented in Section 3.2.1, in this section I investigate whether the CIV factor can be thought of as a proxy for country-level income risk faced by households.[12]

A large literature documents that shocks to individual labor income growth translate into shocks to individual consumption growth because of incomplete risk sharing. This literature usually relates movements in consumption to predicted and unpredictable income changes as well as persistent and non-persistent shocks to economic resources. For example, Blundell et al. (2008) find partial insurance against permanent income shocks but almost full insurance against transitory income shocks. Heathcote et al. (2014) show that permanent shocks to labor income end up in consumption, while transitory shocks are partially insurable.[13] Most closely related to this paper, Lucas (1994), Heaton and Lucas (1996), Krusell and Smith (1997), Marcet and Singleton (1998), Chien and Lustig (2010), Storesletten et al. (2007), Brav et al. (2002), Cogley (2002), Sarkissian (2003), and Herskovic et al. (2016) follow Mankiw (1986) and Constantinides and Duffie (1996) and study models in which assets with higher expected returns do badly at times of higher cross-sectional labor income risk.

My empirical findings suggest that common idiosyncratic return volatility is a plausible proxy for cross-country idiosyncratic risk faced by households. In the previous sec-

[12]For a literature review on how individual household income and individual firm performance are potentially linked, see Herskovic et al. (2016).

[13]Meghir and Pistaferri (2011) and Jappelli and Pistaferri (2010) review the relevant theoretical and empirical literature.

tions, I showed that there is a strong factor structure in idiosyncratic volatilities for currency excess returns and that shocks to the CIV factor are priced. The first interesting empirical finding is that carry trade returns are negatively related to the realization of CIV shocks. A more striking result is that carry trade returns are also negatively related to CIV shocks in consumption growth. However, I do not observe this negative relation between carry trade returns and *Total* volatility shocks in consumption growth. This suggests that carry trade returns are related to the idiosyncratic component of volatilities, not the systematic one. Figure 5 presents two new pieces of evidence connecting CIV risk to country-level income risk that is orthogonal to global income risk faced by households.

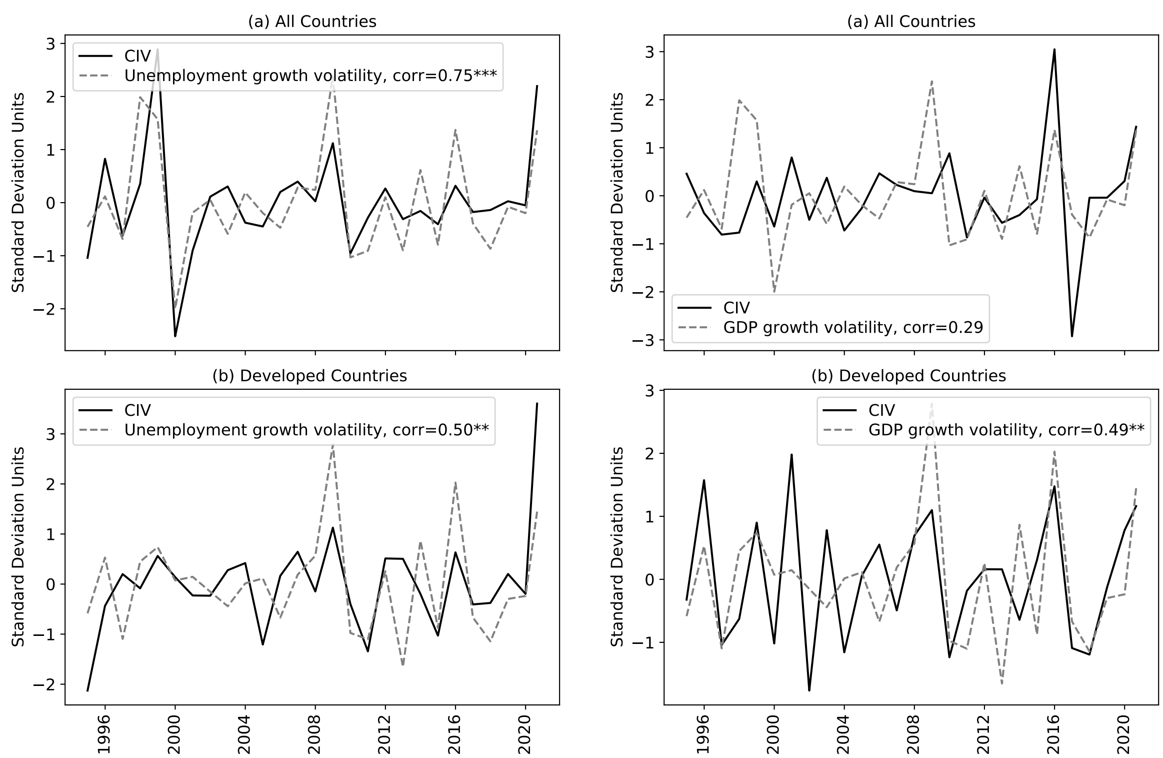

Figure 5: Common idiosyncratic volatility and cross-country income risk.

The figure compares yearly changes in CIV with yearly changes in the cross-sectional standard deviation of unemployment growth (left panels) and GDP growth (right panels). The left panels show results for all countries, while the right panels show results for developed countries. CIV is the equal-weighted average of currency idiosyncratic volatilities based on the MM model each year. Each series is standardized to have equal mean and variance for ease of comparison. The sample period is November 1983 to June 2020.

My first result is that changes in CIV are significantly associated with cross-country unemployment risk. I calculate country-level unemployment growth rates from 1994 to 2020. Then, to proxy for unemployment risk, I compute the cross-sectional standard deviation of unemployment growth rates each year, after removing a common component from unemployment growth.[14] Results are shown in Panels (a) and (b) of Figure 5. Changes in CIV share a correlation of 75% with changes in the cross-sectional standard deviation of unemployment growth for the full sample, and of 50% for the developed country sample.

My second result relates CIV changes to income risk. Panels (c) and (d) of Figure 5 plot yearly changes in CIV alongside yearly changes in the cross-sectional standard deviation of GDP growth for the full sample and for the developed countries sample, respectively. The correlation between changes in CIV with changes in the standard deviation of the GDP growth distribution is 29% for the full sample, and 49% for the subsample of developed countries. This evidence offers further support of a link between the cross-country dispersion in income and currency excess returns.

4 A Heterogeneous-agent model of exchange rates

Motivated by the facts presented in the previous sections, I develop an incomplete markets asset pricing model in which a common idiosyncratic variance factor, denoted $\sigma^2_{w,t}$, is the key state variable driving both residual currency volatility and dispersion in household income growth across countries. Currencies with more negative exposure to this innovation (a more negative CIV-beta) earn a higher risk premium. While this setup abstracts away from trade in the consumption goods market, it constitutes a useful benchmark in the international finance literature, since it allows us to introduce aggregate shocks that are correlated with the variance of idiosyncratic risk as in Constantinides and Duffie (1996) and Storesletten et al. (2004). I show a calibration that quantitatively accounts for the cross-sectional differences in average returns across CIV-beta sorted portfolios. In the interest of space, the model details are relegated to the Appendix.

[14] Results are quantitatively similar and qualitatively the same if using an alternative definition such as variance of factor model residuals. I compute a monthly, quarterly and annual version of the CIV factor to conform with various data sources that are available at one these frequencies. The common component is defined as the cross-sectional average of unemployment (GDP) growth. This average growth rate is approximately equal to the first principal component of a given unemployment (GDP) growth panel but avoids principal components complications arising from unbalanced panels.

4.1 Model setup

Consider a world with N different countries and currencies. In order to take advantage of the law of large numbers, I assume that N is large. In every country j, there is a continuum of agents with preferences defined over a single non-durable consumption good. There is an arbitrary number of traded securities in positive or zero net supply, and all agents are endowed with an equal number of all securities at time zero. The market, defined as the total sum of traded securities in positive net supply, pays net dividends $D_{j,t}$ at time t and has normalized supply of one unit. However, there are no markets for trading the wealth portfolio of agent i – the portfolio with dividend flow equal to agent i's consumption flow. This implies that financial markets are incomplete, therefore preventing agents from insuring their idiosyncratic shocks in consumption growth. Moreover, investors are not able to perfectly hedge against country-specific shocks in consumption growth. As a consequence, consumption heterogeneity exists not only within but also across countries.

Preferences. In every country j, there is a continuum of agents with recursive Epstein and Zin (1989) and Weil (1989) preferences. Let $U_{i,j,t}$ denote the utility for agent i in country j derived from consuming $C_{i,j,t}$. The value function of each agent takes the following recursive form:

$$U_{i,j,t} = \left[(1-\beta)(C_{i,j,t})^{\left(\frac{1-\gamma}{\theta}\right)} + \beta \left(\mathrm{E}_t \left[(U_{i,j,t+1})^{1-\gamma} \right] \right)^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}},$$

where $\theta \equiv (1-\gamma)/(1-1/\psi)$, $0 < \beta < 1$ is the subjective discount factor, $\gamma > 0$ is the risk-aversion parameter, and $\psi > 0$ is the intertemporal elasticity of substitution (IES). When $\psi > 1$ and $\gamma > 1$, $\theta < 0$ and agents prefer early resolution of uncertainty.[15] As shown in Epstein and Zin (1989), for these recursive preferences, the logarithm of the marginal rate of substitution of consumer i in country j is given by

$$m_{i,j,t+1} = \theta \log \beta - \frac{\theta}{\psi} \Delta c_{i,j,t+1} + (\theta - 1) r_{i,j,t+1}, \tag{9}$$

where $r_{i,j,t+1}$ is the log return on agent i's total wealth portfolio.

Consumption shares. Let C_t denote the world aggregate consumption at time t. The

[15] When $\theta = 1$, that is, $\gamma = (1/\psi)$, the above recursive preferences collapse to the standard case of expected utility. Further, when $\theta = 1$ and in addition $\gamma = 1$, we get the standard case of log utility.

consumption of agent i in country j at time t is defined as

$$C_{i,j,t} = \Phi_{i,j,t}\Phi_{j,t}C_t, \tag{10}$$

where $\Phi_{i,j,t}$ is investor i's consumption share in domestic consumption and $\Phi_{j,t}$ is country j's consumption share in world consumption. Agent i in country j is endowed with income $I_{i,j,t} = \Phi_{i,j,t}C_{j,t} - D_{j,t}$ at date t, and aggregate income in country j is defined as $I_{j,t}$.

Following Constantinides and Duffie (1996) and Constantinides and Ghosh (2017), I assume that the law of large numbers holds across investors and countries, and specify $\Phi_{i,j,t}$ and $\Phi_{j,t}$ as

$$\Phi_{i,j,t} = \Phi_{i,j,t-1}\exp\left(\sigma_{j,t-1}\epsilon_{i,j,t} - \frac{1}{2}\sigma^2_{j,t-1}\right) \text{ and } \Phi_{j,t} = \Phi_{j,t-1}\exp\left(\sigma_{w,t-1}\epsilon_{j,t} - \frac{1}{2}\sigma^2_{w,t-1}\right), \tag{11}$$

where $\epsilon_{i,j,t}$ and $\epsilon_{j,t}$ are i.i.d. standard normal variables denoting, respectively, agent i's and country j's consumption shocks at time t. This specification for the household income process implies that $I_{i,j,t}$ is determined by the sum of all past idiosyncratic shocks.

The following Proposition is a direct consequence of the assumption that income shocks are permanent, combined with the assumption of symmetric and homogeneous preferences. In deriving the result that autarchy is an equilibrium, I follow Constantinides and Duffie (1996) and rely on the assumption that the market is incomplete and hence prevents households from insuring any component of their idiosyncratic income shocks. We could interpret $C_{i,j,t}$ as the post-trade consumption that investor i obtains after she has exhausted all insurance options and the temporary innovations to labor income have been smoothed out. Endogenizing the relation between household income and household consumption through a richer model of financial market trading is outside the scope of the present investigation.

Proposition 1 (Autarchy equilibrium)**.** *Agents choose not to trade in equilibrium and* $C_{i,j,t} = \Phi_{i,j,t}C_{j,t}$.

Proof. See Appendix B. □

Autarchy implies that the consumption of agent i at date t is $C_{i,j,t} = \Phi_{i,j,t}C_{j,t}$ and that individual consumption growth $C_{i,j,t+1}/C_{i,j,t} = \Phi_{i,j,t+1}C_{j,t+1}/\Phi_{i,j,t}C_{j,t}$ is independent of the investor's consumption level. This feature, combined with the property that the household's utility is homogeneous in the household's consumption level, implies that the marginal rate of substitution of household i is therefore independent of the household's consumption level – it is specific to household i only through the term $\Phi_{i,j,t+1}/\Phi_{i,j,t}$. In pricing any security, other than households' wealth portfolios, the term $\Phi_{i,j,t+1}/\Phi_{i,j,t}$ is integrated out of the pricing equation and the private valuation of any security is common across households, verifying the conjecture that autarchy is an equilibrium.

Consumption dynamics. Let C_t denote the world aggregate consumption at time t and $c_t \equiv \log(C_t)$. I use lowercase letters to denote logs and $\Delta c_t \equiv c_t - c_{t-1}$ to denote the first difference in log variables. I model the idiosyncratic volatility factor structure on agent i's consumption by assuming that the nominal log consumption growth evolves according to the following law of motion:

$$\Delta c_{i,j,t+1} = \Delta\phi_{i,j,t+1} + \Delta\phi_{j,t+1} + \Delta c_{t+1}, \tag{12}$$

$$\Delta c_{t+1} = \mu_c + \varphi_c \sigma_{w,t} u_{w,t+1}, \tag{13}$$

$$\Delta\phi_{i,j,t+1} = \sigma_{j,t}\epsilon_{i,j,t+1} - \frac{1}{2}\sigma^2_{j,t}, \tag{14}$$

$$\Delta\phi_{j,t+1} = \sigma_{w,t}\epsilon_{j,t+1} - \frac{1}{2}\sigma^2_{w,t}, \tag{15}$$

where volatilities are modeled as square-root processes,

$$\sigma^2_{j,t+1} = \sigma^2_j + \rho_j(\sigma^2_{j,t} - \sigma^2_j) + \tau_j \sigma_{j,t} u_{j,t+1}, \tag{16}$$

$$\sigma^2_{w,t+1} = \sigma^2_w + \rho_w(\sigma^2_{w,t} - \sigma^2_w) + \tau_w \sigma_{w,t} u_{w,t+1}. \tag{17}$$

In this model, there are three types of shocks: (i) global shocks (denoted as $u_{w,t+1}$); (ii) country-specific shocks (denoted as $\epsilon_{j,t+1}$ and $u_{j,t+1}$, uncorrelated across countries); and (iii) individual shocks (denoted as $\epsilon_{i,j,t+1}$, uncorrelated across individuals and countries). All shocks are i.i.d. Gaussian, with zero mean and unit variance. Here, $\sigma^2_{w,t}$ is the cross-sectional dispersion of country-specific shocks, and $u_{w,t+1}$ is the innovation in this cross-country dispersion.[16] The global shock $u_{w,t+1}$ also drives global consumption

[16] An important feature of volatility process (16) is the square-root term in the innovation, whose conditional variance $\tau^2_j \sigma^2_{j,t}$ falls to zero as $\sigma^2_{j,t}$ approaches zero. In discrete time, $\sigma^2_{j,t}$ can turn negative with a large enough negative realization of $u_{j,t+1}$. This happens with positive probability, but the probability approaches zero as the time interval goes to zero (Sun, 1992). The Feller condition, $\frac{2(1-\rho^2_j)}{\tau^2_j} \geqslant 1$, controls the shape of the unconditional distribution of $\sigma^2_{j,t}$, and helps ensure that the $\sigma^2_{j,t}$ process remains positive. The

growth. Increases in cross-country dispersion hurt global growth more when dispersion levels are high. When $\varphi_c < 0$, global consumption growth is negatively correlated with shocks to the cross-country dispersion of consumption growth. Importantly, both the within-country dispersion $\sigma^2_{j,t}$ and the across-country dispersion $\sigma^2_{w,t}$ are countercyclical, increasing during contractions and decreasing during expansions.[17]

Economic interpretation. Let $\mathrm{E}_i[\cdot]$ and $\mathrm{Var}_i[\cdot]$ denote the cross-sectional expectation and variance operators. The cross-sectional variances of the consumption share processes are:

$$\begin{aligned}\mathrm{Var}_j\left(\Delta\phi_{j,t+1}\right) &= \mathrm{Var}_j\left(\Delta c_{j,t+1} - \Delta c_{t+1}\right) = \sigma^2_{w,t},\\ \mathrm{Var}_i\left(\Delta\phi_{i,j,t+1}\right) &= \mathrm{Var}_i\left(\Delta c_{i,j,t+1} - \Delta c_{j,t+1}\right) = \sigma^2_{j,t},\ \forall\, j = 1,\ldots,N.\end{aligned}$$

Here, $\sigma^2_{w,t}$ measures the cross-sectional variance of consumption growth *across* countries, and $\sigma^2_{j,t}$ measures the cross-sectional variance of consumption growth *within* country j. In the model, $\sigma^2_{w,t}$ is also the time series volatility of global consumption growth.

4.2 Equilibrium outcomes

With this model in hand, I first derive closed-form expressions for interest rates, exchange rates, and currency factors. I then turn to currency betas to derive the asset pricing implications of this model.

Pricing kernel. Let M_{t+1} and $M_{j,t+1}$ denote the domestic and foreign SDFs that satisfy the Euler equations for the domestic and foreign bond returns:

$$\mathrm{E}_t\left[M_{t+1}R_{t+1}\right] = 1, \tag{18}$$

$$\mathrm{E}_t\left[M_{j,t+1}R_{j,t+1}\right] = 1,\ \forall\, j = 1\ldots,N, \tag{19}$$

where $R_{j,t+1}$ represents the foreign bond return expressed in units of foreign currency, while R_{t+1} denotes the domestic bond return, expressed in units of the domestic currency ("dollars"). More generally, x_j denotes a foreign variable expressed in units of foreign currency. The subscript j =US is dropped for any variable or parameter that corresponds to the home country.

same logic applies to the square root process given by equation (17).

[17] To derive analytical expressions, I follow the approach of Bansal and Yaron (2004) and Lustig et al. (2011) and assume that the volatility process is conditionally normal. When I solve the model numerically I ensure that the volatility is positive by replacing negative realizations with a very small number.

Using the dynamics presented in the previous section, I am able to derive an analytical expression for the equilibrium stochastic discount factor given by equation (9) in terms of the fundamental state variables and shocks in the economy. In Appendix B, equation (51), I calculate the aggregate stochastic discount factor that determines the sources and the compensations for risk in country j as:

$$-m_{j,t+1} = \alpha_j + \chi_j \sigma_{w,t}^2 + \varphi_j \sigma_{j,t}^2 + \omega_j \sigma_{w,t} u_{w,t+1} + \xi_j \sigma_{j,t} u_{j,t+1} + \gamma \sigma_{w,t} \epsilon_{j,t+1}, \tag{20}$$

where

$$\alpha_j = -\theta \log \beta + \gamma \mu_c - (\theta - 1)\left[\kappa_{0j} + A_{0j}(1-\kappa_{1j}) - A_{1j}(\rho_w - \kappa_{1j})\sigma_w^2 - A_{2j}(\rho_j - \kappa_{1j})\sigma_j^2\right], \tag{21}$$

$$\chi_j = -(\theta-1)A_{1j}(\rho_w - \kappa_{1j}) - \frac{\gamma}{2}, \tag{22}$$

$$\varphi_j = -(\theta-1)A_{2j}(\rho_j - \kappa_{1j}) - \frac{\gamma}{2}(1+\gamma), \tag{23}$$

$$\omega_j = -(\theta-1)\tau_w A_{1j} + \gamma \varphi_c, \tag{24}$$

$$\xi_j = -(\theta-1)\tau_j A_{2j}, \tag{25}$$

and κ_{0j}, κ_{1j}, A_{0j}, A_{1j}, and A_{2j} are defined in Appendix B by equations (38), (39), (47), (48) and (49), respectively.

This exponentially affine pricing kernel shares some features with other models in the literature, such as those proposed by Backus et al. (2001), Lustig et al. (2011), and Verdelhan (2018). There is a global volatility factor, $\sigma_{w,t}^2$, that enters the pricing kernel of all investors in N different countries. There is also a country-specific volatility factor, $\sigma_{j,t}^2$. Countries are ex-ante heterogeneous with regard to their exposure ω_j to the global shock $u_{w,t+1}$ and with regard to their exposure ξ_j to the country-specific shock $u_{j,t+1}$. Country j's exposure to the local shock $\epsilon_{j,t+1}$ is identical across countries. The global volatility factor $\sigma_{w,t}^2$ prices both the global shock $u_{w,t+1}$ and the country-specific shock $\epsilon_{j,t+1}$. The market price of risk of the country-specific shock $\epsilon_{j,t+1}$ is $\gamma\sigma_{w,t}^2$, which is identical across countries. Finally, the market price of the second country-specific shock $u_{j,t+1}$ is $\xi_j \sigma_{j,t}^2$ and the market price of the global shock $u_{w,t+1}$ is $\omega_j \sigma_{w,t+1}$.

Exchange rates. When we consider assets denominated in foreign currency, instead of using the pricing kernel $M_{j,t+1}$ to value them, we could alternatively convert foreign returns into dollars and value them using M_{t+1}. The equivalence between these two procedures gives us a connection between exchange rate movements and pricing kernels in the two currencies. To see this clearly, note that when markets are complete, we can express the dollar return on a foreign asset as $R_{t+1} = (S_{j,t}/S_{j,t+1})R_{j,t+1}$, where $S_{j,t}$ denotes the nominal exchange rate in units of foreign currency per US dollar. When $S_{j,t}$ increases, the foreign currency depreciates and the US dollar appreciates. If the foreign asset and currencies are both traded, the return must satisfy both Euler equations (18) and (19):

$$\mathrm{E}_t\left[M_{j,t+1}R_{j,t+1}\right] = \mathrm{E}_t\left[\frac{S_{j,t}}{S_{j,t+1}}M_{t+1}R_{j,t+1}\right].$$

This equality implies that we can express the implied changes in exchange rates from the stochastic discount factors at home and abroad as:

$$\Delta s_{j,t+1} = m_{t+1} - m_{j,t+1}. \tag{26}$$

With complete markets, the choices of $m_{j,t+1}$ and m_{t+1} satisfying equation (26) are unique. Otherwise, we can choose from a set of admissible kernels so that the relationship remains satisfied (Backus et al., 2001).

If we substitute equation (20) into equation (26), we get

$$\begin{aligned}\Delta s_{j,t+1} = {} & (\alpha_j - \alpha) + (\chi_j - \chi)\sigma_{w,t}^2 + \varphi_j\sigma_{j,t}^2 - \varphi\sigma_t^2 \\ & + \underbrace{(\omega_j - \omega)\sigma_{w,t}u_{w,t+1}}_{\text{global shock}} + \underbrace{\xi_j\sigma_{j,t}u_{j,t+1} + \gamma\sigma_{w,t}\epsilon_{j,t+1}}_{\text{country j-spec.}} - \underbrace{(\xi\sigma_t u_{t+1} + \gamma\sigma_{w,t}\epsilon_{t+1})}_{\text{US-spec.}},\end{aligned} \tag{27}$$

which implies that the endogenous exchange rate changes can be decomposed into a part driven by country-specific shocks and a part that reflects exposure to global risk. If the foreign country has a higher exposure ω_j to global shock $u_{w,t+1}$ than the US, that is, if $\omega_j > \omega$, its currency appreciates against the US dollar when a negative $u_{w,t+1}$ realization occurs.

Risk-free rates. I now turn to risk-free rates. Since the shocks to the log SDF are Gaussian, the risk-free rate is simply $r_{j,t} = -\mathrm{E}_t\left[m_{j,t+1}\right] - \frac{1}{2}\mathrm{Var}_t\left[m_{j,t+1}\right]$. Using the dynamics for the pricing kernel given by equation (20), we can express the model-implied risk-free rates as

$$r_{j,t} = \alpha_j + \left[\chi_j - \frac{1}{2}\left(\omega_j^2 + \gamma^2\right)\right]\sigma_{w,t}^2 + \left(\varphi_j - \frac{1}{2}\xi_j^2\right)\sigma_{j,t}^2. \tag{28}$$

The risk-free rates depend on both country-specific volatilities, $\sigma_{j,t}$, and global volatilities, $\sigma_{w,t}$. They do not depend on US-specific volatilities (except for the US risk-free rate).

For the sake of clarity, I consider the following parameter restriction:

Condition 1. *Risk-free rates decrease when country-specific volatility increases, that is,*

$$\varphi_j - \frac{1}{2}\xi_j^2 < 0. \tag{29}$$

This restriction implies that when the country-specific volatility is high, the risk-free rate tends to be low relative to the rest of the world because of large precautionary savings and increased demand for safety.

Currency excess returns. The USD excess return for investing in the currency of country j satisfies:

$$\begin{aligned} rx_{j,t+1} &= r_{j,t} - r_t - \Delta s_{j,t+1} \\ &= -\frac{1}{2}(\omega_i^2 - \omega^2)\sigma_{w,t}^2 + \frac{\xi^2}{2}\sigma_t^2 - \frac{\xi_j^2}{2}\sigma_{j,t}^2 \\ &\quad - \underbrace{(\omega_j - \omega)\sigma_{w,t}u_{w,t+1}}_{\text{global shock}} + \underbrace{(\xi\sigma_t u_{t+1} + \gamma\sigma_{w,t}\epsilon_{t+1})}_{\text{US-spec.}} - \underbrace{(\xi_j\sigma_{j,t}u_{j,t+1} + \gamma\sigma_{w,t}\epsilon_{j,t+1})}_{\text{country j-spec.}}. \end{aligned} \tag{30}$$

Thus, currency excess returns capture global shocks, US-specific shocks, and country-specific shocks. If the country-specific shocks are diversifiable, only global shocks are priced in the cross-section of currency returns. Intuitively, if a currency appreciates with respect to the US dollar when the global shock $u_{w,t+1}$ increases, this currency is essentially a hedge against this global risk factor. This makes the currency more attractive to investors and yields lower expected returns. However, as will be clear below, the presence of a common component in idiosyncratic volatilities implies that idiosyncratic risk is also priced in the cross-section of currency excess returns.

I summarize my findings in the following five propositions. All details and proofs are reported in Appendix C.

Proposition 2 (Common factor in the idiosyncratic return variance)**.** *The idiosyncratic return variance is given by*

$$\text{Var}_t(rx_{j,t+1}^{idio}) = \xi_j^2\sigma_{j,t}^2 + \gamma^2\sigma_{w,t}^2. \tag{31}$$

Thus, there exists a common factor in idiosyncratic variances.

Proof. See Appendix C. □

Proposition 2 implies that the heterogeneous-agent model presented in this section is capable of generating a factor structure in idiosyncratic volatilities, consistent with my empirical evidence presented in Section 3.1. The intuition behind Proposition 2 comes straight from the dynamics for country j's consumption share given in equation (15). The global volatility factor $\sigma_{w,t}$ is the volatility of the shock that drives the income dispersion across countries. This proposition is important because it implies that the idiosyncratic but not diversifiable risk is proxied by the cross-country variation in consumption growth, $\sigma_{w,t}$. If investors experience persistent idiosyncratic shocks in consumption, then consumption dispersion can have important implications for asset pricing, since persistent shocks significantly decrease the agents' ability to diversify. Without this common factor in idiosyncratic volatilities, I would not be able to replicate the commonality in idiosyncratic volatilities over time that is observed in the data.

As in Section 3, the common idiosyncratic variance (CIV) factor is defined as the equally weighted cross-sectional average of idiosyncratic return volatilities of different currencies at a point in time:

Definition 1. *The common idiosyncratic variance (CIV) factor is*

$$CIV_t \equiv \mathrm{E}_j\left[\mathrm{Var}_t(rx^{idio}_{j,t+1})\right] = \overline{\xi_j^2\sigma_{j,t}^2} + \gamma^2\sigma_{w,t}^2, \tag{32}$$

where $\overline{\xi_j^2\sigma_{j,t}^2} \equiv \mathrm{E}_j\left[\xi_j^2\sigma_{j,t}^2\right]$.

Thus, in terms of dynamics, CIV is proportional to the consumption growth share dispersion process $\sigma_{w,t}^2$, where γ^2 is the constant of proportionality. This result allows us to use innovations in the volatility factor constructed from currency excess returns to test my asset pricing mechanism empirically rather than measure the cross-sectional dispersion of investor consumption growth directly.

Proposition 3 (CIV-beta). *If $\omega < \omega_j$, the slope coefficient $\beta_{CIV,j}$ in the linear regression*

$$rx_{j,t+1} = a_j + \beta_{CIV,j} CIVshocks_{t+1} + \nu_{j,t+1}$$

is negative.

Proof. See Appendix C. □

Proposition 3 shows that the CIV-beta, $\beta_{CIV,j}$, is negative when the US loading on the the global shock $u_{w,t+1}$ is smaller (more negative) than country j's loading on the same global shock. If $\omega > \omega_j$, the result reverses. The economic intuition is as follows: In times when the price of the $u_{w,t+1}$ risk is relatively high (more negative) at home compared to its foreign counterpart, low CIV-beta currencies correspond to high exposures to this global shock. As a result, these currencies tend to depreciate in bad times. An investor going long in low CIV-beta currencies when the price of that risk is relatively high at home will thus bear that depreciation risk: the lower the CIV-beta, the larger the risk and the larger the risk premium.

Next, I solve for the model-implied market price of CIV risk. Proposition 4 summarizes my results.

Proposition 4 (Price of CIV risk)**.** *The expected return on currency j (including the Jensen term) has a beta representation*

$$\mathrm{E}_t\left[rx_{j,t+1}\right] + \frac{1}{2}\mathrm{Var}_t\left(rx_{j,t+1}\right) = \beta_j \lambda_t,$$

where $\beta_j = \left[\beta_{CIV,j}, \xi, \gamma\right]$ *is the vector of risk exposures,* $\lambda_t = \left[\lambda_{CIV,t}, \xi\sigma_t^2, \gamma\sigma_{w,t}^2\right]'$ *is the vector of market prices of risk, and* $\lambda_{CIV,t} \equiv \omega\tau_w\gamma^2\sigma_{w,t}^2 < 0$ *is the market price of CIV risk.*

Proof. See Appendix C. □

Proposition 4 implies that the currency risk premium has two components: a global component and a dollar component. The global component is the part that reflects compensation for exposure to the global shock $u_{w,t+1}$, whereas the dollar component reflects exposures to the US-specific shocks u_{t+1} and ϵ_{t+1}. This new global component compensates investors for movements in the cross-sectional (income and) consumption distribution, today and in the future. Currencies that have low returns when the cross-sectional volatility of consumption growth increases ($\beta_{CIV,j} < 0$) are risky and carry high expected returns because the market price of CIV risk, $\lambda_{CIV,t}$, is negative. The dollar component is identical across currencies, so all cross-sectional variation in currency risk premiums is solely due to heterogeneity in exposure to $u_{w,t+1}$, i.e., heterogeneity in ω_j.

Proposition 5 characterizes the relationship between carry trade returns and CIV shocks in the model.

Proposition 5 (Covariance of carry trade and CIV)**.** *If $\overline{\omega_j}^H > \overline{\omega_j}^L$, the covariance between CIV shocks and carry trade innovations is negative, that is,*

$$\text{Cov}_t\left(Carry_{t+1}, CIV_{t+1}\right) = -\tau_w(\overline{\omega_j}^H - \overline{\omega_j}^L)\gamma^2\sigma^2_{w,t} < 0. \tag{33}$$

Proof. See Appendix C. □

Proposition 5 is important because it corroborates the results presented in Section 3.2.1 with regard to the negative correlation between CIV shocks and carry trade returns. If the high-interest rate countries have a higher (less negative) average exposure $\overline{\omega_j}^H$ to the global shock $u_{w,t+1}$ than the low-interest rate countries, that is, if $\overline{\omega_j}^H > \overline{\omega_j}^L$, their currencies depreciate against the US dollar when a positive $u_{w,t+1}$ realization occurs. In this case, investors demand a risk premium for holding high-interest rate currencies because these currencies perform poorly during periods of unexpectedly high CIV shocks, which are bad times for investors.

Finally, Proposition 6 shows that sorting foreign countries by the level of their short-term interest rate is similar to sorting countries by their exposure to CIV shocks.

Proposition 6 (CIV-beta and interest rate differential)**.** *If $\omega_j < 0$, then*

(i) CIV-beta is decreasing in ω_j;

(ii) Interest rate differentials are increasing in ω_j.

Therefore, sorting currencies on CIV-beta is equivalent to sorting currencies on interest rate differentials.

Proof. See Appendix C. □

In the model, sorting countries by their interest rate differentials or by their CIV-betas can be interpreted as sorting countries by their exposure ω_j to the global shock

$u_{w,t+1}$; high-interest rate countries are high (less negative) ω_j countries. During bad global shocks, $u_{w,t+1} > 0$, these currencies depreciate, as can be verified in equation (27). Therefore, carry trades are risky because negative CIV-beta currencies depreciate in bad times.

4.3 Quantitative implications of the CIV model

In this section, I use my model to evaluate whether the average return spreads across CIV-beta-sorted portfolios documented in Section 3.2.2 are quantitatively consistent with the extent of idiosyncratic volatility co-movement documented in Section 3.1.

4.3.1 Calibration

The model has eight global parameters: the mean, μ_c, and the exposure to time-varying volatility shocks, φ_c, of global consumption growth; three parameters governing the dynamics of the global state variable, σ_w^2, ρ_w, and τ_w; and the three preference parameters, namely, the subjective discount factor, β, the risk-aversion coefficient, γ, and the elasticity of intertemporal substitution (IES), ψ. I assume that the parameter governing the volatility of pricing factors is the same across countries, that is, $\tau \equiv \tau_w = \tau_j$. The model also has two country-specific parameters governing the dynamics of the country-specific state variable: σ_j^2 and ρ_j. Table 8 shows my benchmark parameter values; the model is calibrated to data for the 1983-2020 period at an annual frequency.

The risk aversion γ is set to 6.5. This is a common value in the literature and also the value chosen by Colacito et al. (2018) in their simulations. The IES parameter, which governs consumers' willingness to trade off consumption over time, plays an important role in determining the asset pricing implications of my framework. There is considerable debate in the macroeconomics and finance literature about the value of the IES. Hall (1988) estimates the IES to be close to zero. This estimate is obtained by analyzing the response of aggregate consumption growth to movements in the real interest rate over time. However, as noted by Bansal and Yaron (2004) and Gruber (2013), it is difficult to estimate the causal effect of interest rates on consumption growth without strong structural assumptions because interest rates and consumption/saving decisions are the joint equilibrium outcome of the capital market. These concerns are sometimes addressed by using instruments for movements in the current interest rate. As a consequence, a wide variety of parameter values are used in the literature. On the one hand, Campbell and Mankiw (1989), Campbell (2003), and Guvenen (2009) advocate values for the IES well

Table 8: Benchmark calibration

This table lists the parameters of the benchmark calibration. The preference parameters are intertemporal discount (β), risk-aversion (γ), and the intertemporal elasticity of substitution (ψ). The global consumption growth and the consumption share processes are described by equations (12)-(16). Panel A reports parameters calibrated against observational data and values commonly used in the literature. Panel B discusses the calibration of the five currency portfolios sorted from lowest volatility (P1) to highest volatility (P5) based on the generalized method of moments (GMM) estimates. US-specific parameters are reported in the last column. Finally, Panel C presents implied country-specific parameters.

Panel A: Preferences, global consumption growth, and consumption share parameters

Parameter	Calibration	Parameter	Calibration	Parameter	Calibration
γ	7	φ_c	-2.00	ρ_w	0.65
τ	0.02	β	0.98	σ_w^2	0.009
ψ	1.5	μ_c	0.02		

Panel B: Portfolio-specific parameters

Parameter	P1	P2	P3	P4	P5	US
ρ_j	0.80	0.76	0.72	0.66	0.62	0.60
σ_j^2	0.22	0.37	0.86	0.90	0.40	0.07

Panel C: Implied country-specific parameters

Parameter	P1	P2	P3	P4	P5	US
α_j	24.66	26.09	24.77	16.41	18.74	23.47
ω_j	-7.72	-14.60	-18.57	-19.78	-26.42	-18.59
φ_j	6.19	4.40	3.56	2.15	18.85	44.46
χ_j	0.35	-0.09	-0.29	-0.47	-0.29	-1.61
ξ_j	-13.54	-10.92	-7.14	-2.97	-10.25	-24.60
κ_{0j}	0.66	0.39	0.38	0.38	0.44	0.38
κ_{1j}	1.27	1.12	1.12	1.12	1.15	1.12
A_{0j}	1.55	2.22	2.23	2.23	2.05	2.23
A_{1j}	-13.37	-33.69	-37.81	37.92	-23.79	-36.87
A_{2j}	-140.53	-107.61	-119.81	-137.12	-157.98	-155.62

below 1, while Bansal and Yaron (2004), Barro (2009), and Gruber (2013) argue for substantially higher values of the IES. Here, I assume that investors have a relatively high willingness to substitute consumption over time (at least during periods of unexpectedly high CIV shocks). I therefore focus on parameterizations with an IES equal to $\psi = 1.5$ as my baseline case.

The time discount factor β is set at 0.98. Differences in the discount factor β have only minimal effects on the currency risk premium in my model, affecting mostly the risk-free rates. Mean consumption growth μ_c is 2% per year. I set φ_c equal to -2 to capture the negative correlation between global consumption growth and the cross-country dispersion in consumption growth.

The persistence of the cross-sectional dispersion process, ρ_w, is set to 0.65 per year, a value equal to the persistence of the CIV factor in the data. This choice implies that my main state variable moves at business cycle frequencies instead of much lower frequencies. I set the mean of the cross-sectional dispersion in consumption growth, σ_w^2, to 0.009, and the parameter that governs the conditional variance of the dispersion process, τ, is set to 0.02 to ensure that $\sigma_{w,t}^2$ remains positive. The model results in a market price of CIV risk, λ_{CIV}, of -0.014.

As shown in Section 3.2.2, currencies whose returns have a more negative exposure to CIV shocks earn higher average returns. To represent the typical country in each of the CIV-beta-sorted quintile portfolios, I solve my model for five currencies that differ in terms of their country-specific parameters $\Theta_j \equiv (\rho_j, \sigma_j^2)$. That is, I choose five sets of country-specific parameters Θ_j and price the resulting returns inside the model. Since I am considering currency returns from the perspective of a US investor, I also need to estimate US-specific parameters, $\Theta \equiv (\rho, \sigma^2)$.

I estimate these two country-specific parameters for each quintile portfolio, in addition to the two US-specific parameters, using GMM to simultaneously match three moments in the data. The first is the CIV beta, $\beta_{CIV,j}$, in equation (54). The second is the volatility of interest rates. The third moment is the slope from a regression of idiosyncratic currency return variance on the CIV factor:

$$\mathrm{Var}_t\left(rx_{j,t+1}^{idio}\right) = a_j + b_j CIV_t + \eta_{j,t}. \tag{34}$$

I therefore have 15 moment restrictions in 12 parameters, providing overidentifying restrictions to test the model specification. I use a diagonal weighting matrix with a weight of one on all the moments.

Interestingly, the GMM estimates reported in Panel B of Table 8 show that there are substantial cross-sectional differences in the persistency of country-specific pricing factors, $\sigma_{j,t}^2$. The GMM estimates for ρ_j demonstrate that the shock that drives income dispersion within high-interest rate countries (P1) is much more persistent than the shock that drives income dispersion within low-interest rate countries (P5). Thus, as in Ready et al. (2017), persistent interest rate differentials account for much of the currency carry trade profitability.

Simulation results. Table 9 summarizes the quantitative results. The main result of the calibration exercise is that the model is able to match the monotonic pattern in excess returns on the CIV-beta-sorted portfolios. Panel A shows that the model generates a monotonically declining pattern in equally weighted currency excess returns from P1 to P5. The return spread between portfolio 1 and portfolio 5 of 3.79% per year in the model is also very similar to its counterpart in the data, 4.18%. The currencies in portfolio P1 (P5) have negative (positive) exposure to the CIV factor. Their returns fall (increase) when the cross-sectional volatility increases, making them risky (a hedge). As a result, they carry the highest (lowest) risk premia.

The simulated volatilities of excess returns also resemble total return volatilities of the CIV-beta-sorted portfolios, shown in Panel B. Annual return volatilities (standard deviations) for the typical currency in each of the quintile portfolios range from 9.54% to 9.61%. Furthermore, the model is able to match the decreasing pattern in Sharpe ratios and interest rates.

Panel D of Table 9 shows average interest-rate differentials for the five portfolios. The results suggest that CIV-beta portfolios are similar to the carry trade portfolios in that forward discounts monotonically decline when moving from portfolio 1 to portfolio 5. Thus, sorting on the exposure to CIV risk is similar to sorting on interest rate differentials and hence the carry trade portfolios themselves.

5 Concluding Remarks

I have provided new evidence that incomplete consumption risk sharing across countries is an important determinant of the cross-section of expected currency returns. My results suggest a link between the cross-sectional average of idiosyncratic volatilities of currency excess returns and the cross-country volatility of consumption growth. The economic intuition is simple: If cross-country differences in currency risk premiums are driven by local, country-specific factors that are not diversifiable at the global level, then there must be a relation between currency returns and the idiosyncratic consumption risk. Using the CIV factor as a proxy for such consumption risk, I find a strong relationship between CIV shocks and currency excess returns in the data.

My results support the prediction that cross-country variability in consumption growth

Table 9: Simulated moments

This table reports moments from the model and compares them with the data. Panels A and B report the mean and standard deviation of currency excess returns, respectively. Panel C reports annualized Sharpe ratios. Panel D reports the average interest rate differential and Panel E presents the average interest rate. Finally, Panel E reports CIV-betas. The model is simulated at annual frequency for 10,000 periods. All moments in the data are expressed as annual quantities and computed from the 1983-2020 sample.

Moments	P1	P2	P3	P4	P5	US
Panel A: Average Excess Returns (%)						
Data	3.11	0.34	-0.43	-0.39	-1.07	
Model	2.94	0.15	-0.42	-0.33	-0.85	
Panel B: Volatility of Excess Returns (%)						
Data	9.82	9.46	10.52	9.27	8.73	
Model	9.54	9.66	9.61	9.56	9.54	
Panel C: Sharpe Ratio						
Data	0.30	0.03	-0.06	-0.06	-0.13	
Model	0.31	0.02	-0.04	-0.03	-0.09	
Panel D: Average Interest Rate Differential (%)						
Data	2.36	1.18	1.10	-0.08	-1.24	
Model	2.33	1.25	0.80	-0.01	-1.27	
Panel E: Average Interest Rate (%)						
Data	5.46	4.43	3.98	3.39	2.22	3.48
Model	5.64	4.56	4.11	3.29	2.04	3.31
Panel F: CIV-beta						
Data	-12.48	-4.26	0.81	2.30	10.32	
Model	-12.48	-4.26	0.81	2.30	10.32	

is a key driver of risk premia in FX markets. First, I show an extraordinary degree of co-movement among the idiosyncratic volatilities of currency excess returns. The common component – the CIV factor – is defined as the cross-sectional average of idiosyncratic volatilities of different currencies at a point in time. Shocks to this common component are priced. High-interest rate currencies are negatively related to CIV shocks, and thus deliver low returns during bad times for investors, when low-interest rate currencies provide a hedge by yielding positive returns. Next, I provide evidence linking the CIV factor to cross-country income risk faced by investors. My findings are consistent with an incomplete markets heterogeneous-agent model with persistent, uninsurable shocks in consumption growth. Consistent with my empirical results, CIV risk is also priced in the model: Increases in CIV lead to an increase in the dispersion of consumption growth across countries and are associated with high marginal utility for the average investor. Currencies that have higher exposure to CIV shocks (positive CIV-beta) hedge against deterioration in future investment opportunities. When sorting currencies into portfolios based on CIV-betas, I find that currencies with more negative betas carry higher average returns. Finally, the calibrated model quantitatively matches the observed return spread and volatility facts for plausible parameter values.

Extending this framework to a fully fledged general equilibrium analysis represents an important direction for future research. In future work, researchers could explore alternative theoretical mechanisms that can explain the link between cross-country dispersion in consumption growth and the cross-section of currency excess returns reported in the paper. The model presented here is only one of potentially several frameworks that can predict these facts. The next step would be to allow for trade. Empirical researchers may also wish to explore alternative proxies for cross-country income risk faced by households, using richer financial and economic data sets.

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A Proof that the identity $I_{j,t} = C_{j,t} - D_{j,t}$ is respected

Given that, within every country, the agents are symmetric and their number is normalized to one, I apply the law or large numbers as in Constantinides and Ghosh (2017) and claim that $I_{j,t} = \mathrm{E}[I_{i,j,t} \mid C_{j,t}, D_{j,t}]$. Furthermore, since household shocks are assumed to be normally distributed and independent of anything else in the economy, we obtain:

$$\begin{aligned}
I_{j,t} &= \mathrm{E}[I_{i,j,t} \mid C_{j,t}, D_{j,t}] \\
&\overset{(11)}{=} \mathrm{E}\left[\exp\left(\sum_{s=1}^{t}\left[\sigma_{j,s-1}\epsilon_{i,j,s} - \frac{1}{2}\sigma_{j,s-1}^2\right]\right)\right] C_{j,t} - D_{j,t} \\
&= \mathrm{E}\left[\mathrm{E}\left[\exp\left(\sum_{s=1}^{t}\left[\sigma_{j,s-1}\epsilon_{i,j,s} - \frac{1}{2}\sigma_{j,s-1}^2\right]\right)\middle| \{\sigma_{j,\tau}\}_{\tau=0}^{t-1}\right]\right] C_{j,t} - D_{j,t} \\
&= C_{j,t} - D_{j,t}.
\end{aligned}$$

□

B Proof that autarchy is an equilibrium

I conjecture and verify that autarchy is an equilibrium. The proof follows several steps. First, I calculate agent i's private valuation of its wealth portfolio $r_{i,j,t+1}$. To do that, I conjecture that the log wealth-consumption ratio of agent i in country j is linear in the state variables $\sigma_{j,t}^2$ and $\sigma_{w,t}^2$, and does not depend on any agent-specific characteristics, that is,

$$wc_{i,j,t} = A_{0j} + A_{1j}(\sigma_{w,t}^2 - \sigma_w^2) + A_{2j}(\sigma_{j,t}^2 - \sigma_j^2). \tag{35}$$

Next, to verify this conjecture, I substitute this return in the agent i's SDF, as stated in equation (9), and evaluate the Euler equation for the consumption claim of agent i in country j: $\mathrm{E}_t[M_{i,j,t+1}R_{i,j,t+1}] = 1$.

Finally, I integrate out of this SDF the agent's idiosyncratic income shocks and show that agents have a common SDF. This implies that the individual wealth-consumption ratio does not depend on agent-specific attributes, only on aggregate objects, verifying the conjecture that autarchy is an equilibrium.

B.0.1 Return on agent i's wealth portfolio

The beginning-of-period (or cum-dividend) total wealth $W_{i,j,t}$ that is not spent on consumption $C_{i,j,t}$ earns a gross return $R_{i,j,t+1}$ and leads to beginning-of-next period total wealth $W_{i,j,t+1}$. The return on a claim to consumption, the total wealth return, can be written as

$$R_{i,j,t+1} = \frac{W_{i,j,t+1}}{(W_{i,j,t} - C_{i,j,t})} = \frac{C_{i,j,t+1}}{C_{i,j,t}} \frac{WC_{i,j,t+1}}{(WC_{i,j,t} - 1)},$$

where $WC_{i,j,t+1} \equiv \frac{W_{i,j,t+1}}{C_{i,j,t+1}}$ is defined as the wealth-consumption ratio. Taking logs, we get

$$r_{i,j,t+1} = wc_{i,j,t+1} + \Delta c_{i,j,t+1} - \log\left(e^{wc_{i,j,t}} - 1\right). \tag{36}$$

Following Campbell and Shiller (1988), we can approximate the last term in equation (36) by using a first-order Taylor series approximation around the long-run average log wealth-consumption ratio $\overline{wc}_j \equiv \mathrm{E}[w_{i,j,t} - c_{i,j,t}]$:

$$\log\left(e^{wc_{i,j,t}} - 1\right) \approx -\kappa_{0j} + \kappa_{1j} wc_{i,j,t}, \tag{37}$$

where the linearization constants are given by

$$\kappa_{0j} = -\log\left(e^{\overline{wc}_j} - 1\right) + \kappa_{1j}\overline{wc}_j, \tag{38}$$

$$\kappa_{1j} = \frac{e^{\overline{wc}_j}}{e^{\overline{wc}_j} - 1} > 1. \tag{39}$$

By substituting equation (37) into equation (36), we can rewrite the total wealth return as

$$r_{i,j,t+1} = \kappa_{0j} + \Delta c_{i,j,t+1} + wc_{i,j,t+1} - \kappa_{1j} wc_{i,j,t}. \tag{40}$$

Finally, we can substitute the conjecture for the log wealth-consumption ratio given by equation (35) into equation (40) and solve for the return on agent i's private wealth portfolio in terms of shocks and state variables as follows:

$$\begin{aligned} r_{i,j,t+1} = & \left[\mu_c + \kappa_{0j} + A_{0j}(1-\kappa_{1j}) - \frac{1}{2}\left(\sigma_w^2 + \sigma_j^2\right)\right] \\ & + \left[A_{1j}(\rho_w - \kappa_{1j}) - \frac{1}{2}\right]\left(\sigma_{w,t}^2 - \sigma_w^2\right) \\ & + \left[A_{2j}(\rho_j - \kappa_{1j}) - \frac{1}{2}\right]\left(\sigma_{j,t}^2 - \sigma_j^2\right) \\ & + \left(\varphi_c + A_{1j}\tau_w\right)\sigma_{w,t}u_{w,t+1} + \left(A_{2j}\tau_j\right)\sigma_{j,t}u_{j,t+1} + \sigma_{w,t}\epsilon_{j,t+1} + \sigma_{j,t+1}\epsilon_{i,j,t+1}. \end{aligned} \tag{41}$$

B.0.2 Individual stochastic discount factor

We can solve for the individual stochastic discount factor by substituting the dynamics given in Section 4.1, the return on agent i's private wealth portfolio given by equation (41), and the conjecture given by (35) into equation (9):

$$\begin{aligned} m_{i,j,t+1} = & \left[\theta \log\beta - \gamma\mu_c + (\theta-1)\left[\kappa_{0j} + A_{0j}(1-\kappa_{1j})\right] + \frac{\gamma}{2}(\sigma_w^2 + \sigma_j^2)\right] \\ & + \left[(\theta-1)A_{1j}(\rho_w - \kappa_{1j}) + \frac{\gamma}{2}\right]\left(\sigma_{w,t}^2 - \sigma_w^2\right) \\ & + \left[(\theta-1)A_{2j}(\rho_j - \kappa_{1j}) + \frac{\gamma}{2}\right]\left(\sigma_{j,t}^2 - \sigma_j^2\right) \\ & + \left[(\theta-1)\tau_w A_{1j} - \gamma\varphi_c\right]\sigma_{w,t}u_{w,t+1} + \left[(\theta-1)\tau_j A_{2j}\right]\sigma_{j,t}u_{j,t+1} \\ & - \gamma\sigma_{w,t}\epsilon_{j,t+1} - \gamma\sigma_{j,t}\epsilon_{i,j,t+1}. \end{aligned} \tag{42}$$

The return on a claim to the consumption stream of agent i in country j, $R_{i,j}$, satisfies the Euler equation under her stochastic discount factor:

$$\begin{aligned} 1 &= \mathrm{E}_t\left[M_{i,j,t+1}R_{i,j,t+1}\right] \\ &= \mathrm{E}_t\left[\mathrm{E}_i[M_{i,j,t+1}R_{i,j,t+1}]\right] \\ &= \mathrm{E}_t\left[\mathrm{E}_i\left[\exp\left(m_{i,j,t+1} + r_{i,j,t+1}\right)\right]\right] \\ &= \mathrm{E}_t\left[\exp\left(\mathrm{E}_i[m_{i,j,t+1} + r_{i,j,t+1}] + \frac{1}{2}\mathrm{Var}_i\left(m_{i,j,t+1} + r_{i,j,t+1}\right)\right)\right], \end{aligned} \tag{43}$$

where the second equality uses the fact that if any individual IMRS is a valid discount factor, the average IMRS across agents is also a valid discount factor, and the last equality applies the cross-sectional normality of consumption share growth. By using the properties of the log-normal distribution, we can show that the Euler equation is satisfied only if

$$0 = \mathrm{E}_t \left[\mathrm{E}_i[m_{i,j,t+1} + r_{i,j,t+1}] + \frac{1}{2} \mathrm{Var}_i \left(m_{i,j,t+1} + r_{i,j,t+1} \right) \right] + \frac{1}{2} \mathrm{Var}_t \left[\mathrm{E}_i[m_{i,j,t+1} + r_{i,j,t+1}] + \frac{1}{2} \mathrm{Var}_i \left(m_{i,j,t+1} + r_{i,j,t+1} \right) \right]. \quad (44)$$

Given the dynamics for the return on agent i's wealth portfolio in equation (41) and the dynamics for individual SDF in equation (43), we can now solve for the coefficients A_{0j}, A_{1j}, and A_{2j} in equation (35) by imposing the Euler equation for consumption claim (43).

First, using equations (41) and (43), we can express the sum of $m_{i,j,t+1}$ and $r_{i,j,t+1}$ in terms of shocks and state variables as follows:

$$\begin{aligned} m_{i,j,t+1} + r_{i,j,t+1} = & \left[\theta \log \beta + (1-\gamma)\mu_c + \theta \kappa_{0j} + \theta A_{0j}(1-\kappa_{1j}) - \frac{1}{2}(1-\gamma)(\sigma_w^2 + \sigma_j^2) \right] \\ & + \left[\theta A_{1j}(\rho_w - \kappa_{1j}) - \frac{1}{2}(1-\gamma) \right] \left(\sigma_{w,t}^2 - \sigma_w^2 \right) \\ & + \left[\theta A_{2j}(\rho_j - \kappa_{1j}) - \frac{1}{2}(1-\gamma) \right] \left(\sigma_{j,t}^2 - \sigma_j^2 \right) \\ & + \left[\theta \tau_w A_{1j} + \varphi_c(1-\gamma) \right] \sigma_{w,t} u_{w,t+1} + \left(\theta \tau_j A_{2j} \right) \sigma_{j,t} u_{j,t+1} \\ & + (1-\gamma)\sigma_{w,t}\epsilon_{j,t+1} + (1-\gamma)\sigma_{j,t}\epsilon_{i,j,t+1}. \end{aligned}$$

Then, by using the cross-sectional normality of the consumption share growth, we get:

$$\begin{aligned} \mathrm{E}_i \left[m_{i,j,t+1} + r_{i,j,t+1} \right] = & \left[\theta \log \beta + (1-\gamma)\mu_c + \theta \kappa_{0j} + \theta A_{0j}(1-\kappa_{1j}) - \frac{1}{2}(1-\gamma)(\sigma_w^2 + \sigma_j^2) \right] \\ & + \left[\theta A_{1j}(\rho_w - \kappa_{1j}) - \frac{1}{2}(1-\gamma) \right] \left(\sigma_{w,t}^2 - \sigma_w^2 \right) \\ & + \left[\theta A_{2j}(\rho_j - \kappa_{1j}) - \frac{1}{2}(1-\gamma) \right] \left(\sigma_{j,t}^2 - \sigma_j^2 \right) \\ & + \left[\theta \tau_w A_{1j} + \varphi_c(1-\gamma) \right] \sigma_{w,t} u_{w,t+1} + \left(\theta \tau_j A_{2j} \right) \sigma_{j,t} u_{j,t+1} \\ & + (1-\gamma)\sigma_{w,t}\epsilon_{j,t+1}, \quad (45) \\ \mathrm{Var}_i \left[m_{i,j,t+1} + r_{i,j,t+1} \right] = & (1-\gamma)^2 \sigma_{j,t}^2. \quad (46) \end{aligned}$$

Finally, we can use equations (45)-(46) to solve the Euler equation (44). Using log-normal properties, we can take the expected value conditional on time t information and compute the Euler equation:

$$
\begin{aligned}
0 = &\left[\theta \log \beta + (1-\gamma)\mu_c + \theta\kappa_{0j} + \theta A_{0j}(1-\kappa_{1j}) - \theta A_{1j}(\rho_W - \kappa_{1j})\sigma_w^2 - \theta A_{2j}(\rho_j - \kappa_{1j})\sigma_j^2\right] \\
&+ \left[\frac{1}{2}\theta^2\tau_w^2 A_{1j}^2 + \theta\left[(\rho_w - \kappa_{1j}) + \varphi_c\tau_w(1-\gamma)\right] A_{1j} - \frac{1}{2}(1-\gamma)\left[\gamma - (1-\gamma)\varphi_c^2\right]\right]\left(\sigma_{w,t}^2 - \sigma_w^2\right) \\
&+ \left[\frac{1}{2}\theta^2\tau_j^2 A_{2j}^2 + \theta(\rho_j - \kappa_{1j})A_{2j} - \frac{\gamma}{2}(1-\gamma)\right]\left(\sigma_{j,t}^2 - \sigma_j^2\right).
\end{aligned}
$$

The previous equation holds for all realizations of the state variables only if the constant term and the terms that multiply $\sigma_{w,t}^2$ and $\sigma_{j,t}^2$ are equal to zero. Applying the method of undetermined coefficients, we get

$$
A_{0,j} = \frac{\theta \log \beta + \theta\kappa_{0,j} + (1-\gamma)\mu_c - \theta A_{1j}(\rho_w - \kappa_{1j})\sigma_w^2 - \theta A_{2j}(\rho_j - \kappa_{1j})\sigma_j^2}{\theta(\kappa_{1j} - 1)}, \tag{47}
$$

$$
A_{1,j}^{+,-} = \frac{-\left[\rho_w - \kappa_{1j} + (1-\gamma)\varphi_c\tau_w\right]\theta \pm \sqrt{\theta^2\left[\rho_w - \kappa_{1j} + (1-\gamma)\varphi_c\tau_w\right]^2 + (1-\gamma)\left[\gamma - (1-\gamma)\varphi_c^2\right]\tau_w^2\theta^2}}{\theta^2\tau_w^2}, \tag{48}
$$

$$
A_{2,j}^{+,-} = \frac{-(\rho_j - \kappa_{1j})\theta \pm \sqrt{\theta^2(\rho_j - \kappa_{1j})^2\theta^2 + \gamma(1-\gamma)\tau_j^2\theta^2}}{\theta^2\tau_j^2}. \tag{49}
$$

There are two roots, $A_{1j}^{+,-}$ and $A_{2j}^{+,-}$, which are real as long as ϕ^2 is sufficiently small. Similarly to the stochastic volatility model presented in Tauchen (2011), the roots A_{1j}^{-} and A_{2j}^{-} have the unappealing property that

$$
\lim_{\tau_w^2 \to 0} A_{1j}^{-}\tau_w^2 \neq 0,
$$

$$
\lim_{\tau_j^2 \to 0} A_{2j}^{-}\tau_j^2 \neq 0,
$$

which would mean the impact of $\sigma_{w,t}$ and $\sigma_{j,t}$ would grow without bound as stochastic volatility becomes unimportant. Thus, I take A_{1j}^{+} and A_{2j}^{+} as the economically meaningful roots and set $A_{1j} = A_{1j}^{+}$ and $A_{2j} = A_{2j}^{+}$.

B.0.3 Aggregate stochastic discount factor

Finally, because all agents can invest in all risky assets, the Euler equation has to be satisfied for any two agents i and i' in country j, and for every asset k:

$$1 = \mathrm{E}_t \left[M_{i,j,t+1} R_{k,j,t+1} \right], \ \ \forall \, k.$$

This implies that the average SDF must also price every financial asset k:

$$\begin{aligned} 1 &= \mathrm{E}_t \left[\mathrm{E}_i \left[M_{i,j,t+1} R_{k,j,t+1} \right] \right] \\ &= \mathrm{E}_t \left[\mathrm{E}_i \left[M_{i,j,t+1} \right] R_{k,j,t+1} \right] \\ &= \mathrm{E}_t \left[M_{j,t+1} R_{k,j,t+1} \right], \ \ \forall \, k. \end{aligned}$$

where

$$M_{j,t+1} \equiv \mathrm{E}_i \left[M_{i,j,t+1} \right].$$

Taking logs and using the properties of the log-normal distribution,

$$m_{j,t+1} = \mathrm{E}_i \left[m_{i,j,t+1} \right] + \frac{1}{2} \mathrm{Var}_i \left[m_{i,j,t+1} \right]. \tag{50}$$

Finally, by substituting equation (42) into equation (50), the common log stochastic discount factor for country j is given by:

$$-m_{j,t+1} = \alpha_j + \chi_j \sigma_{w,t}^2 + \varphi_j \sigma_{j,t}^2 + \omega_j \sigma_{w,t} u_{w,t+1} + \xi_j \sigma_{j,t} u_{j,t+1} + \gamma \sigma_{w,t} \epsilon_{j,t+1}, \tag{51}$$

where

$$\begin{aligned} \alpha_j &= -\theta \log \beta + \gamma \mu_c - (\theta - 1) \left[\kappa_{0j} + A_{0j}(1 - \kappa_{1j}) - A_{1j}(\rho_w - \kappa_{1j}) \sigma_w^2 - A_{2j}(\rho_j - \kappa_{1j}) \sigma_j^2 \right], \\ \chi_j &= -(\theta - 1) A_{1j}(\rho_w - \kappa_{1j}) - \frac{\gamma}{2}, \\ \varphi_j &= -(\theta - 1) A_{2j}(\rho_j - \kappa_{1j}) - \frac{\gamma}{2}(1 + \gamma), \\ \omega_j &= -(\theta - 1) \tau_w A_{1j} + \gamma \varphi_c, \\ \xi_j &= -(\theta - 1) \tau_j A_{2j}. \end{aligned}$$

Therefore, each agent's private valuation of any security, other than agent's wealth portfolios, is common, implying that no-trade is an equilibrium. □

C Proofs in Section 4

C.1 Proof of Proposition 2

Let $rx^{idio}_{j,t+1}$ denote the residual or country j-specific part of currency excess returns in equation (30), that is,

$$rx^{idio}_{j,t+1} \equiv -\xi_j \sigma_{j,t} u_{j,t+1} - \gamma \sigma_{w,t} \epsilon_{j,t+1}. \tag{52}$$

The idiosyncratic variance is then given by

$$\mathrm{Var}_t(rx^{idio}_{j,t+1}) = \xi_j^2 \sigma_{j,t}^2 + \gamma^2 \sigma_{w,t}^2.$$

Thus, there is a common factor $\sigma^2_{w,t}$ in idiosyncratic variances of currency excess returns. □

C.2 Proof of Proposition 3

Given the expression for the CIV factor in equation (32), we can express CIV shocks as follows:

$$\begin{aligned} CIVshock_{t+1} &\equiv CIV_{t+1} - \mathrm{E}_t\left[CIV_{t+1}\right] \\ &= \overline{\tau_j \xi_j^2 \sigma_{j,t} u_{j,t+1}} + \tau_w \gamma^2 \sigma_{w,t} u_{w,t+1}. \end{aligned}$$

By taking the limit when $N \to \infty$ and applying the law of large numbers, we get

$$CIVshock_{t+1} = \tau_w \gamma^2 \sigma_{w,t} u_{w,t+1}. \tag{53}$$

Then, using the expression for currency excess returns from equation (30), we can compute the CIV-beta as follows:

$$\beta_{CIV,j} \equiv \frac{\mathrm{Cov}_t\left(rx_{j,t+1}, CIVshock_{t+1}\right)}{\mathrm{Var}_t\left(CIVshock_{t+1}\right)} = \frac{\omega - \omega_j}{\tau_w \gamma^2}. \tag{54}$$

Therefore, if $\omega_j > \omega$, we have $\beta_{CIV,j} < 0$. $\square$

C.3 Proof of Proposition 4

The expected return on currency j (including the Jensen term) is given by

$$\begin{aligned}
\mathrm{E}_t\left[rx_{j,t+1}\right] + \frac{1}{2}\mathrm{Var}_t\left(rx_{j,t+1}\right) &= -\mathrm{Cov}_t\left(m_{t+1}, rx_{j,t+1}\right) \\
&\overset{(1)}{=} -\mathrm{Cov}_t\left(m_{t+1}, -\Delta s_{j,t+1}\right) \\
&\overset{(27)+(20)}{=} \left[\omega(\omega_j - \omega) + \gamma^2\right]\sigma_{w,t}^2 + \xi^2\sigma_t^2 \\
&= \beta_j \lambda_t,
\end{aligned}$$

where $\beta_j = \left[\beta_{CIV,j}, \xi, \gamma\right]$ is the vector of risk exposures, $\lambda_t = \left[\lambda_{CIV,t}, \xi\sigma_t^2, \gamma\sigma_{w,t}^2\right]'$ is the vector of market prices of risk, and $\lambda_{CIV,t} \equiv \omega\tau_w\gamma^2\sigma_{w,t}^2$ is the price of CIV risk.

Finally, since $\tau_w > 0$ and $\omega < 0$, the market price of CIV risk is negative, that is, $\lambda_{CIV,t} < 0$. $\square$

C.4 Proof of Proposition 5

The carry risk factor is, by definition, the average exchange rate of high-versus-low interest rate currencies,

$$Carry_{t+1} \equiv \frac{1}{N_H}\sum_{j\in H} rx_{j,t+1} - \frac{1}{N_L}\sum_{j\in L} rx_{j,t+1} \tag{55}$$

where N_H (N_L) denotes the number of high (low) interest rate currencies in the sample. In the model, the carry factor is equal to

$$\begin{aligned}
Carry_{t+1} \overset{(30)+(55)}{=} & -\frac{1}{2}\left(\overline{\omega_j}^H - \overline{\omega_j}^L\right)\sigma_{w,t}^2 - \frac{1}{2}\left(\overline{\xi_j^2\sigma_{j,t}^2}^H - \overline{\xi_j^2\sigma_{j,t}^2}^L\right) \\
& - \left(\overline{\omega_j}^H - \overline{\omega_j}^L\right)\sigma_{w,t}u_{w,t+1} - \left(\overline{\xi_j\sigma_{j,t}u_{j,t+1}}^H - \overline{\xi_j\sigma_{j,t}u_{j,t+1}}^L\right) \\
& - \gamma\sigma_{w,t}\left(\overline{\epsilon_{j,t+1}}^H - \overline{\epsilon_{j,t+1}}^L\right).
\end{aligned} \tag{56}$$

By equations (56) and (53), we have

$$\mathrm{Cov}_t\left(Carry_{t+1}, CIVshock_{t+1}\right) = -\tau_w \gamma^2 \left(\overline{\omega_j}^H - \overline{\omega_j}^L\right) \sigma_{w,t}^2,$$

Thus, if $\overline{\omega_j}^H > \overline{\omega_j}^L$, the covariance between CIV shocks and carry trade returns is negative. □

C.5 Proof of Proposition 6

Using equation (28), we can express the interest rate differential as follows:

$$r_{j,t} - r_t = (\alpha_j - \alpha) + \left[(\chi_j - \chi) - \frac{1}{2}(\omega_j^2 - \omega^2)\right]\sigma_{w,t}^2 + \left(\varphi_j - \frac{1}{2}\xi_j^2\right)\sigma_{j,t}^2 - \left(\varphi - \frac{1}{2}\xi^2\right)\sigma_t^2. \tag{57}$$

If $\omega_j < 0$:

(i) Equation (54) implies that $\beta_{CIV,j}$ is decreasing in ω_j; and
(ii) Equation (57) implies that $r_{j,t} - r_t$ is increasing in ω_j.

Therefore, sorting on interest rate differentials is equivalent to sorting on CIV-betas. □